Dynamics of Social Diversity

Sidney Redner, CNLS, LANL (on leave from Boston University)
collaborator: Eli Ben-Naim, LANL

thanks to Dietrich Stauffer

Question: How do different social classes emerge in an interacting population?

Model ingredients: Competition for status

Decline in the absence of interaction

Results: *decline predominates → single destitute class*

competition predominates → multiclass society

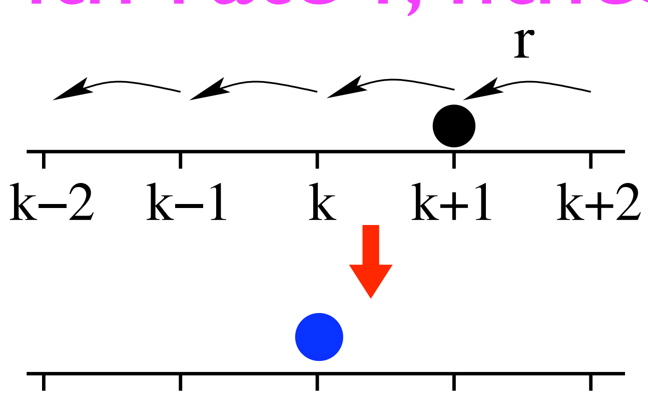
Bose condensate lower class
Fermionic middle class
boundary layer upper class

Diversity Model

related work: Bonabeau et al (1995)

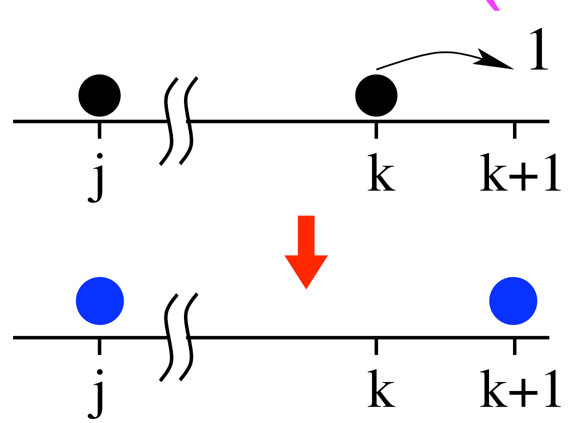
Each agent is endowed with fitness $k \geq 0$

With rate r, fitness of random agent $k \to k-1$



decay in the absence of stimulation

With rate 1, $(k,j) \to (k+1,j)$ if $k \geq j$



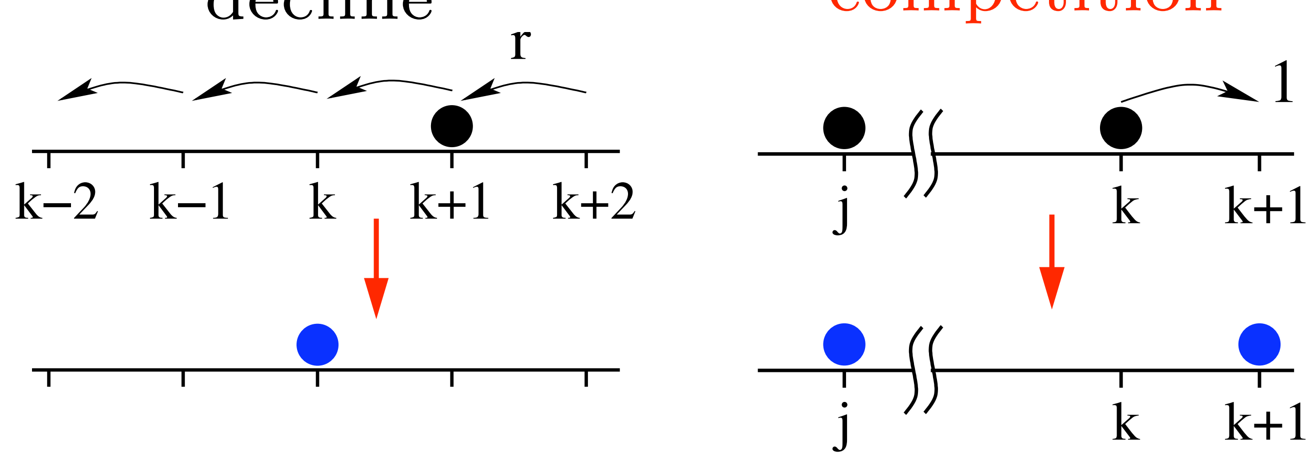
stronger agent gains by competition

What type of social structure emerges from these interactions?

