PY 542: Non-Equilibrium Statistical Physics   Problem Set 1

Reading: Before the first lecture, please read chapter 1 of the text for a general perspective. For this week, please also read chapter 2, sections 1–3 and 5.

Problems: Due Friday, September 10 by 5:00pm

These problems are meant to be somewhat preliminary in nature and reply on some basic techniques in mathematical physics. Please come see me for advice if you are unfamiliar with the methods that you will need for this assignment.

1. Use Stirling's approximation to derive Eq. (2.3) from (2.2). As a preliminary, first work out the case of a symmetric random walk, $p = q$, and the generalize to the biased random walk. *Note:* If you unfamiliar with Stirling's approximation, please become familiar. Stirling's approximation can be derived by starting with

$$n! = \int_0^\infty t^n \, e^{-t} \, dt \,,$$

   expanding the integrand about its maximum, and performing the resulting Gaussian integral. This technique is known as the *Laplace method*.

   Beware: Eq. (2.3) is incorrect in the text, but this formula is correctly stated in the text errata which are posted on the main page of my website `physics.bu.edu/~redner`.

2. Text 2.5.

3. The problem is a basic exercise in the use of transform methods

   (a) Solve for the Fourier transform of the probability distribution for symmetric diffusion with the initial condition $P(x, t\!=\!0) = \delta(x)$. That is, solve the diffusion equation in the Fourier domain. You will have to figure out the Fourier transform of the initial condition $P(k, t\!=\!0)$ to make the problem well posed. Use this Fourier transform solution to reconstruct the Gaussian form for $P(x, t)$.

   (b) Repeat the same calculation for the mixed Fourier-Laplace transform, $P(k, s)$. Again, you will have to figure out how to express the initial condition in the Fourier-Laplace domain to make the problem well posed.