Fractionalization of the electron

Claudio Chamon

PY 482 Lecture

Boston, April 4, 2013

BOSTON UNIVERSITY

Fractionalization in Polyacetelene

R. Jackiw and C. Rebbi, Phys Rev. D13, 3398 (1976)
W. P. Su, J. R. Schrieffer and A. J. Heeger, PRL 42, 1698 (1979); PRB 22, 2099 (1980)

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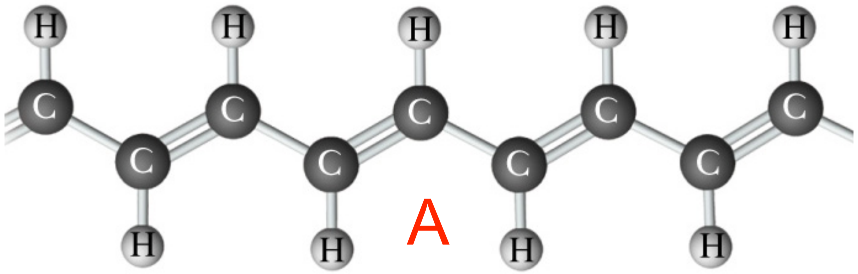

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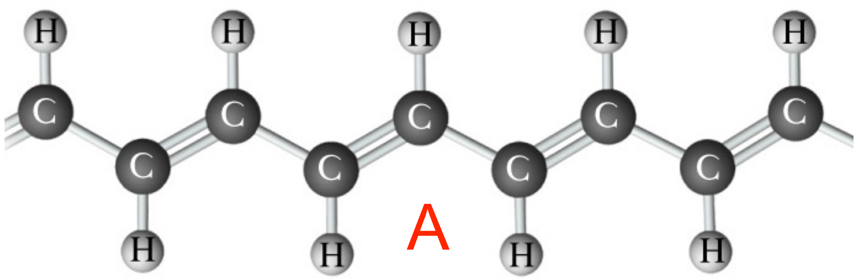

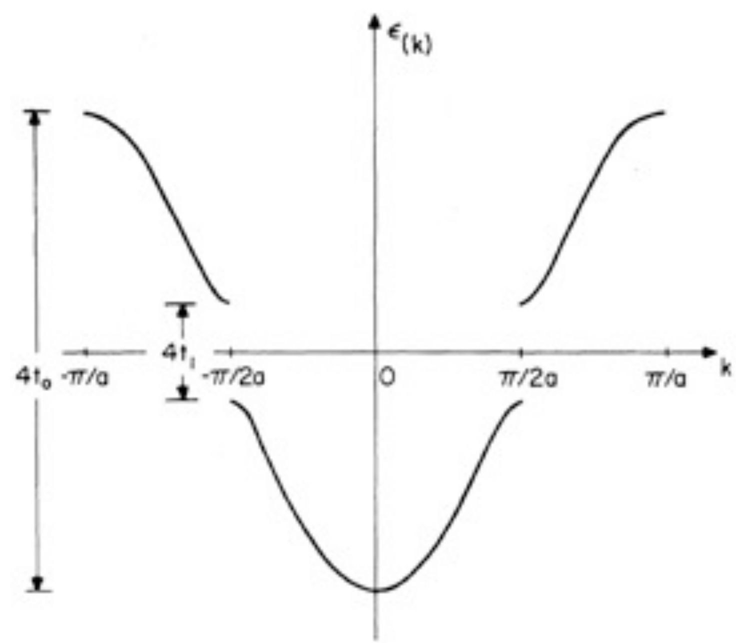

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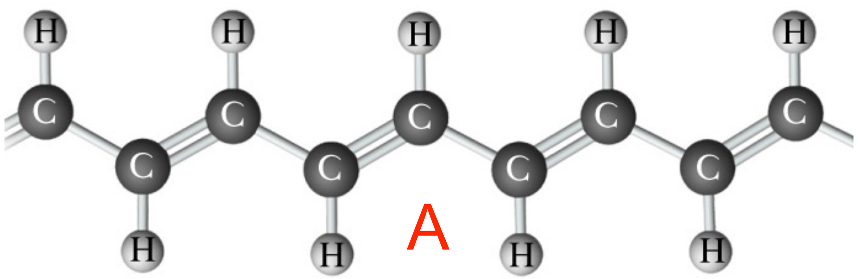

