Coarsening & Freezing in the Kinetic Ising Model

collaborators: P. Krapivsky, V. Spirin

Basic question: What is the final state of the Ising-Glauber model @ T=0 with symmetric initial conditions?

We might expect: Ground state is approached as $t \to \infty$

Basic results: 1.

dimension	expectation
1	correct
2	correct "sort of"
>2	wrong

2. Multiscale relaxation, freezing, & related strange features

The System

Ising Hamiltonian $\mathcal{H} = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j \qquad \sigma_i = \pm 1$

Initial state:

$$\uparrow \qquad \text{with probability } p$$
$$\downarrow \qquad \text{with probability } 1-p$$

Lattice: even co-ordination number, periodic boundaries

Dynamics: Glauber at T=0: Pick a random spin and consider outcome of a reversal

$$\text{if} \quad \Delta E < 0 \qquad \text{do it}$$
$$\text{if} \quad \Delta E > 0 \qquad \text{don't do it}$$
$$\text{if} \quad \Delta E = 0 \qquad \text{do it with prob. } 1/2$$

Results in d=1

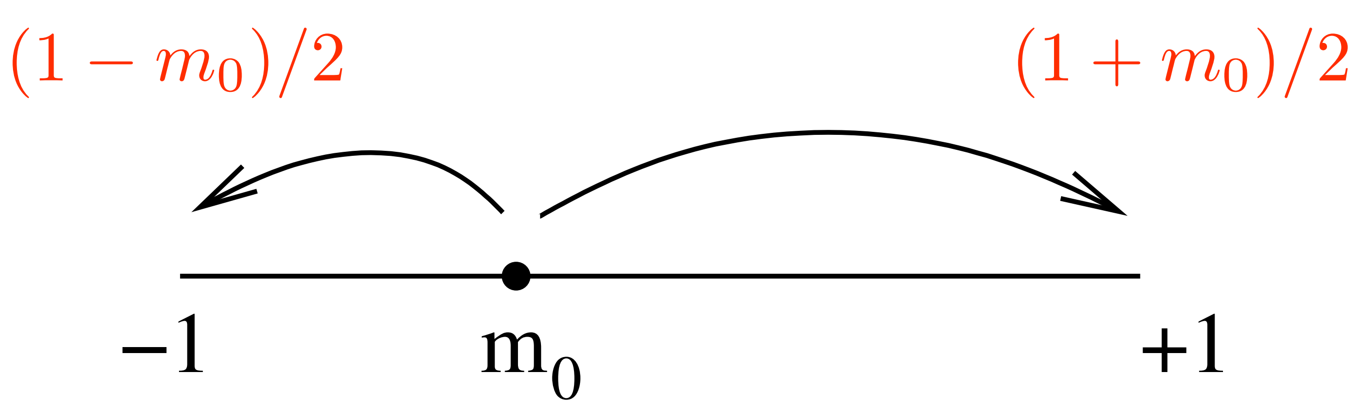
Equation of motion at T=0 (with Glauber kinetics):

$$\dot{s}_j = -s_j + \frac{1}{2}(s_{j-1} + s_{j+1}), \qquad \text{where } s_j = \langle \sigma_j \rangle$$

Hence $\langle \dot{m} \rangle = \sum_j \dot{s}_j = 0 \quad \rightarrow \quad \langle m \rangle$ conserved

m diffuses

Pictorial representation:



ultimate hitting probabilities

Summary of results in d=2

(Spirin, Krapvisky, & SR 01)

Final state:

$$\begin{cases} \text{ground state} & \text{prob.} \approx 2/3 \\ \text{stripe} & \text{prob.} \approx 1/3 \end{cases}$$

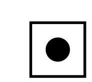
Survival probability: 2 time scales!

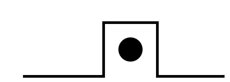
$$M_k \equiv \langle t^k \rangle^{1/k} \sim \begin{cases} L^{3.5} & k > 1 \\ L^{2^+} & k < 1 \end{cases}$$

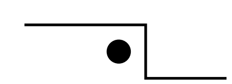
Energy evolution:

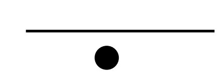
$$E(t) \sim t^{-1/2}$$

$$n_E(t) \sim t^{-\mu(E)}$$

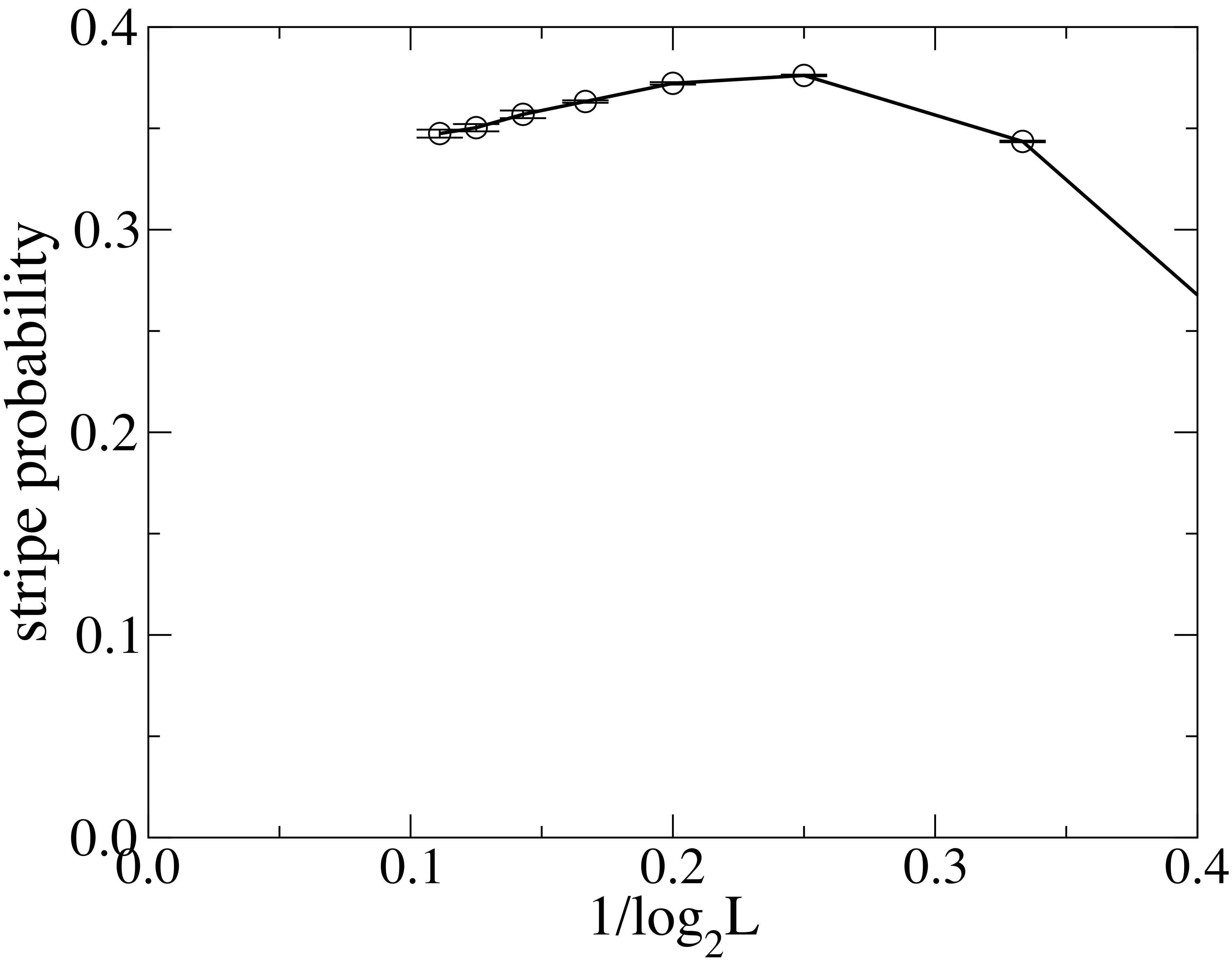
$\mu(+4) \approx 2.1$ 

$\mu(+2) \approx 1.4$ 

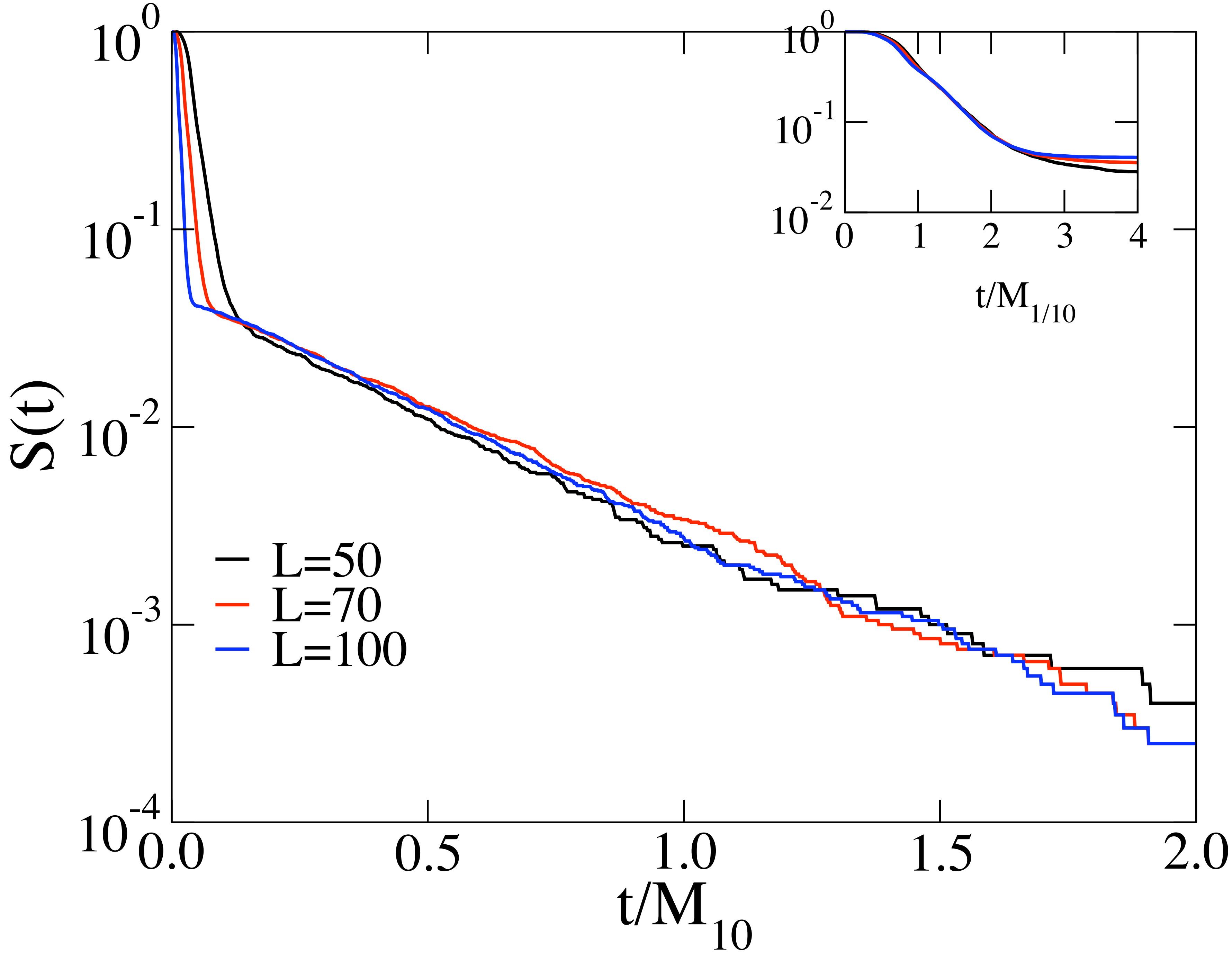
$\mu(0) \approx 0.5$ 

