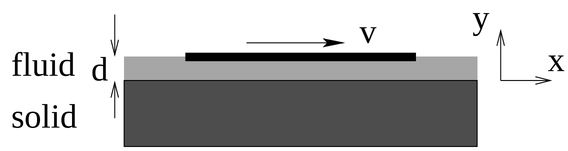
MIDTERM EXAM PY542 November 6, 2007

No notes or books are allowed or needed. Please be sure to **explain your work clearly** to maximize the probability of receiving the appropriate partial credit.

1. A flat plate of area A and mass m slides on top of a fluid of thickness d above a stationary solid. Assume that viscosity is the only mechanism that slows the object and that the fluid velocity beneath the object is $v_x(y) = v\,y/d$.



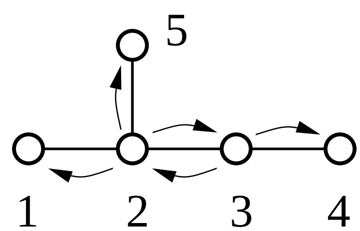
   (a) As a preliminary, derive estimate the viscosity coefficient of the fluid from first principles (you may ignore constants or order 1).

   (b) Numerically estimate the viscosity coefficient for a simple molecular liquid (such as water) with density (1 gm/cc^3). Clearly state the units of viscosity.

   (c) Determine the frictional drag force on the plate and thereby find its velocity as a function of time.

   (d) Estimate the characteristic time for the plate to come to rest when its area is 100 cm^2 its mass is 100 gm, and the fluid layer has thickness 1 cm. (*Note:* If you are unable to solve part (b), try to determine the time by dimensional analysis.

2. Consider a continuous-time nearest-neighbor random walk on the 5-site branched structure shown. The arrows indicate unit hopping rates. When the walk reaches sites 1, 4, or 5, it remains there permanently. The particle is initially at site 2.



   (a) Write the master equations for $P_i(t)$, the occupation probabilities at site i at time t.

   (b) Solve for the Laplace transforms of the occupation probabilities at each site.

   (c) Determine how $P_2(t)$ decays with time in the *long-time* limit. You can find the exact behavior if you wish, but only the asymptotic behavior as $t \to \infty$ is needed.

   (d) What are the probabilities that the walker *eventually* hits sites 1, 4, and 5? You may use the results of part (c), or you may give an independent physical argument.

3. Consider the Langevin equation for the position $x(t)$ of a particle in one dimension in which the acceleration equals a random noise $\eta(t)$:

$$\ddot{x}(t) = \eta(t),$$

   where $\eta(t)$ has zero mean, $\langle \eta(t) \rangle = 0$, and correlation function $\langle \eta(t)\eta(t') \rangle = \Gamma\delta(t - t')$.

   (a) Use dimensional analysis to find the dependence of $\langle v(t)^2 \rangle$ and $\langle x(t)^2 \rangle$ on t.

   (b) Determine $\langle v(t)^2 \rangle$ and $\langle x(t)^2 \rangle$ exactly by solving the Langevin equation. Assume that $x(0) = v(0) = 0$.