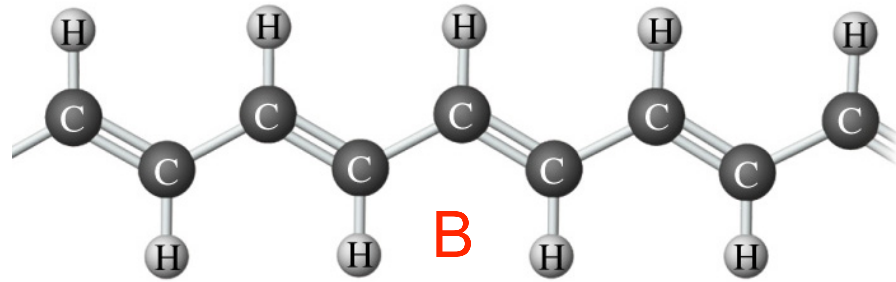

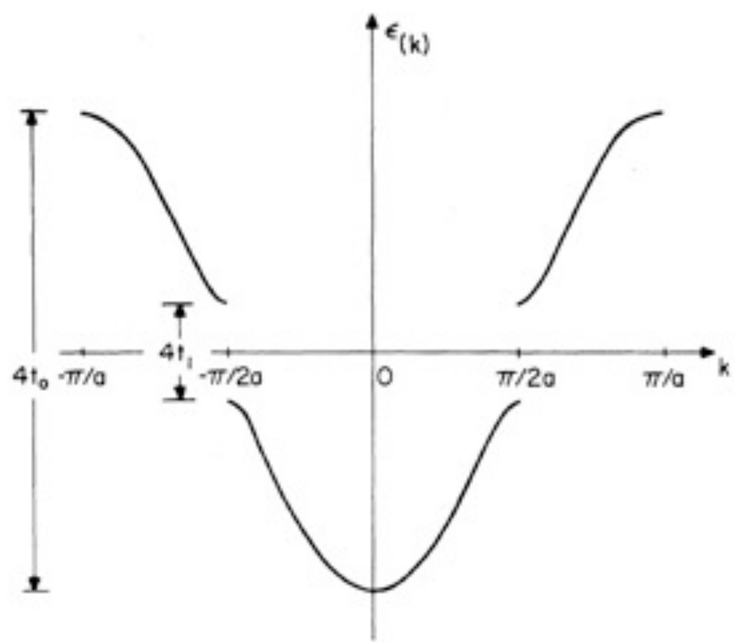

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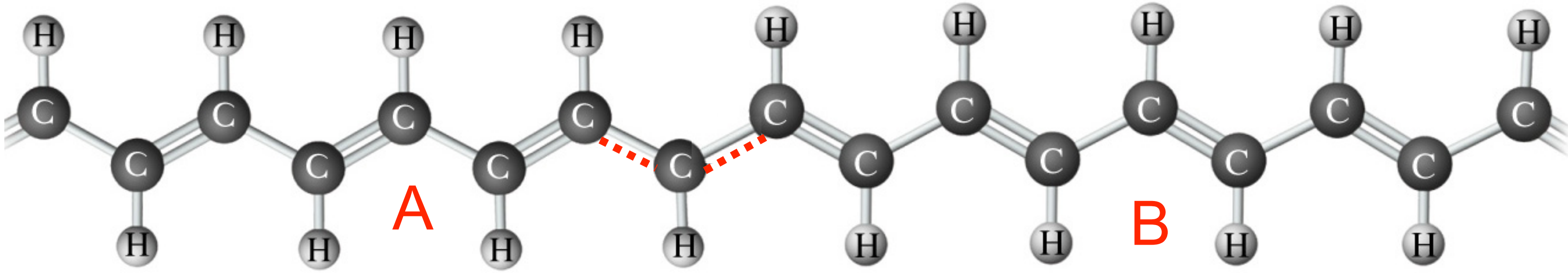

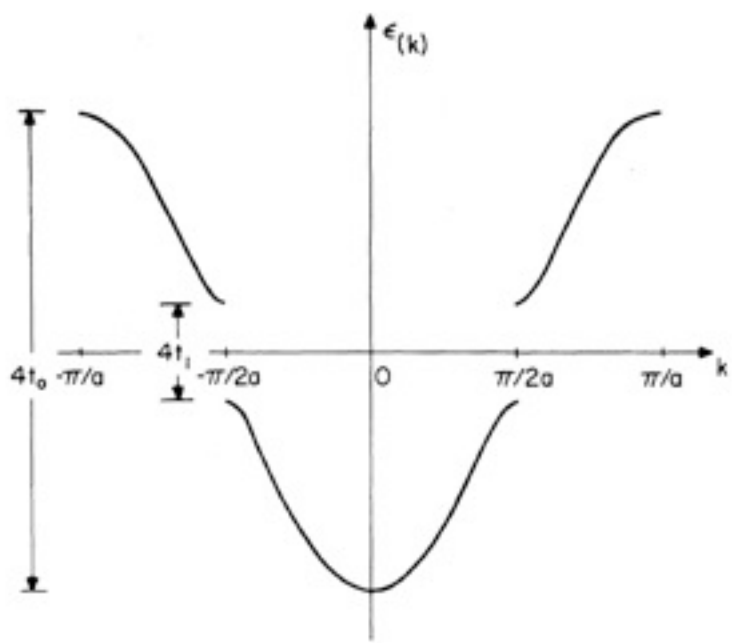

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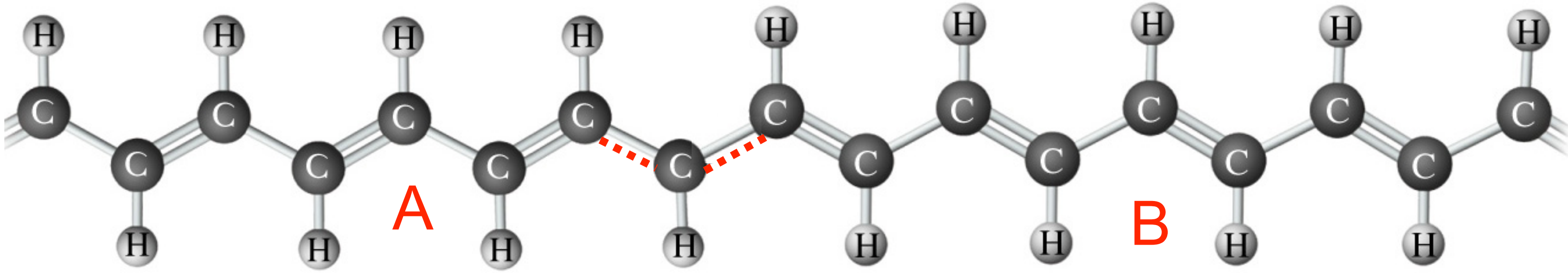

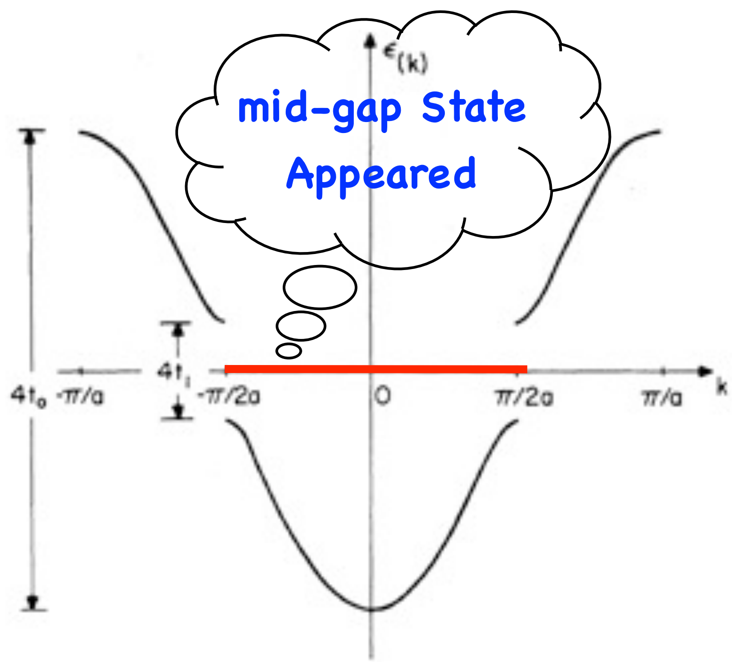

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Counting the charge

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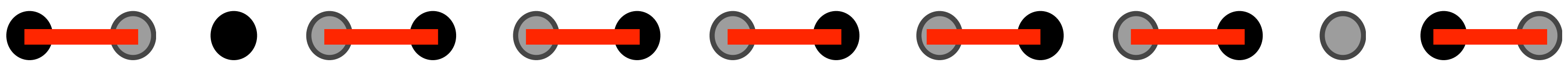

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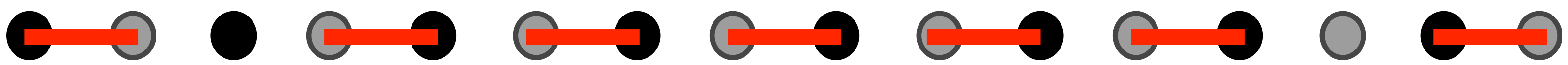