Master Equation Description

$f_k \equiv$ fraction of agents with fitness k

$$\frac{df_k}{dt} = \underbrace{r(f_{k+1} - f_k)}_{\text{decline}} + \underbrace{f_{k-1}F_{k-1} - f_k F_k}_{\text{competition}} \quad \text{for } k \geq 0, \text{with } f_{-1} = 0$$



$$F_k \equiv \sum_{j=0}^{k} f_j = \text{fraction of agents with fitness} \leq \text{k}$$

$$\frac{dF_k}{dt} = r(F_{k+1} - F_k) + F_k(F_{k-1} - F_k) \quad \text{for } k \geq 0, \text{with } F_{-1} = 0$$

Dynamical Behavior

$$\frac{dF_k}{dt} = r(F_{k+1} - F_k) + F_k(F_{k-1} - F_k)$$

continuum limit: $$\frac{\partial F}{\partial t} = (r - F)\frac{\partial F}{\partial k}$$

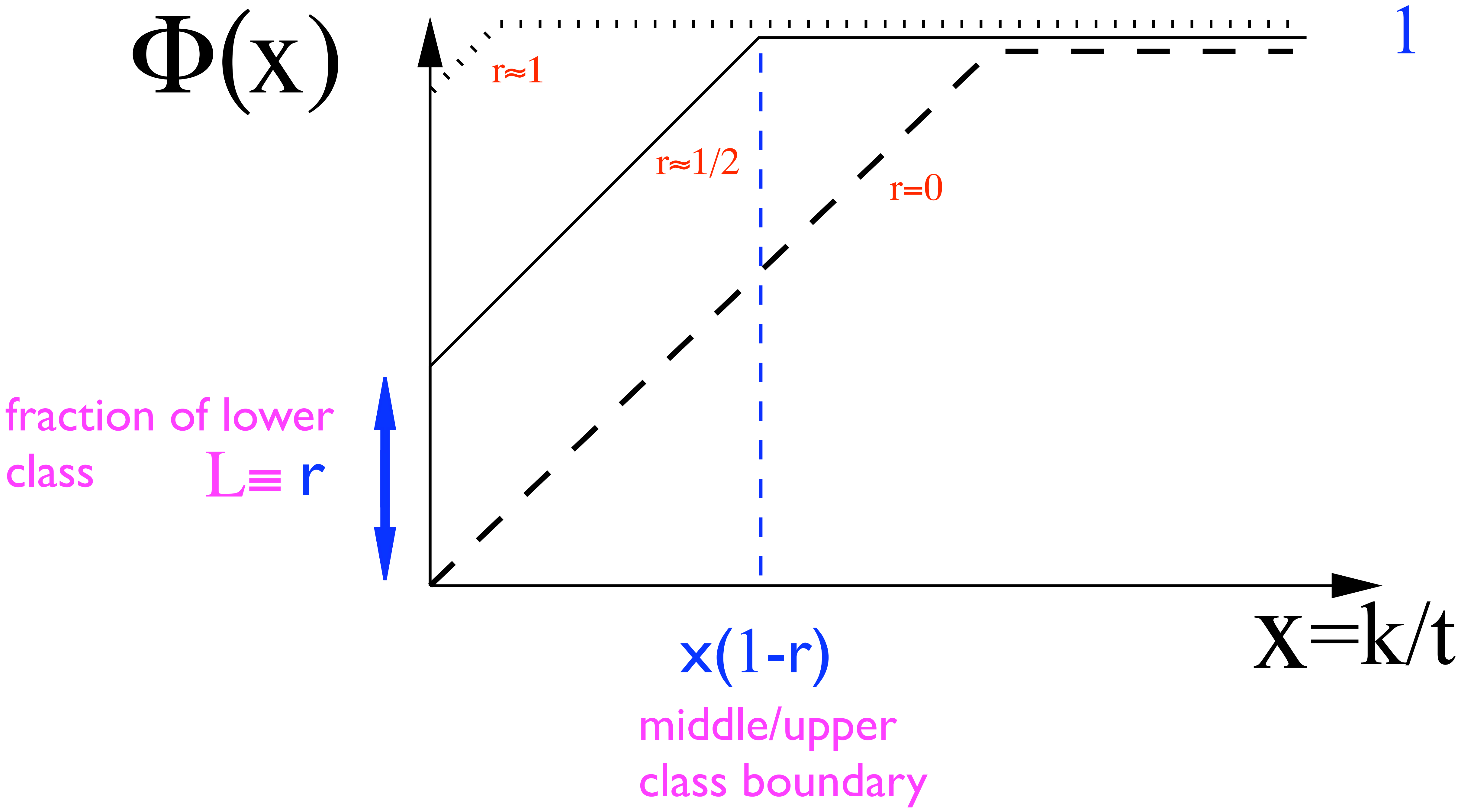
scaling ansatz: $$F(k,t) \sim \Phi(k/t)$$

