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Preface

Statistical physics is an unusual branch of physics because it is not really a well-defined field in a formal sense, but rather, statistical physics is a viewpoint — indeed, the most appropriate viewpoint — to investigate systems with many degrees of freedom. Part of the appeal of statistical physics is that it can be applied to a disparate range of systems that are in the mainstream of physics, as well as to problems that might appear to be outside physics, such as econophysics, quantitative biology, and social organization phenomena. Many of the basic features of these systems involve fluctuations or explicit time evolution rather than equilibrium distributions. Thus the approaches of non-equilibrium statistical physics are needed to discuss such systems.

While the tools of equilibrium statistical physics are well-developed, the statistical description of systems that are out of equilibrium is still relatively primitive. In spite of more than a century of effort in constructing an over-arching approach for non-equilibrium phenomena, there still does not exist canonical formulations, such as the Boltzmann factor or the partition function in equilibrium statistical physics. At the present time, some of the most important theoretical approaches for non-equilibrium systems are either technical, such as deriving hydrodynamics from the Boltzmann equation, or somewhat removed from the underlying phenomena that are being described, such as non-equilibrium thermodynamics.

Because of this disconnect between fundamental theory and applications, our view is that it is more instructive to illustrate non-equilibrium statistical physics by presenting a number of current and paradigmatic examples of systems that are out of equilibrium, and to elucidate, as completely as possible, the range of techniques available to solve these systems. By this approach, we believe that readers can gain general insights more quickly compared to formal approaches, and, further, will be well-equipped to understand many other topics in non-equilibrium statistical physics. We have attempted to make our treatment as self-contained and user-friendly as possible, so that an interested reader can work through the book without encountering unresolved methodological mysteries or hidden calculational pitfalls. Thus while much of the material is mathematical in nature, we have tried to present it as pedagogically as possible. Our target audience is graduate students with a one-course background in equilibrium statistical physics. Each of the main chapters is intended to be self-contained. We also made an effort to supplement the chapters with research exercises and open questions, in the hopes of stimulating further research.

The specific examples presented in this book are primarily based on irreversible stochastic processes. This branch of statistical physics is in many ways the natural progression of kinetic theory that was initially used to describe dynamics of simple gases and fluids. We will discuss the development of basic kinetic approaches to more complex and contemporary systems. Among the large menu of stochastic and irreversible processes, we chose the ones that we consider to be among the most important and most instructive in leading to generic understanding. Our main emphasis is on exact analytical results, but we also send time developing heuristic and scaling methods. We largely avoid presenting numerical simulation results because these are less definitive and instructive than analytical results. An appealing (at least to us) aspect of these examples is that they are broadly accessible. One needs little background to appreciate the systems being studied and the ideas underlying the methods of solution. Many of these systems naturally suggest new and non-trivial questions that an interested reader can easily pursue.

We begin our exposition with a few "aperitifs" — an abbreviated qualitative discussion of basic problems and a general hint at the approaches that are available to solve these systems. Chapter 2 provides a basic introduction to diffusion phenomena because of the central role played by diffusion in many non-equilibrium statistical systems. These preliminary chapters serve as an introduction to the rest of the book.

The main body of the book is then devoted to working out specific examples. In the next three chapters, we discuss the fundamental kinetic processes of aggregation, fragmentation, and adsorption (chapters 3–5).

Aggregation is the process by which two clusters irreversibly combine in a mass-conserving manner to form a larger cluster. This classic process that very nicely demonstrates the role of conservation laws, the utility of exact solutions, the emergence of scaling in cluster-size distributions, and the power of heuristic derivations. Many of these technical lessons will be applied throughout this book.

We then turn to the complementary process of fragmentation, which involves the repeated breakup of an element into smaller fragments. While this phenomenon again illustrates the utility of exact and scaling solutions, fragmentation also exposes important new concepts such as multiscaling, lack of self-averaging, and methods such as traveling waves and velocity selection. All of these are concepts that are used extensively in non-equilibrium statistical physics. We then discuss the phenomenon of irreversible adsorption where, again, the exact solutions of underlying master equations for the occupancy probability distributions provides a comprehensive picture of the basic phenomena.

The next two chapters (6 & 7) discuss the time evolution of systems that involve the competition between multiple phases. We first treat classical spin systems, in particular, the kinetic Ising model and the voter model. The kinetic Ising model occupies a central role in statistical physics because of its broad applicability to spins systems and many other dynamic critical phenomena. The voter model is perhaps not as well known in the physics literature, but it is an even simpler model of an evolving spin system that is exactly soluble in all dimensions. In chapter 7, we study phase ordering kinetics. Here the natural descriptions are in terms of continuum differential equations, rather than master equations.

In chapter 8, we discuss collision-driven phenomena. Our aim is to present the Boltzmann equation in the context of explicitly soluble examples. These include traffic models, and aggregation and annihilation processes in which the particles move at constant velocity between collisions. The final chapter (#9) presents several applications to contemporary problems, such as the structure of growing networks and models of self-organized criticality.

In an appendix, we present the fundamental techniques that are used throughout our book. These include various types of integral transforms, generating functions, asymptotic analysis, extreme statistics, and scaling approaches. Each of these methods is explain fully upon its first appearance in the book itself, and the appendix is a brief compendium of these methods that can be used either as a reference or a study guide, depending on one's technical preparation.