$Q = -1/2$ $\qquad\qquad$ $Q = -1/2$

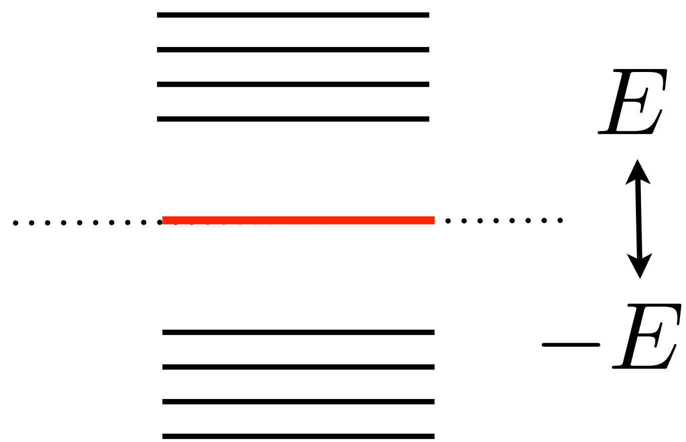
Counting the charge

$Q = -1/2$ $Q = -1/2$

Fractional charge!

What are zero modes?

$$H \; \Psi(x) = 0$$



$$\Psi_E(x) \overset{C}{\longleftrightarrow} \Psi_{-E}(x)$$

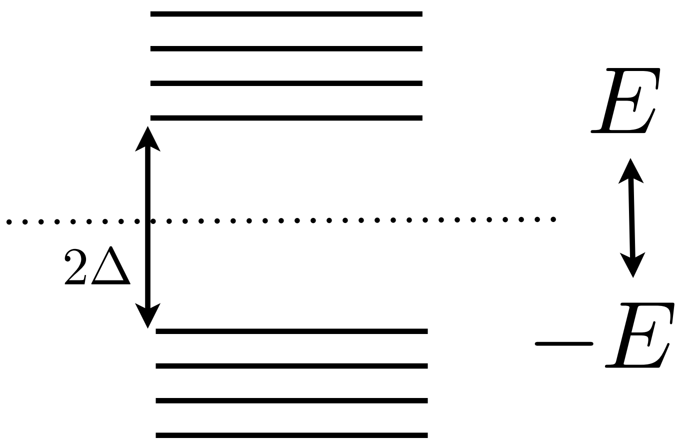
Energy eigenvalues always come in pairs.
So unpaired states are only allowed at

$$\boxed{E = 0}$$

Dirac Hamiltonian

$$H = p\,\sigma_1 + \Delta\,\sigma_2 = \begin{pmatrix} 0 & p - i\Delta \\ p + i\Delta & 0 \end{pmatrix}$$

$$E = \pm\sqrt{p^2 + \Delta^2}$$



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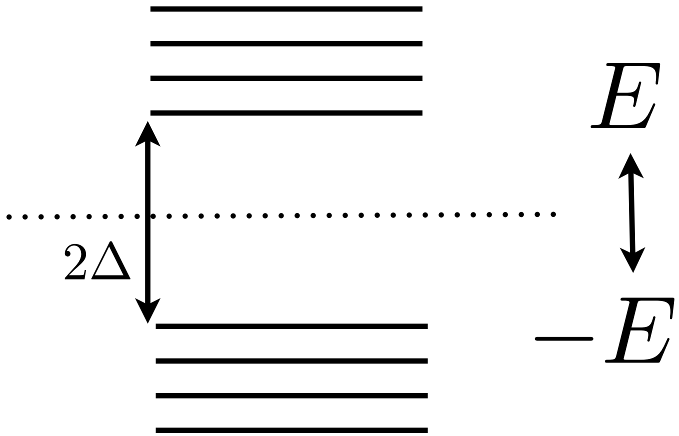
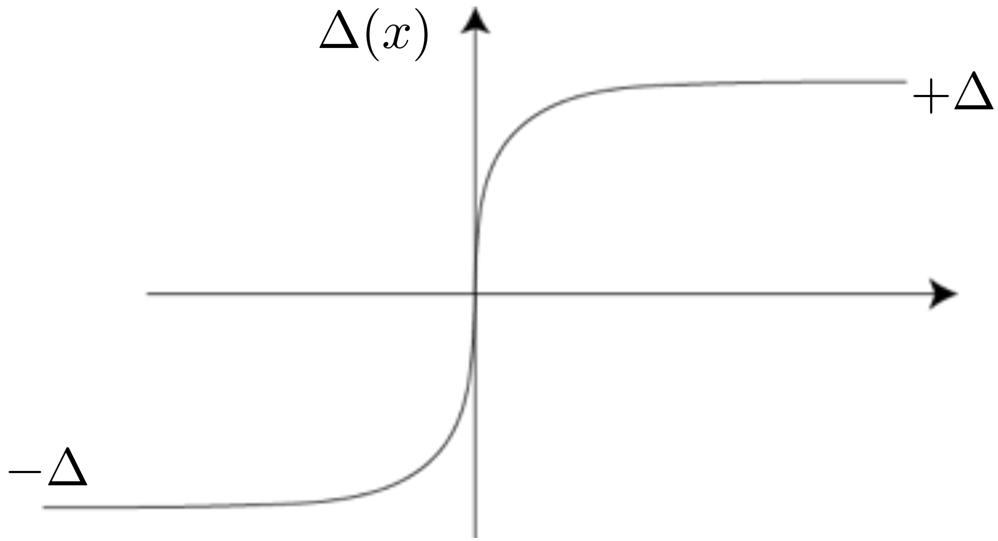