leads to: $$x\Phi' = (\Phi - r)\Phi'$$

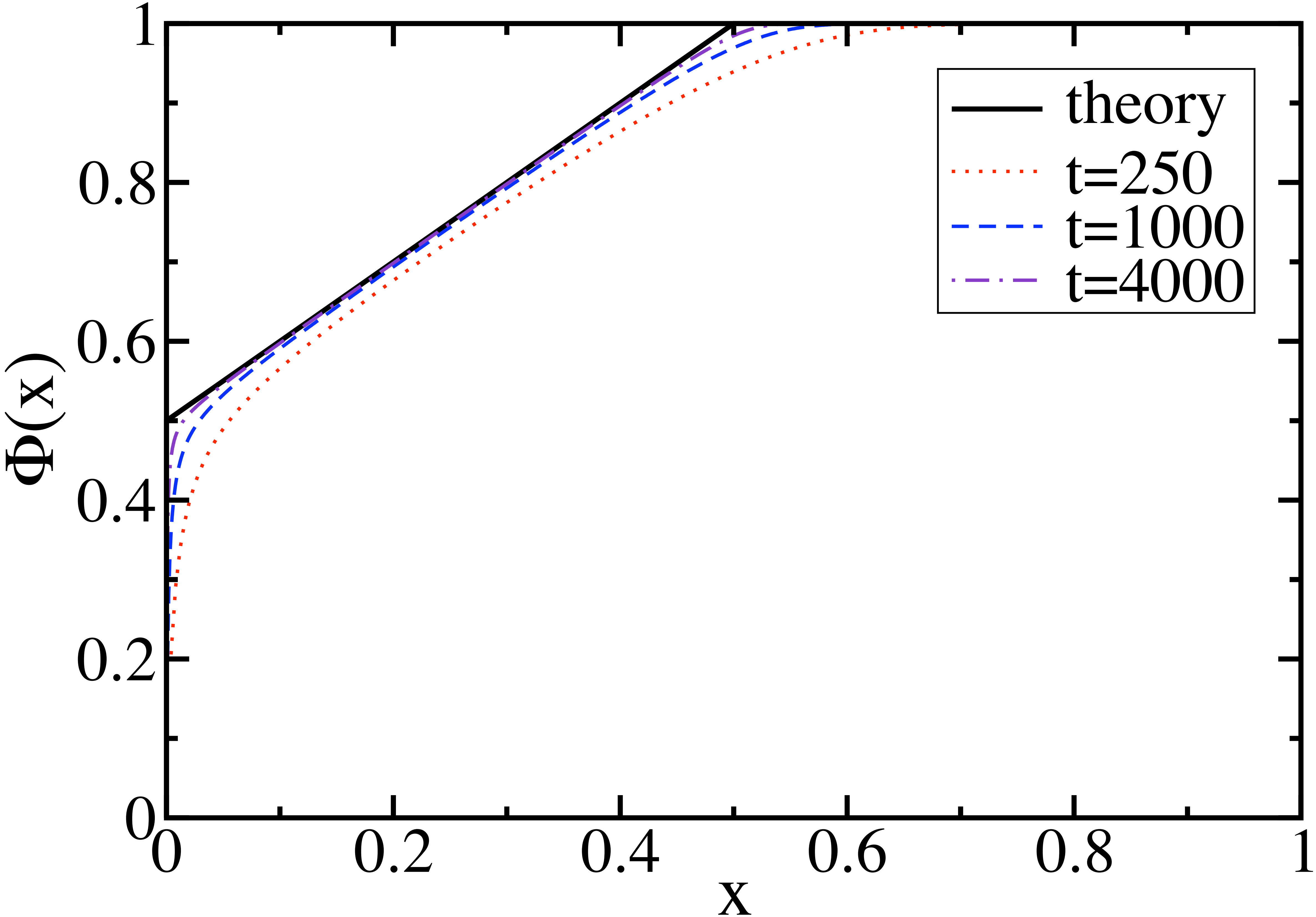
solution: $$\Phi(x) = \begin{cases} r + x & x < 1 - r; \\ 1 & x \geq 1 - r. \end{cases}$$