$\mu(-2) \approx 0.45$ 

Final state in 2d: *stripes*



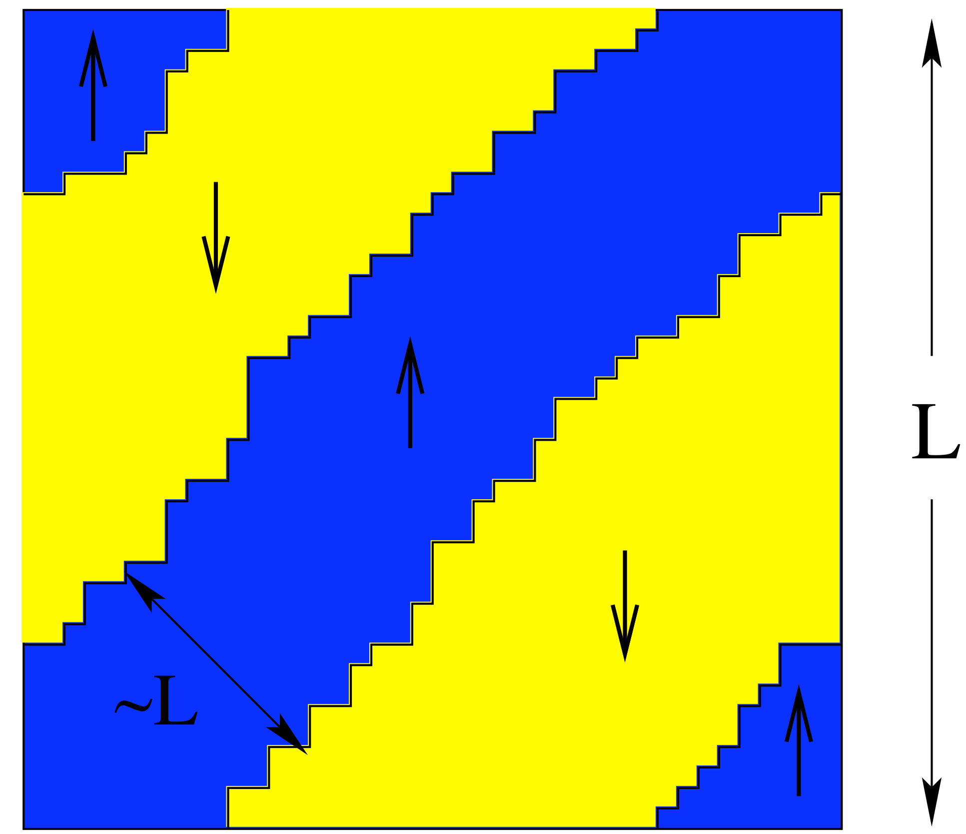
Survival probability: *two time scales*



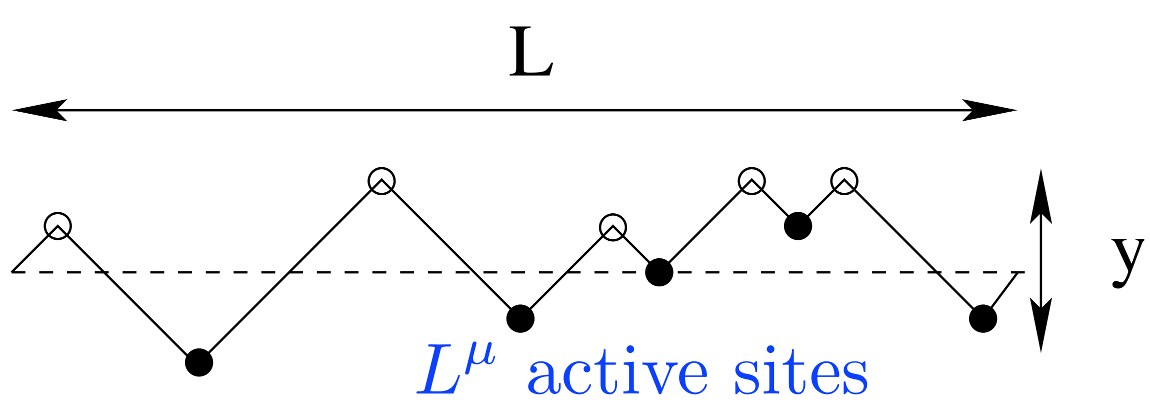
Understanding the two time-scale relaxation