$$\sigma_3\ H\ \sigma_3 = -H$$

$$\Psi_{-E}(x) = \sigma_3\ \Psi_E(x)$$

Spatially dependent masses and zero modes

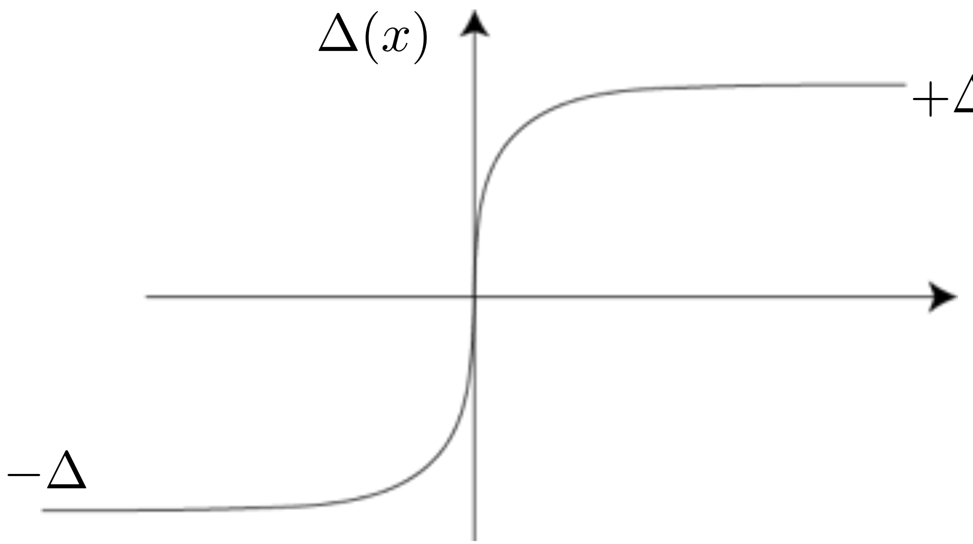
R. Jackiw and C. Rebbi, Phys Rev. D13, 3398 (1976)

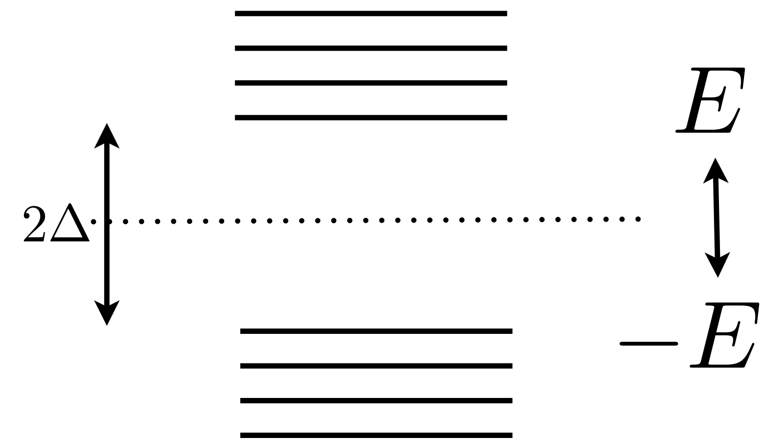


$$[-i\,\sigma_1\,\partial_x + \Delta(x)\,\sigma_2]\,\Psi = E\;\Psi$$

$E = 0 \Rightarrow$ $$\begin{pmatrix} 0 & -i\frac{\partial}{\partial x} - i\Delta(x) \\ -i\frac{\partial}{\partial x} + i\Delta(x) & 0 \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = 0$$

Zero mode is localized





$$\begin{pmatrix} 0 & -i\frac{\partial}{\partial x} - i\Delta(x) \\ -i\frac{\partial}{\partial x} + i\Delta(x) & 0 \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = 0$$

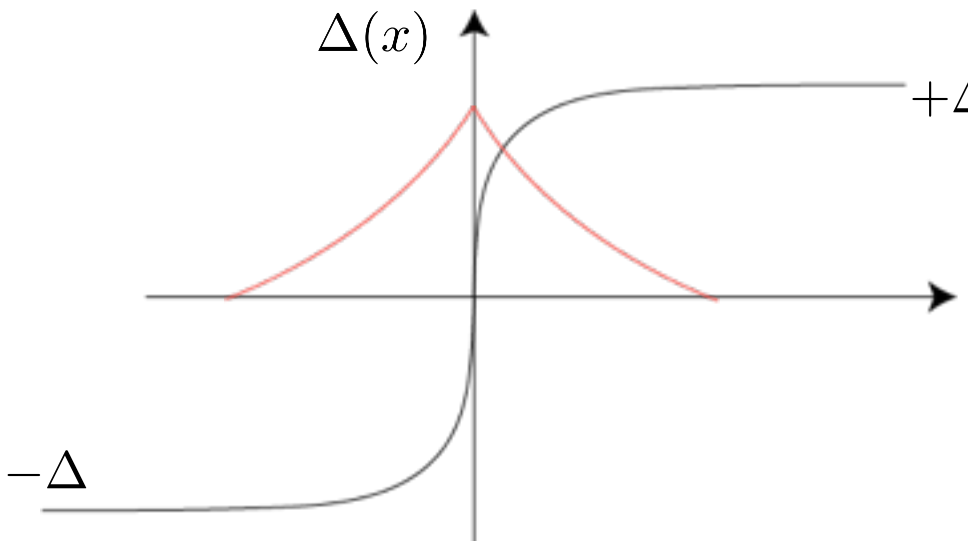
solution I

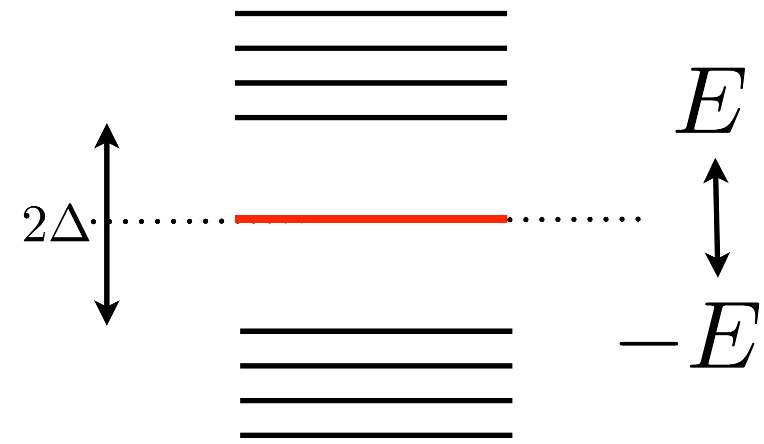
$$u(x) \propto e^{\int_0^x dx' \, \Delta(x')}$$
$$v(x) = 0$$

solution II

$$u(x) = 0$$
$$v(x) \propto e^{-\int_0^x dx' \, \Delta(x')}$$

Zero mode is localized





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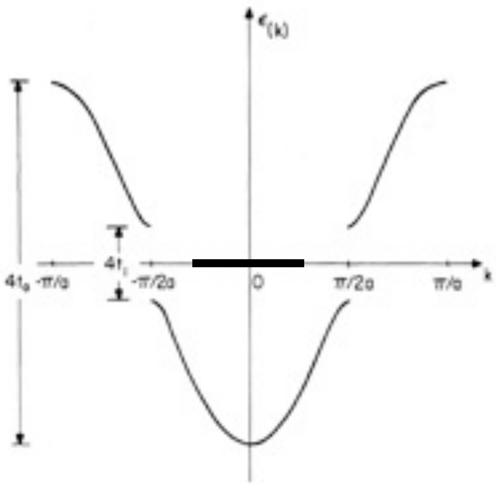
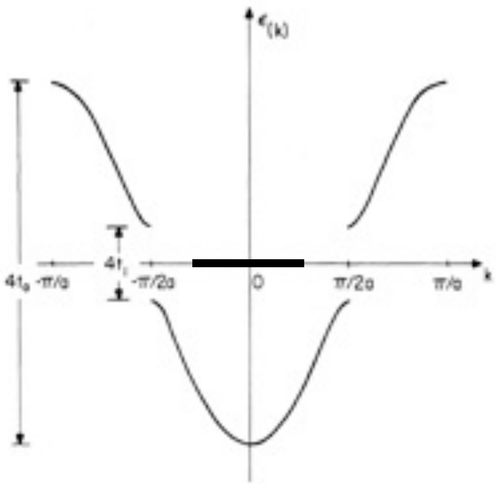