Schematics of the Solution

$$\Phi(x) = \begin{cases} r + x & x < 1 - r; \\ 1 & x \geq 1 - r. \end{cases}$$

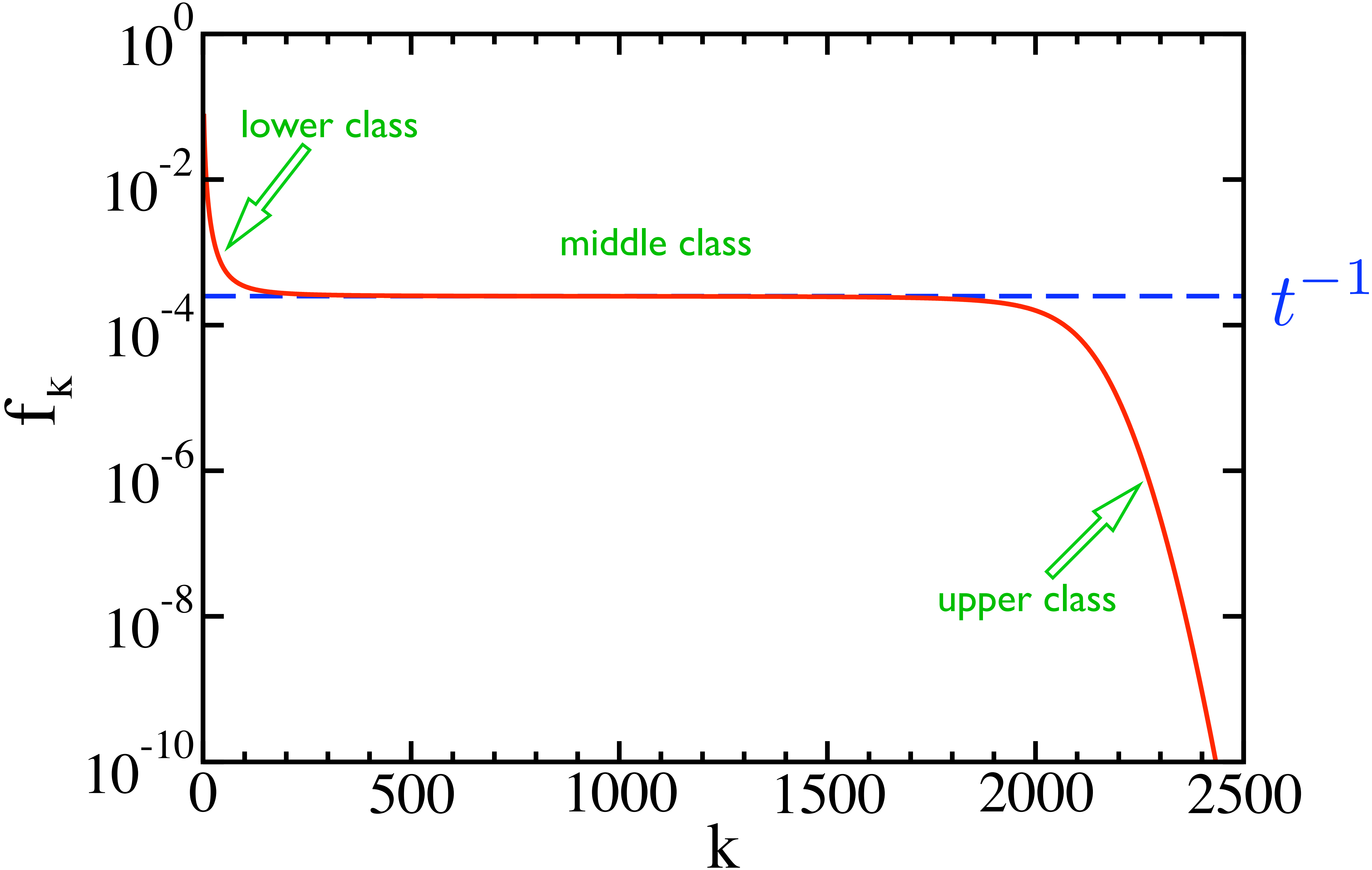


Cumulative fitness distribution for r=1/2

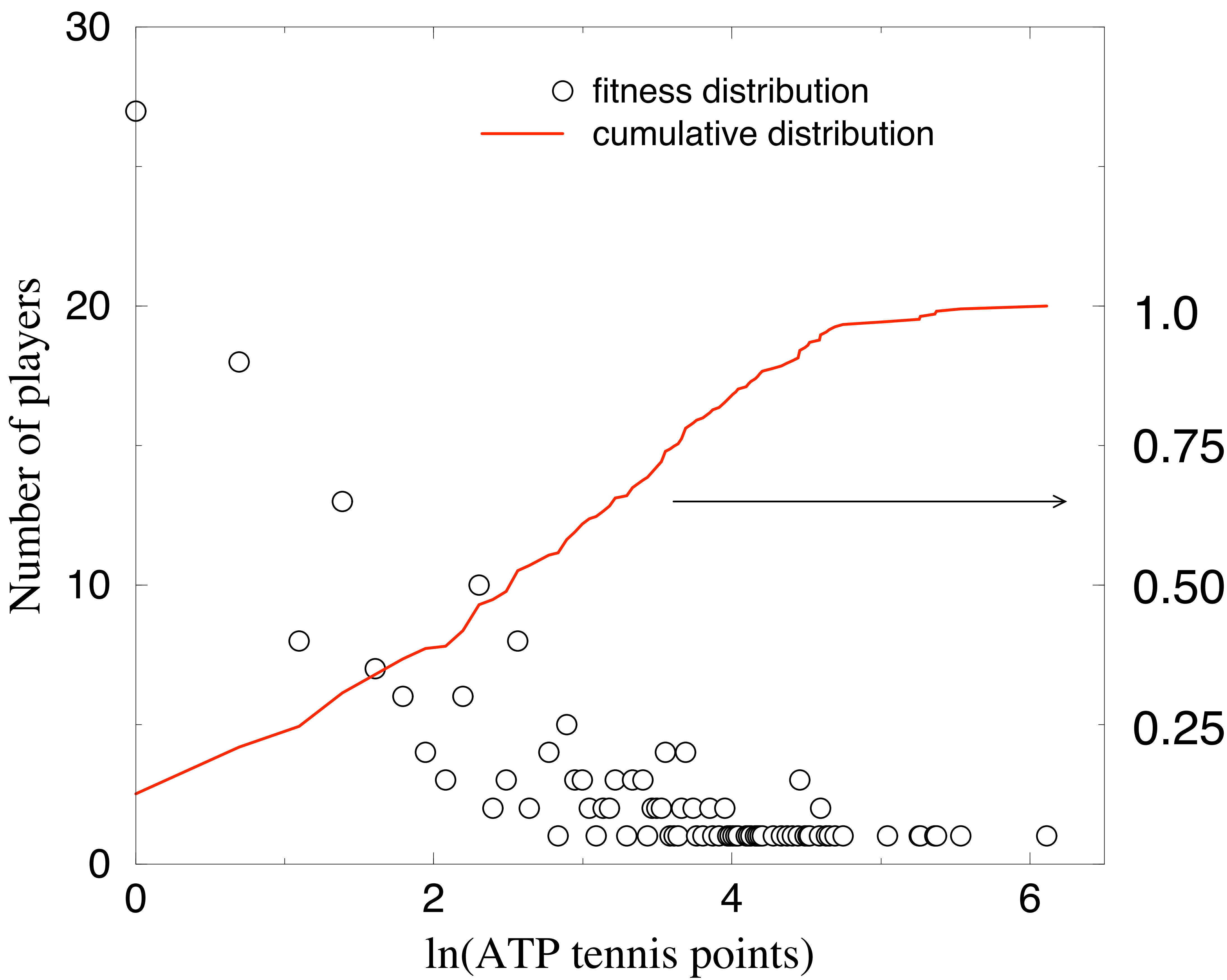


Fitness distribution for r=1/2

(t=4000)



ATP Fitness Distribution



Dynamics of the Lower Class

master equation: $$\frac{dF_k}{dt} = r(F_{k+1} - F_k) + F_k(F_{k-1} - F_k)$$

static ansatz: $$\frac{dF_k}{dt} = 0, \qquad F_k = L(1 - G_k)$$