We observe: 95% short-lived, 5% long lived!

Why? Diagonal stripe!



Diagonal stripe dynamics: (Plischke et al 87)



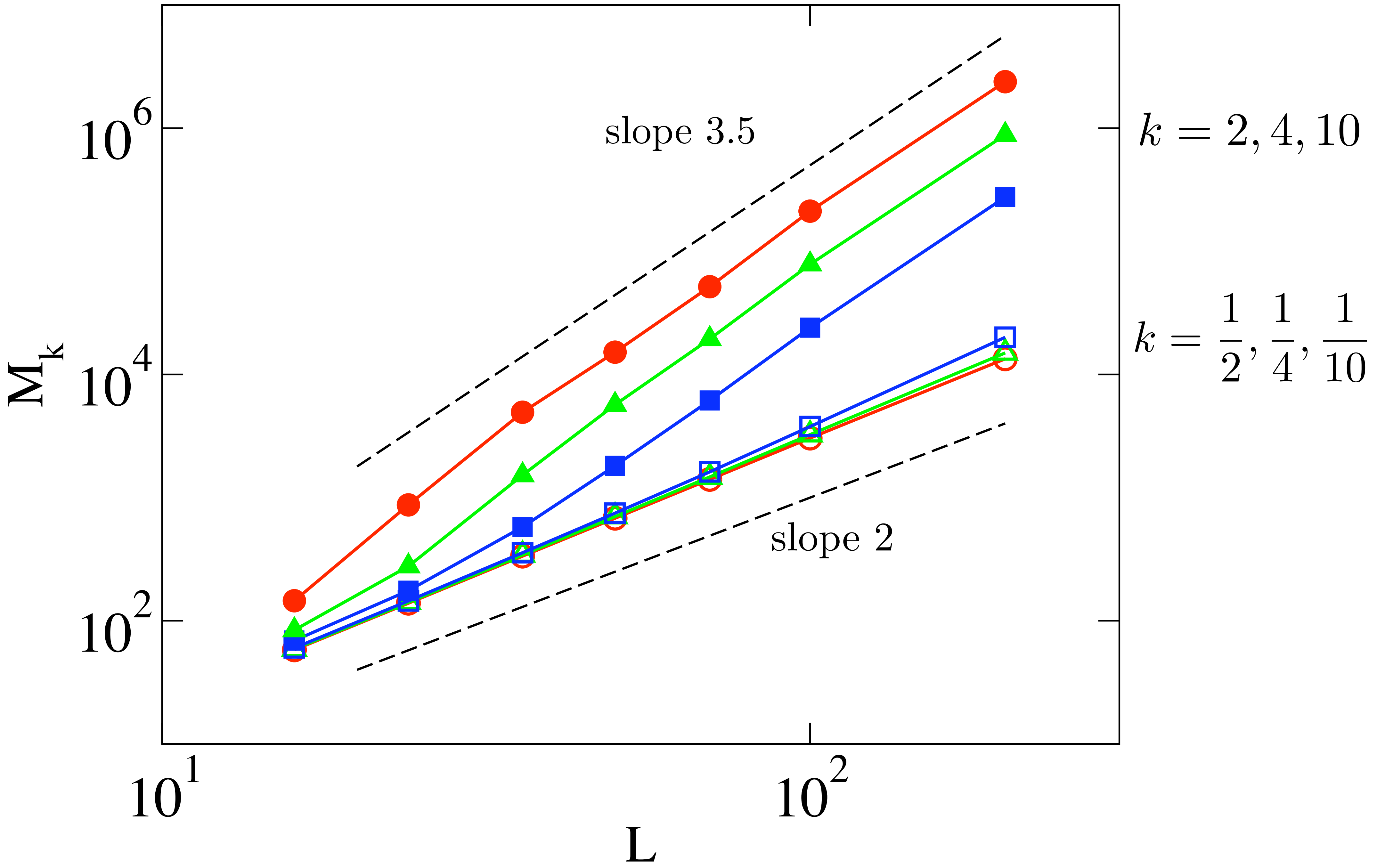
$$\Delta t = 1, \quad L^\mu \text{ events} \quad \rightarrow \quad \Delta y_{\text{cm}} \sim L^{\mu/2}/L$$