Counting the charge

$$\sum_E \rho(E,x) = \sum_E \psi_E^\dagger(x)\,\psi_E(x) = 1 \qquad \left(\text{i.e. } \sum_E \langle x|E\rangle\,\langle E|x\rangle = 1\right)$$



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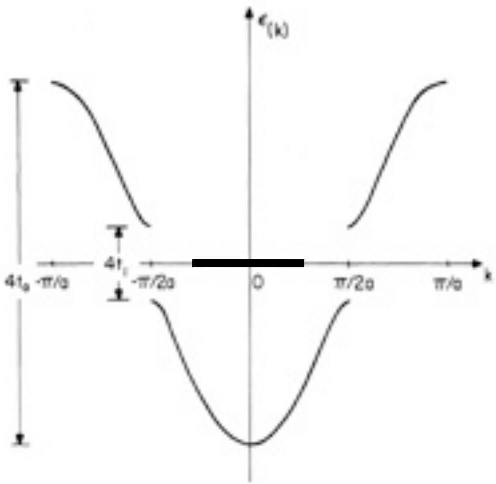
$$\sum_E \rho^{\text{kink}}(E,x) = \sum_E \rho^{\text{no kink}}(E,x)$$



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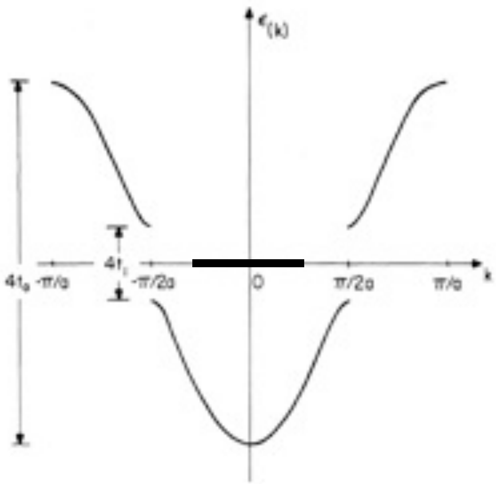
$$\sum_E \rho^{\text{kink}}(E,x) = \sum_E \rho^{\text{no kink}}(E,x)$$

$$\sum_{E\neq 0} \rho^{\text{kink}}(E,x) + |\psi_0(x)|^2 = \sum_{E\neq 0} \rho^{\text{no kink}}(E,x)$$

$$\sum_{E\neq 0} \delta\rho(E,x) = -|\psi_0(x)|^2$$



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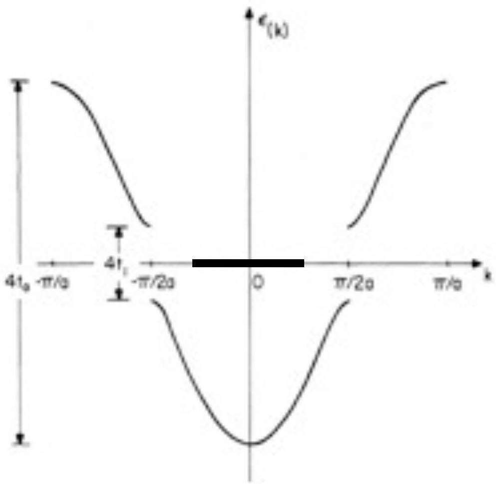
$$2\sum_{E<0} \delta\rho(E,x) = -|\psi_0(x)|^2$$



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$$2\sum_{E<0} \delta\rho(E,x) = -|\psi_0(x)|^2$$

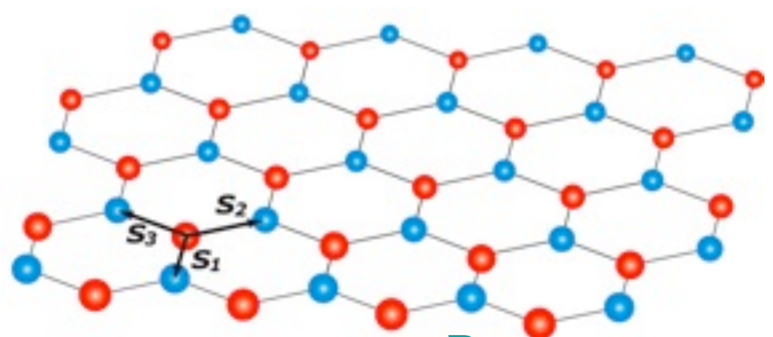
$$\delta\rho(x) = -\frac{1}{2}\,|\psi_0(x)|^2 \quad \Rightarrow \quad \boxed{Q = -1/2}$$

Fractionalization in 2D Dirac fermion systems

C.-Y. Hou, C. Chamon, M. Mudry, PRL 98, 186809 (2007)

2D Dirac fermions in condensed matter systems

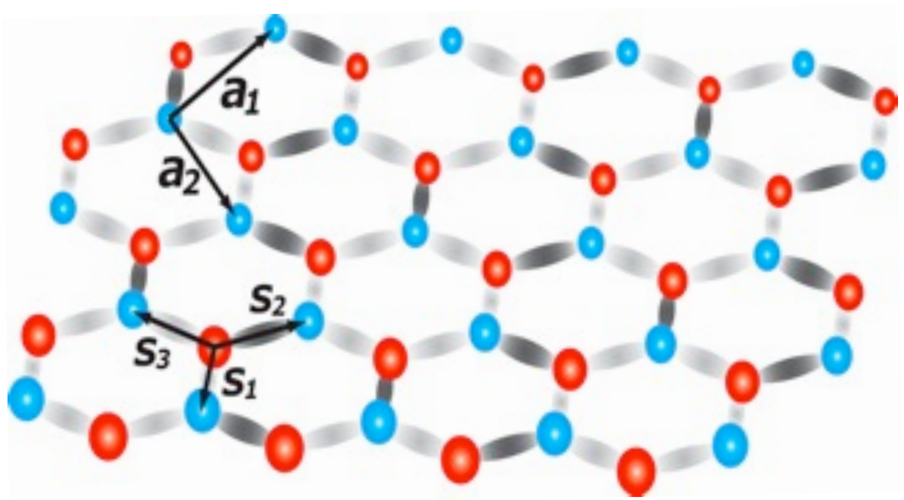
Bipartite lattices A and B - hopping between these