with $G_k \to 0$, as $k \to \infty$

static solution: $$r\,\frac{G_{k+1} - G_k}{G_k - G_{k-1}} = L(1 - G_k)$$

Fitness Distribution in the Lower Class

static solution: $$r\,\frac{G_{k+1}-G_k}{G_k-G_{k-1}} = L(1-G_k)$$

for $r \geq 1, L = 1,$ then $G_k \sim f_k \sim r^{-k}$

for $r < 1, L = r, \longrightarrow$ continuum approximation

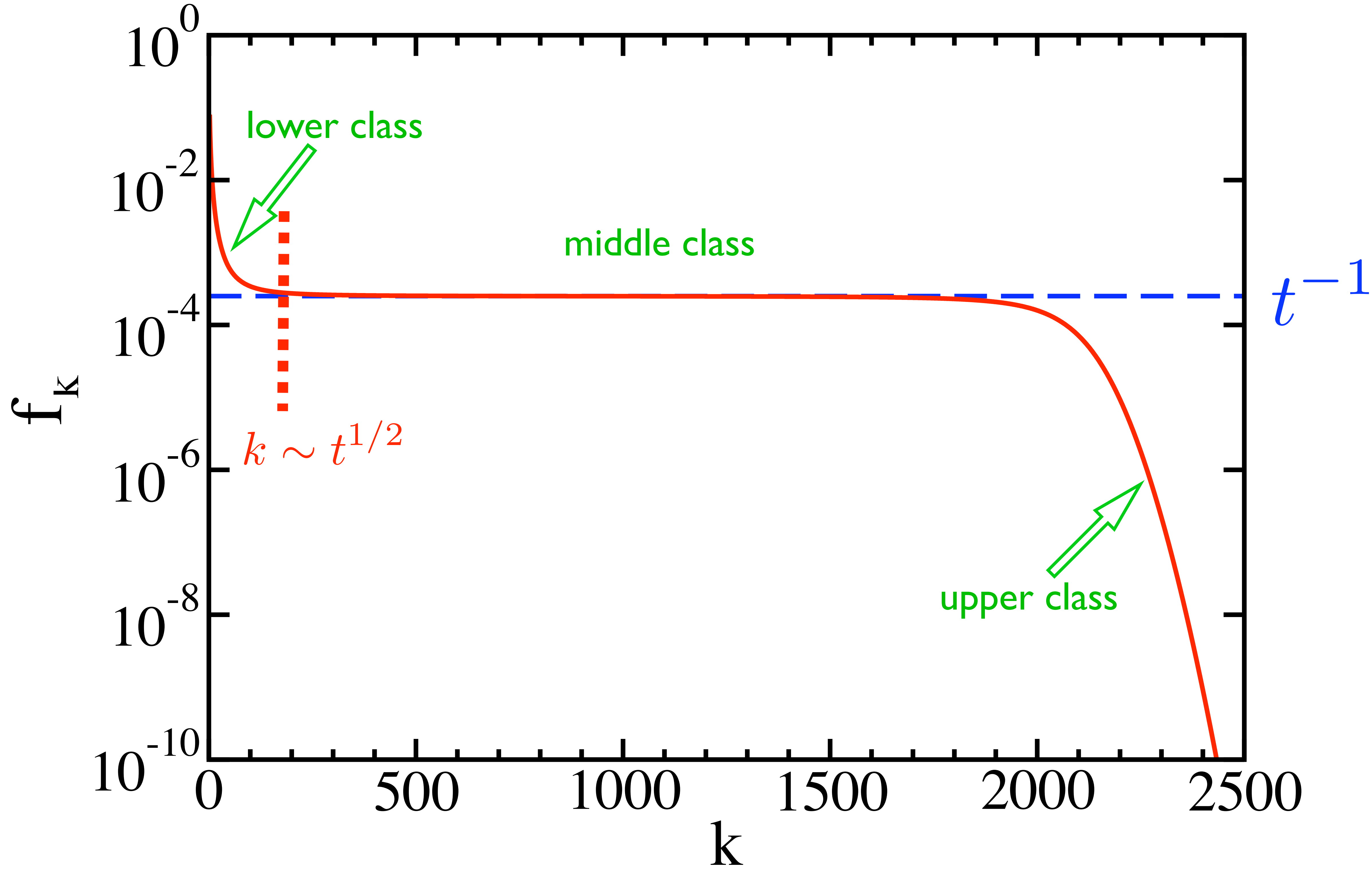
$$\frac{G' + \frac{1}{2}G''}{G' - \frac{1}{2}G''} = 1 - G \rightarrow G'' + GG' = 0$$