$$\rightarrow \quad D(L) \sim L^{\mu-2}$$

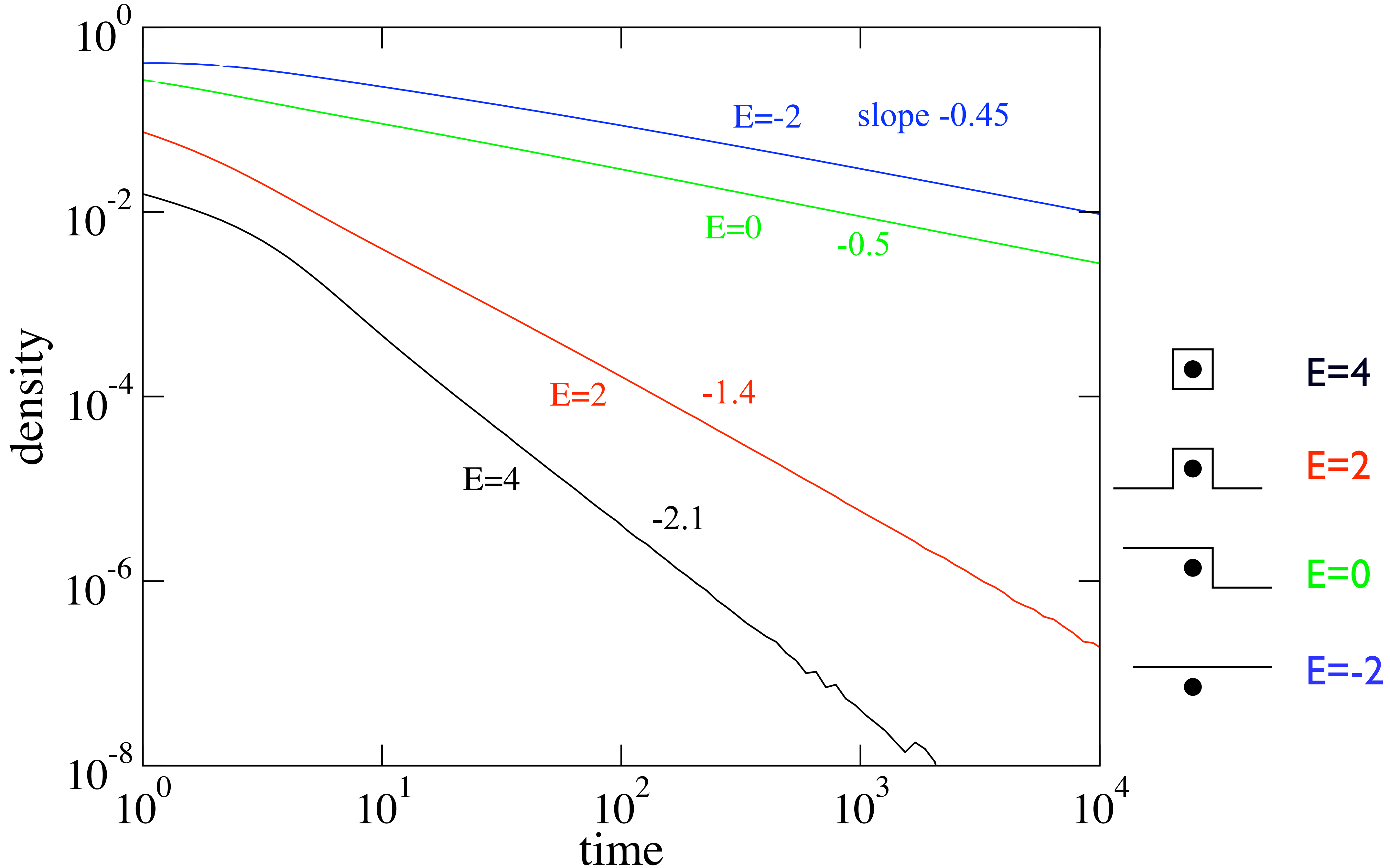
$$\text{survival time } \tau \sim L^2/D \quad \sim \quad L^{4-\mu} \quad \text{but } \mu = 1/2$$