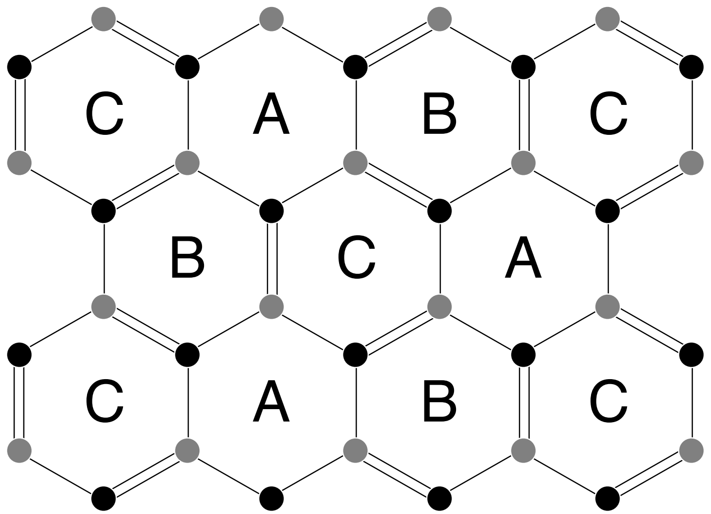


A B

The hopping texture leading to Δ:

Kekule Distortions:



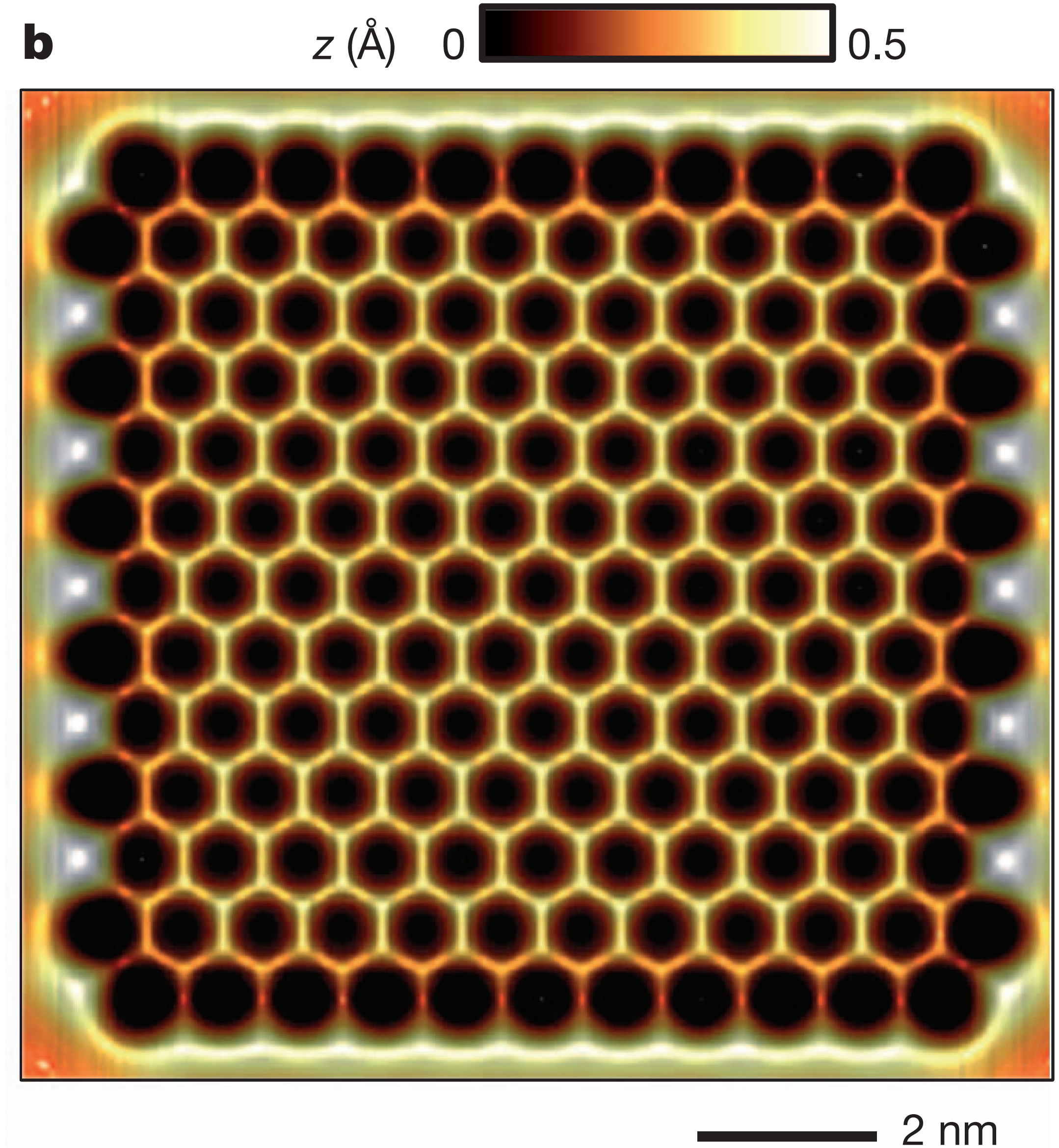


KEKULE

C. Chamon, PRB 62, 2806 (2000)

"Molecular graphene"