gives $G_k \sim \dfrac{2}{k} \;\rightarrow\; f_k \sim \dfrac{2r}{k^2}$

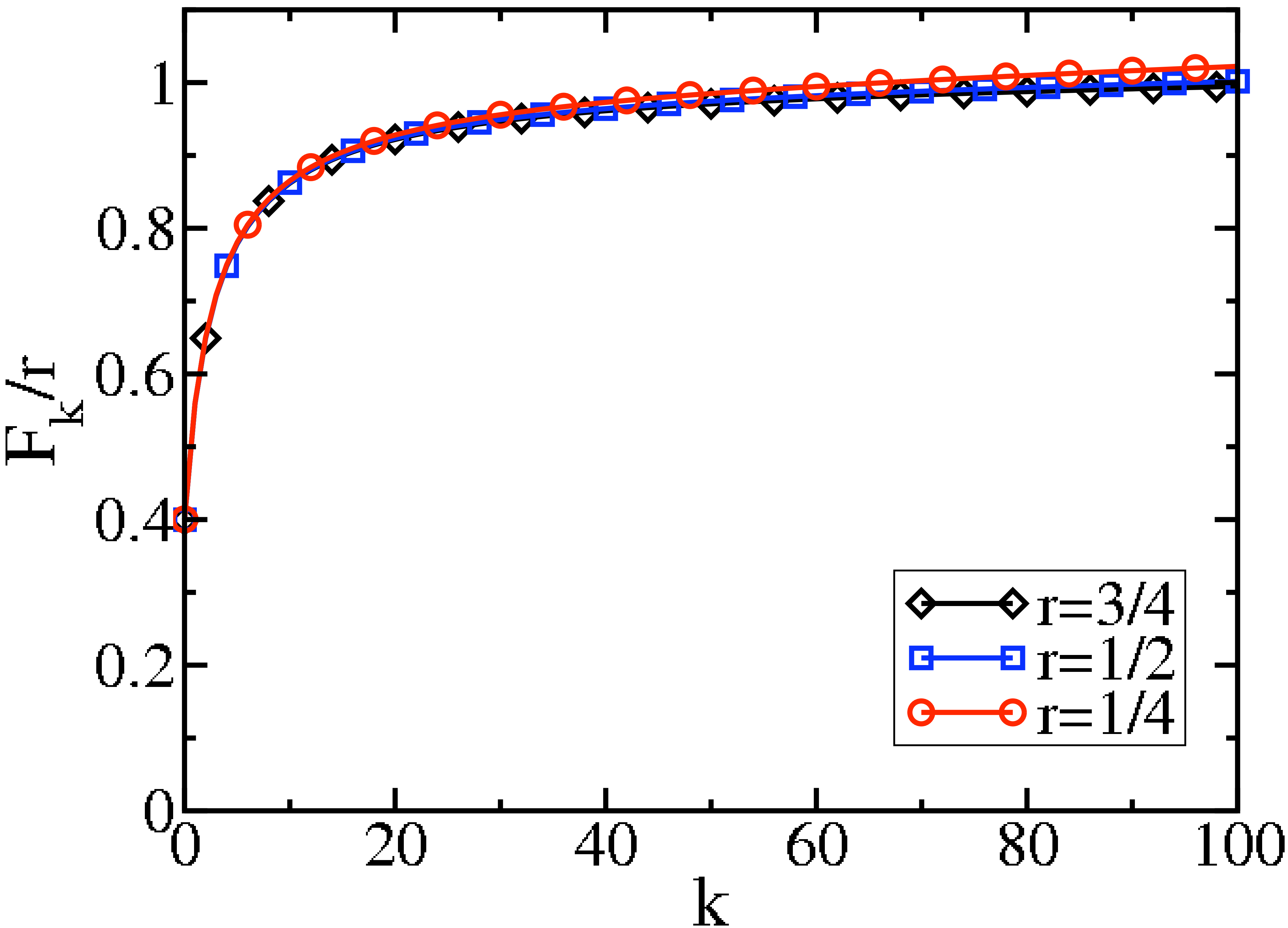
lower/middle class boundary: $$\frac{2r}{k^2} \approx \frac{1}{t} \;\rightarrow k_{\text{lower}} \sim (2rt)^{1/2}$$

Fitness distribution for r=1/2

(t=4000)



Cumulative Distribution in the Lower Class



Dynamics of the Upper Class

master equation: $$\frac{dF_k}{dt} = r(F_{k+1} - F_k) + F_k(F_{k-1} - F_k)$$

continuum: $$\frac{\partial G_k}{\partial t} + v\frac{\partial G_k}{\partial k} = D\frac{\partial^2 G_k}{\partial k^2}$$