$$\sim \quad L^{3.5}$$

Multiscaling in moments of the stopping time

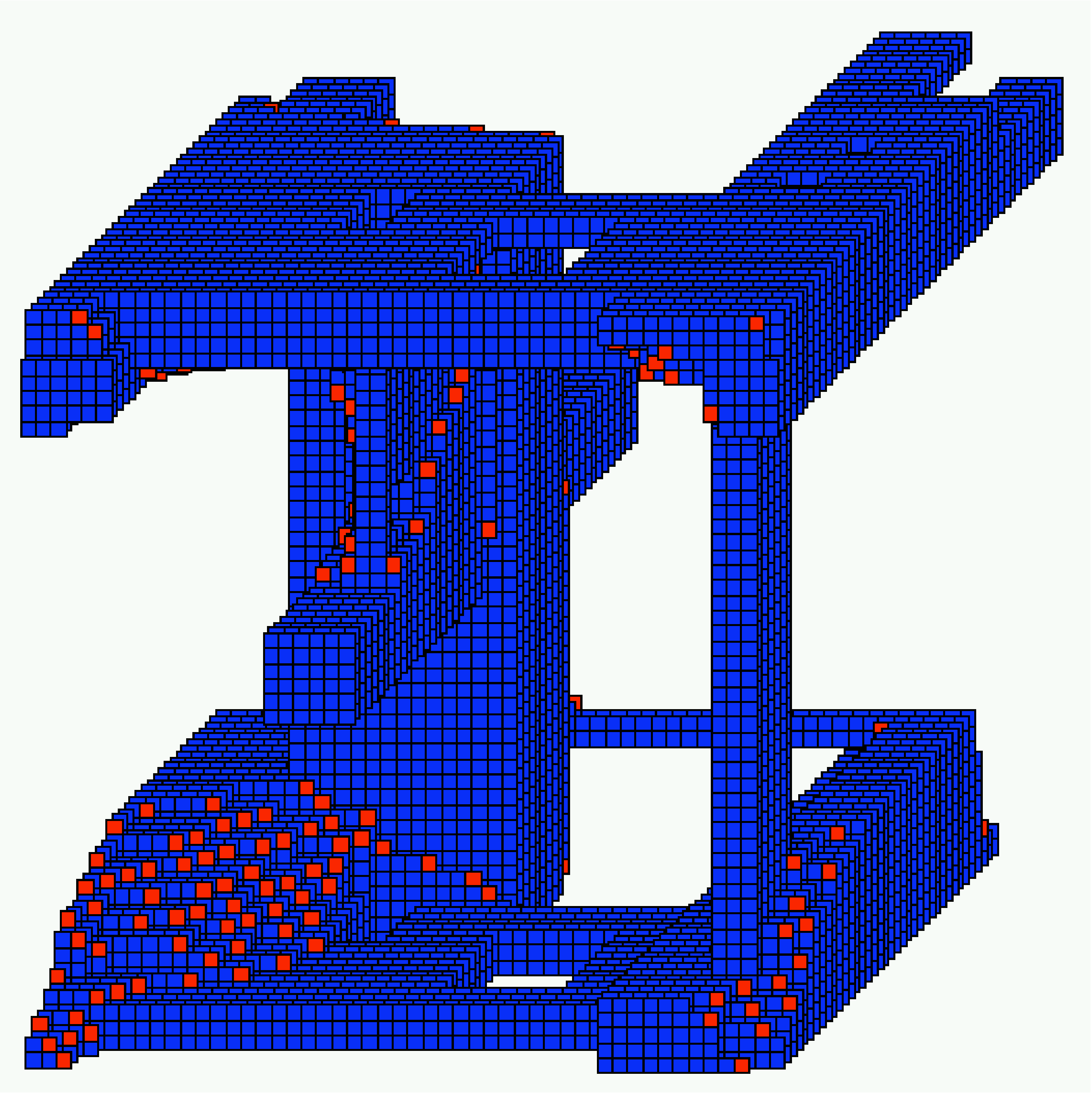


Densities of fixed-energy spins

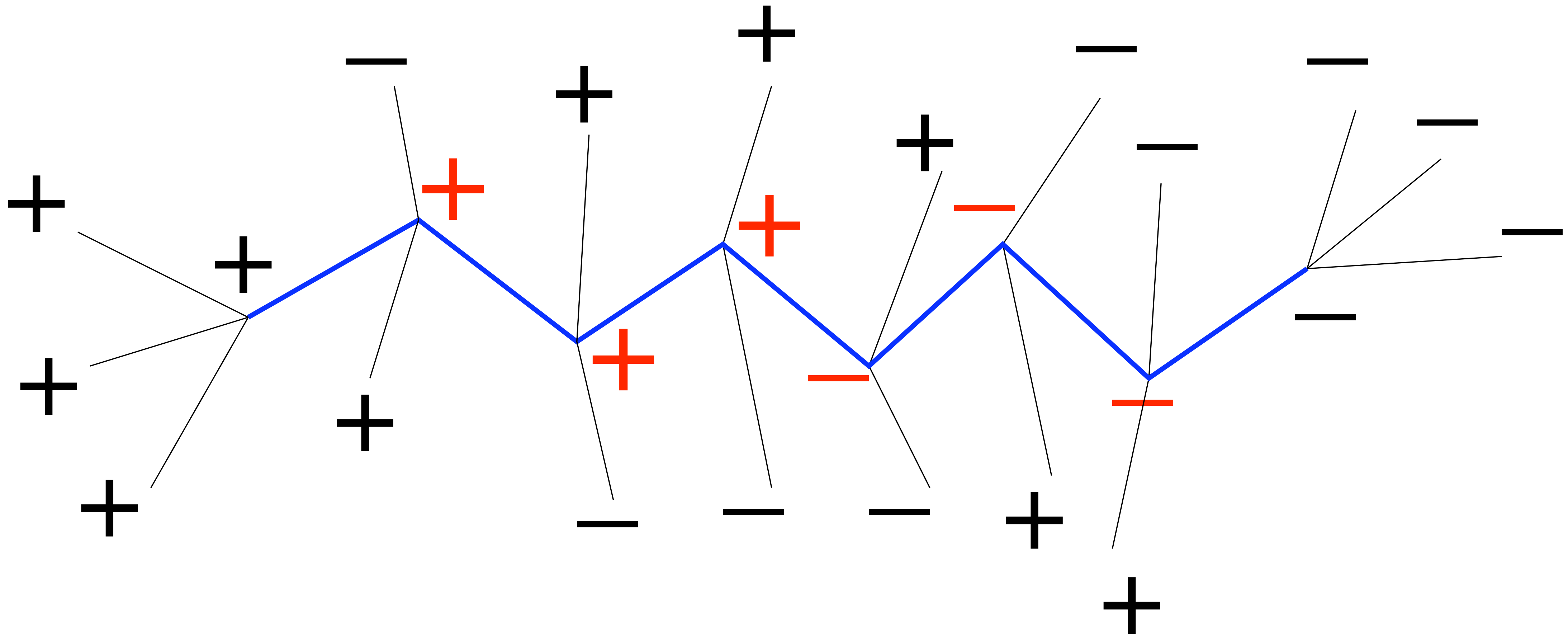