H. Manoharan 's lab:
K. K. Gomes et al., Nature 483, 306 (2012)



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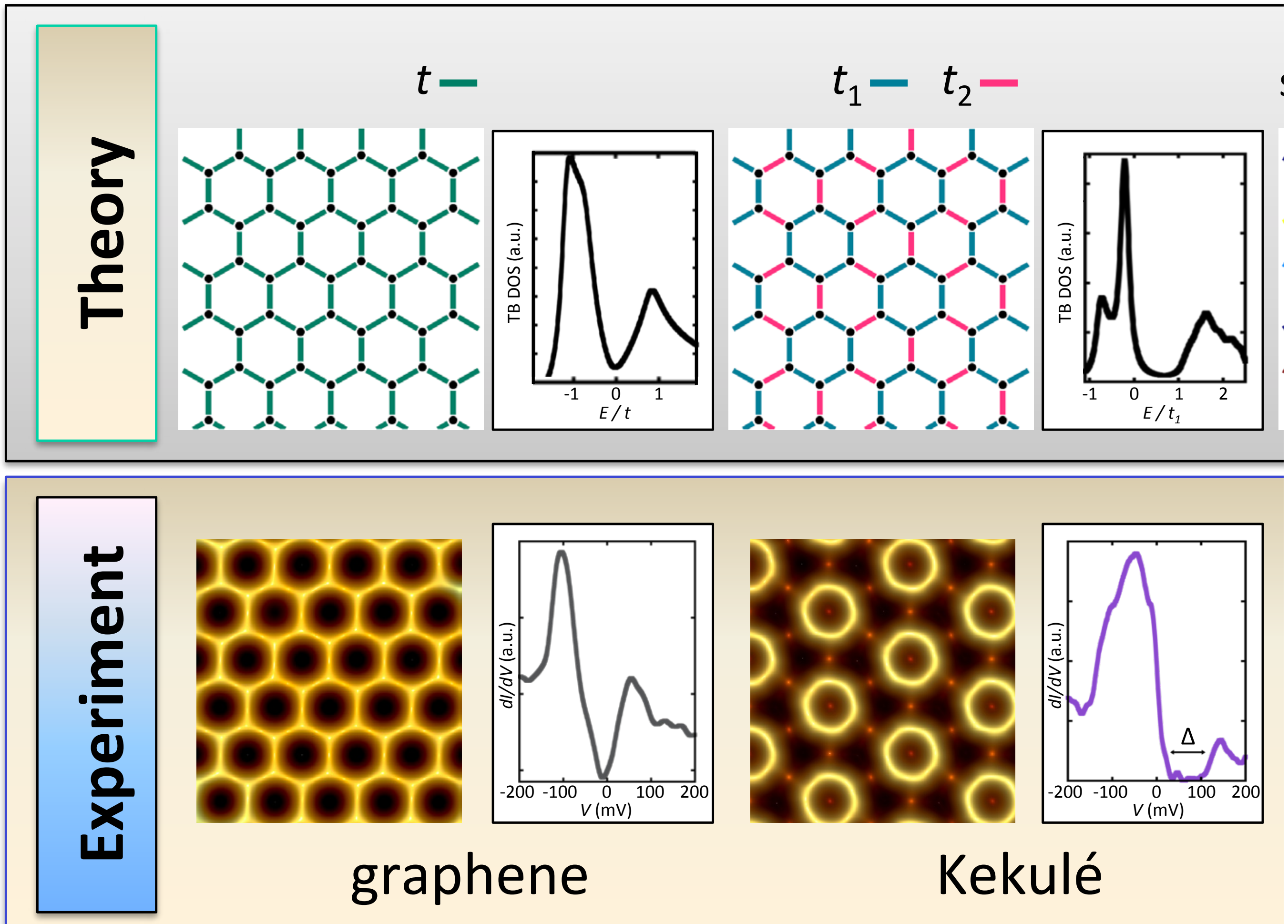
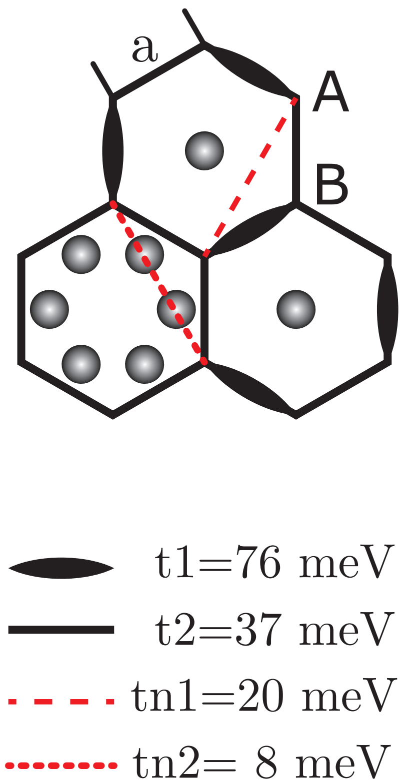
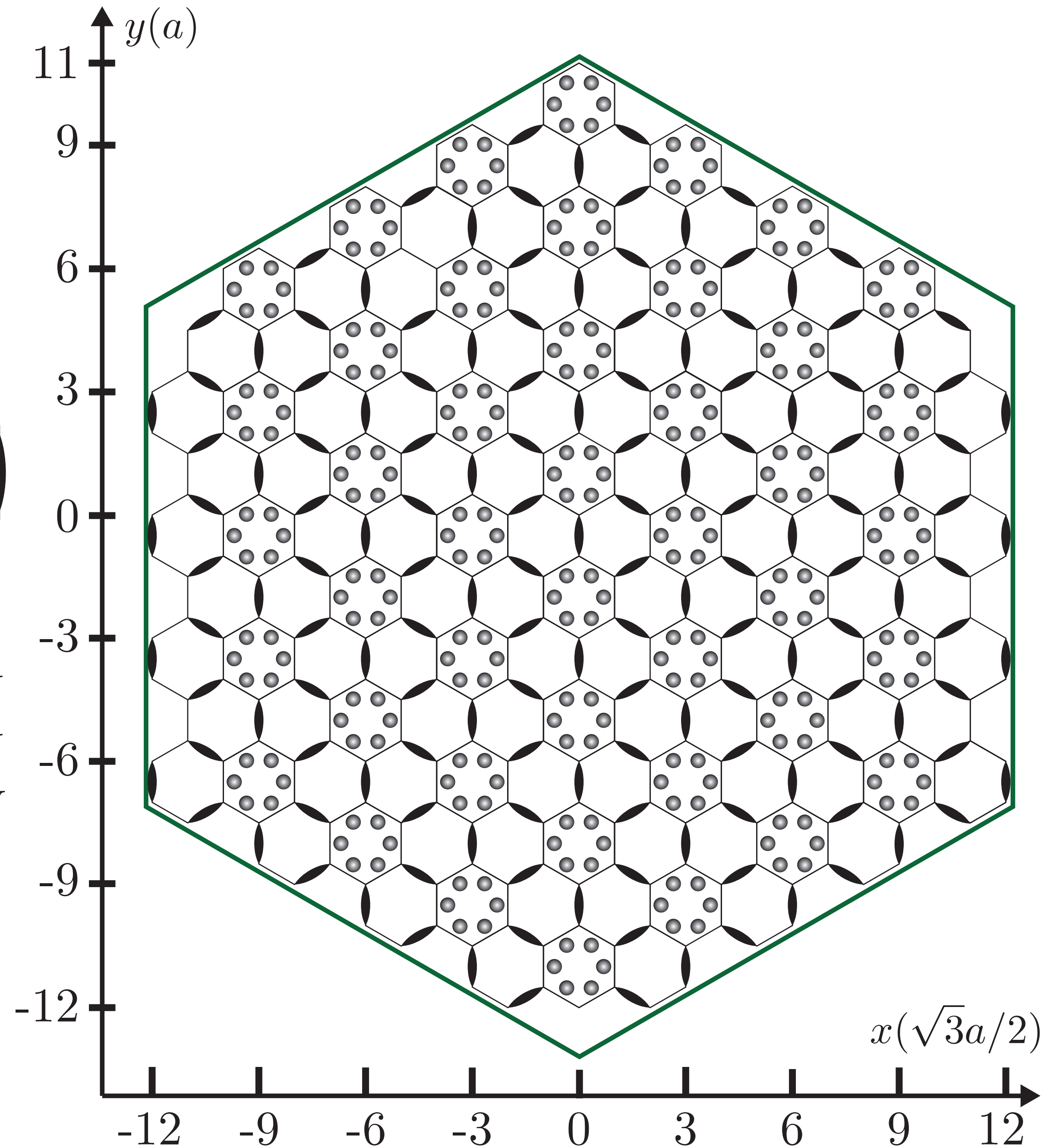