$v = 1 - r$

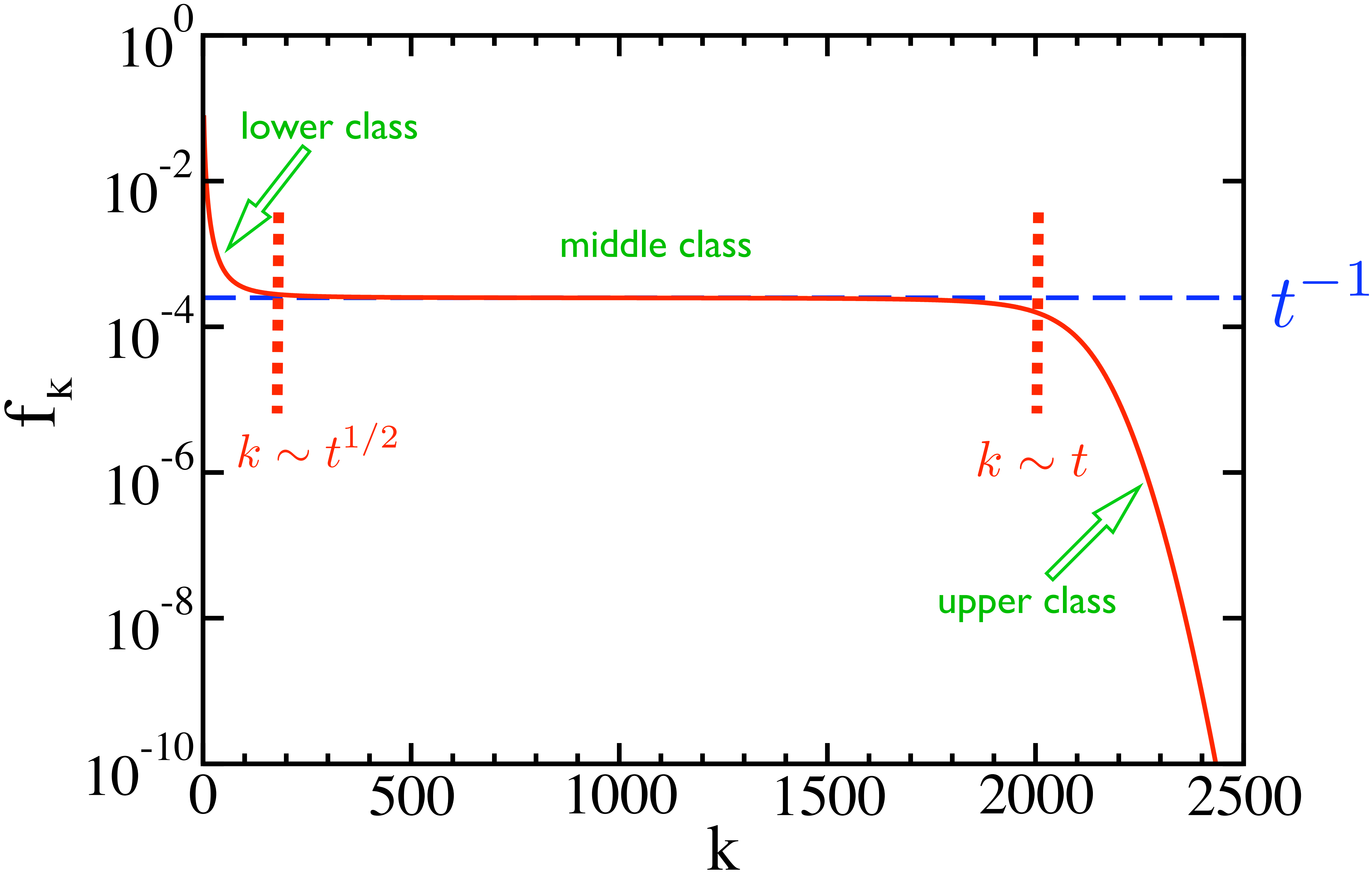
$D = (1+r)/2$

solution: $$f_k \sim \frac{1}{t}\psi\left(\frac{k - vt}{\sqrt{Dt}}\right)$$

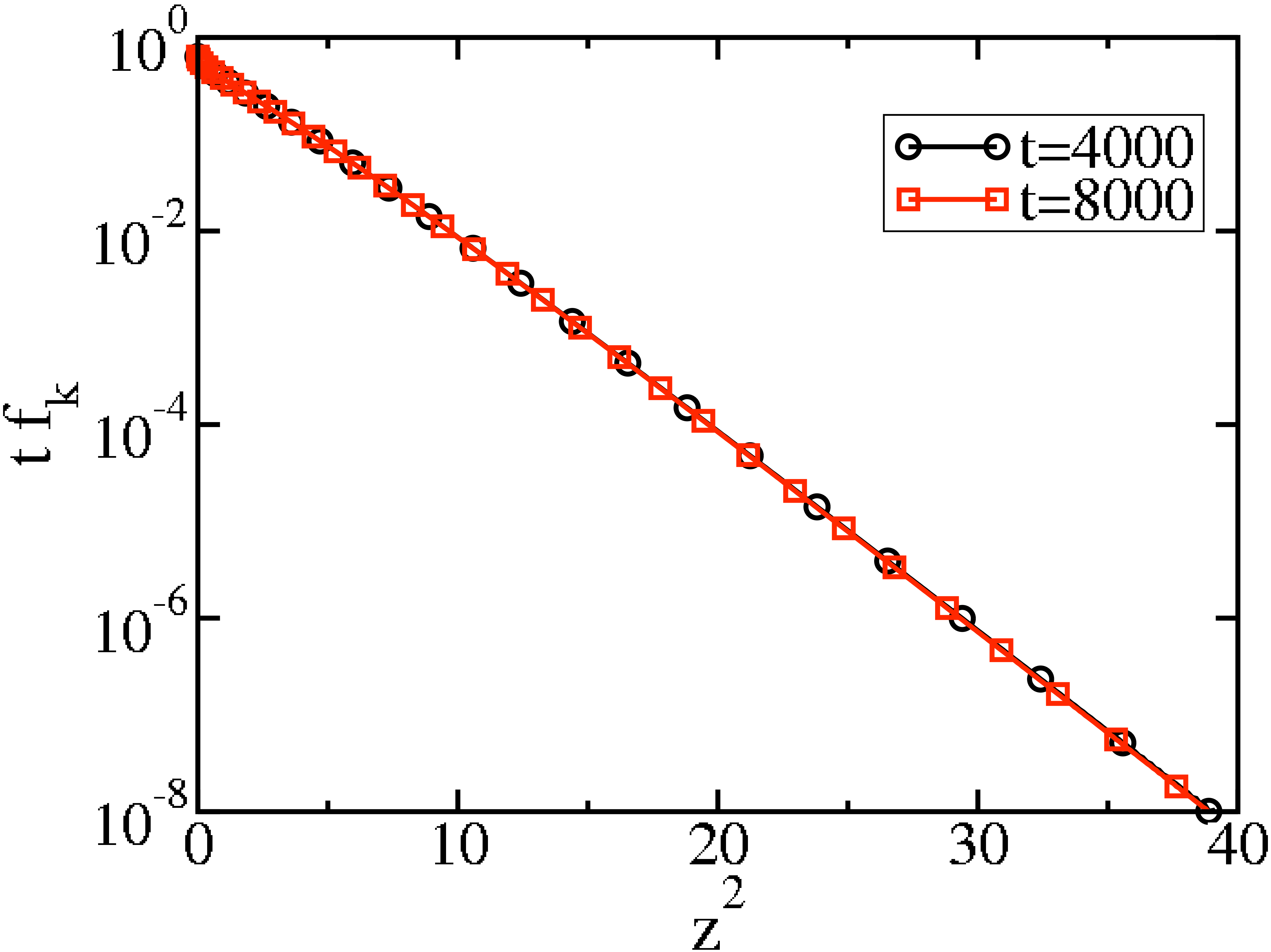
$\psi(z) \sim e^{-z^2/2}, z \to \infty$

Fitness distribution for r=1/2

(t=4000)



Fitness Distribution in the Upper Class



Exact Solution for r=0

rate equation: $$\frac{dF_k}{dt} = F_k(F_{k-1} - F_k)$$

hypothesis: $$F_k = \frac{P_k}{P_{k+1}} \quad \rightarrow \quad \frac{dP_k}{dt} = P_{k-1} \begin{cases} P_0(t) = 1 \\ P_k(t=0) = 1 \end{cases}$$

solution: $$P_k(t) = \sum_{j=0}^{k} \frac{t^j}{j!}$$

for small k:

$$F_0 = \frac{1}{1+t}$$

$$F_1 = \frac{1+t}{1+t+\frac{1}{2}t^2}$$

$$F_2 = \frac{1+t+\frac{1}{2}t^2}{1+t+\frac{1}{2}t^2+\frac{1}{6}t^3}$$

Upper Class Asymptotics for r=0

limiting behavior:

$$F_k = \frac{P_k}{P_{k+1}} \qquad P_k(t) = \sum_{j=0}^{k} \frac{t^j}{j!} \qquad F_k \sim \begin{cases} \frac{k+1}{t} & \frac{k}{t} \to 0; \\ 1 & \frac{k}{t} \to \infty. \end{cases}$$

leading correction:

$$P_k(t) = \sum_{j=0}^{k} \frac{t^j}{j!} \sim \int_{-\infty}^{k} e^{j \ln t - j \ln j + j - \frac{1}{2}\ln(2\pi j)}\, dj$$

$$\sim \frac{e^t}{\sqrt{2\pi t}} \int_{-\infty}^{k} e^{-\frac{1}{2}(j-t)^2/t}\, dj$$

asymptotics:

$$1 - F_k \sim \sqrt{\frac{2}{\pi}} \frac{e^{-(k-t)^2/2t}}{\mathrm{erfc}[(k-t)/\sqrt{2t}]}$$

Outlook & Questions

Basic result: Variegated social structure emerges through competition & decay

Questions:

can everyone get rich? can a rich person become poor?
transience vs. recurrence of single agent trajectory

role of intrinsic fitness vs. luck

more general interactions: symbiosis, deleterious competition, exogenous effects

fluctuations in low spatial dimension