Higher dimensions: Ground state (almost) never reached!

Final "sponge" state in 3d:



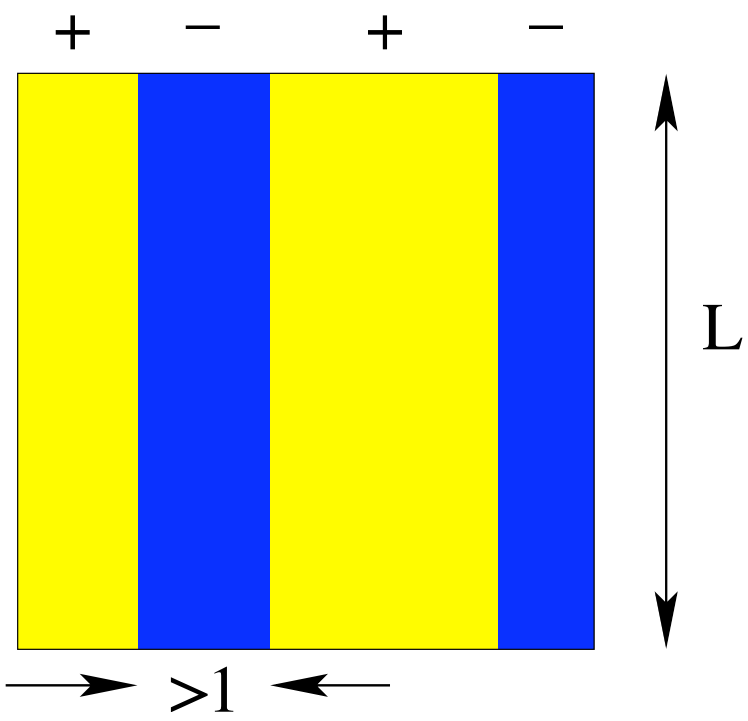
Schematic of blinker states



Why does the system get stuck?

Proliferation of metastable states as d increases.

d=2: stripe packing

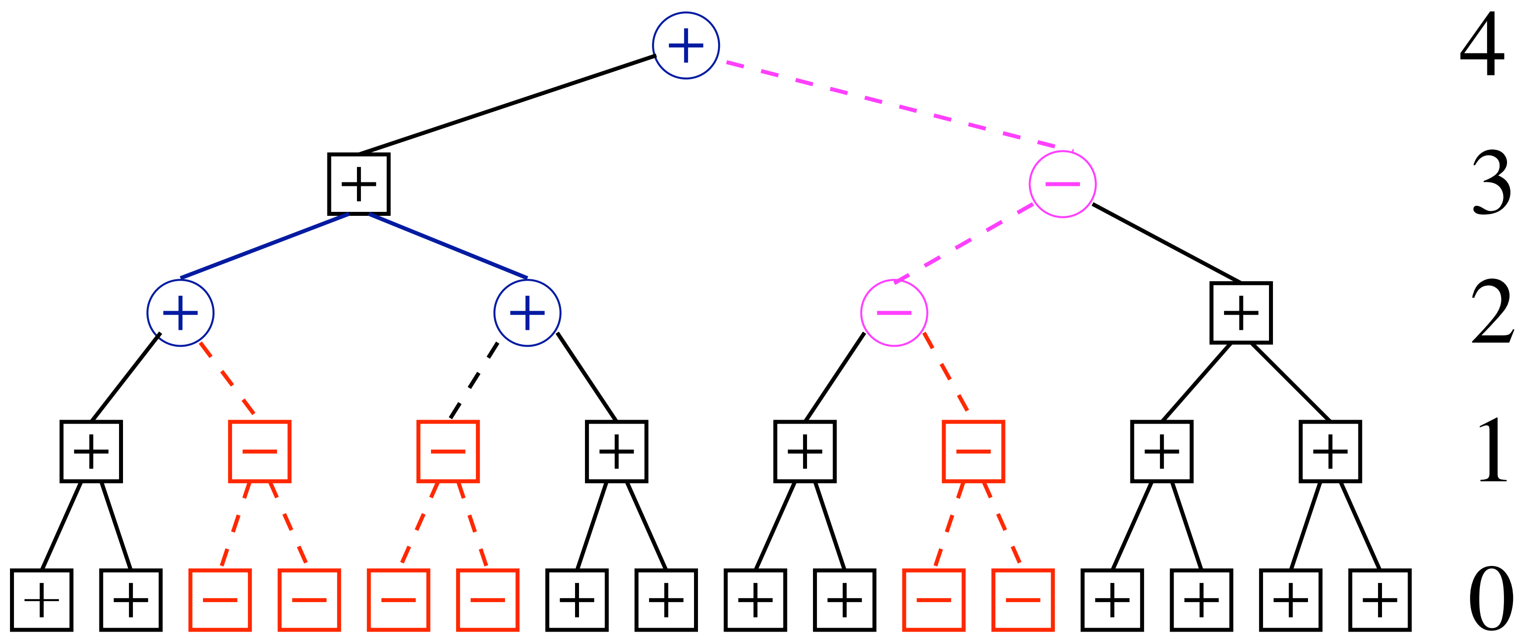


$$M_L \sim e^{aL} \sim e^{aV^{1/2}}$$

d=3: filament packing

$$M_L \sim e^{bL^2} \sim e^{bV^{2/3}}$$

d= ∞: Cayley tree



Recursion formulae for degeneracy

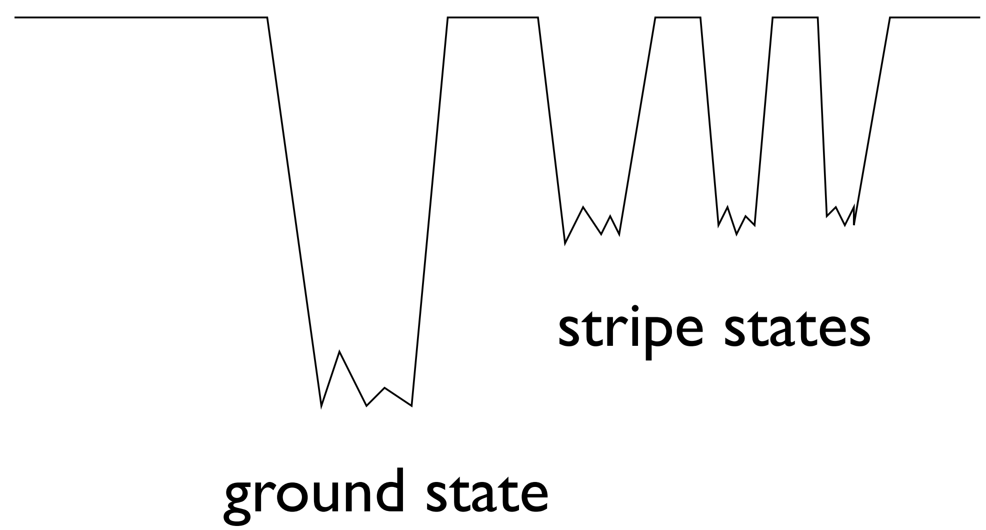
○ $$U_{N+1} = 2D_N \times \frac{1}{2} D_N = D_N^2$$ undetermined

□ $$D_{N+1} = \frac{1}{2} D_N^2 + \frac{1}{2} U_N^2 + 2 D_N U_N$$ determined

$$\rightarrow \ln M_{\mathcal{N}} \sim \ln(U_N + D_N) = \text{const.} \times \mathcal{N}$$

Conclusions & Outlook

1. Even the simplest Ising model has a complex evolution landscape



cartoon for d=2

2. How to characterize & quantify frozen states in d=2 & d>2?

3. What happens for non-symmetric initial conditions?