Kekulé

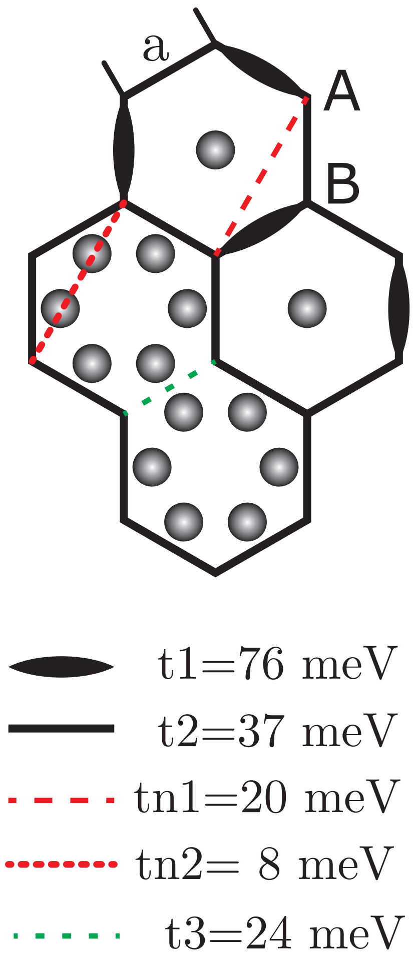


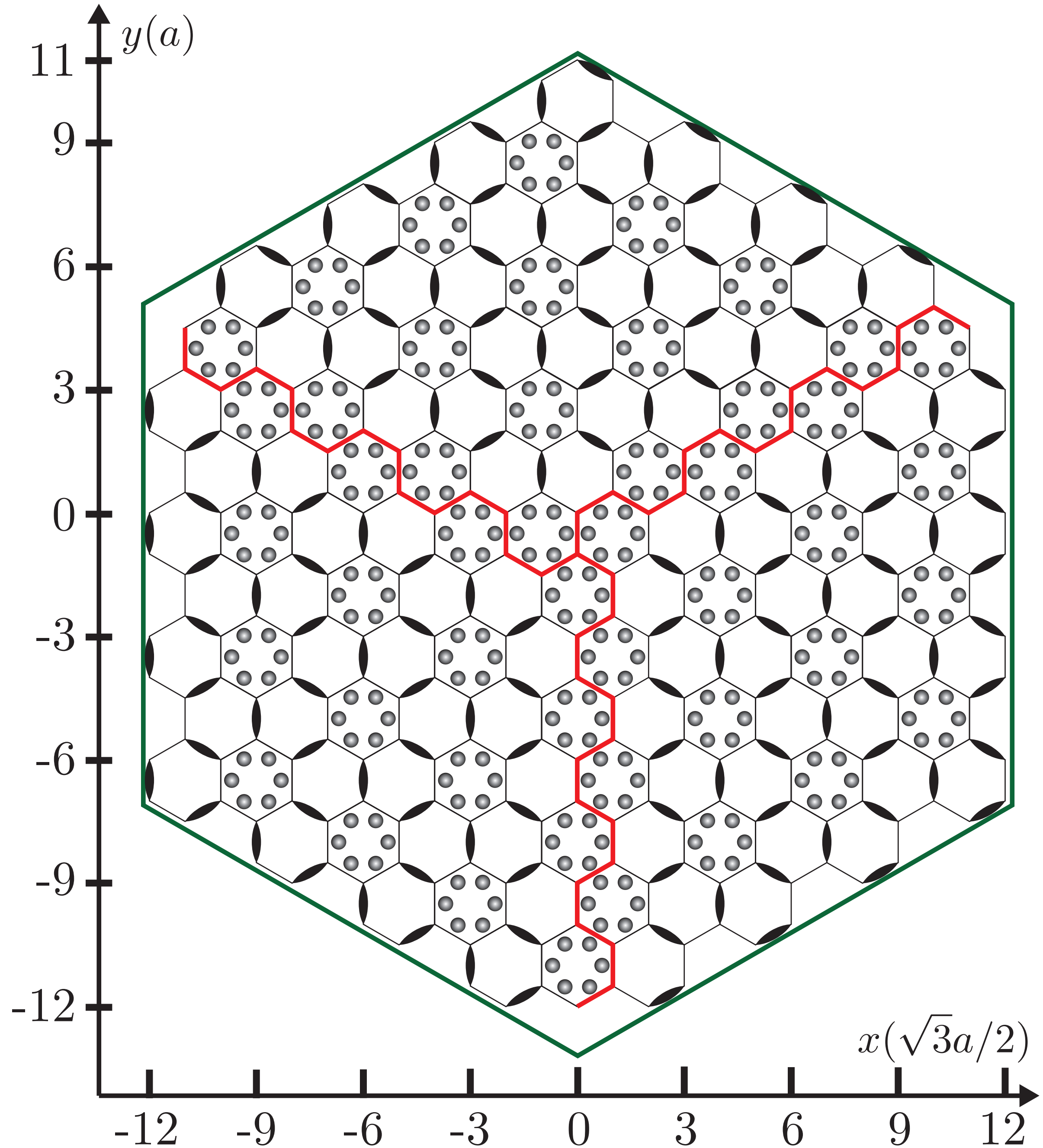
a
A
B
t1=76 meV
t2=37 meV
tn1=20 meV
tn2= 8 meV



$y(a)$
11
9
6
3
0
-3
-6
-9
-12
$x(\sqrt{3}a/2)$
-12
-9
-6
-3
0
3
6
9
12

Kekulé-vortex





A “picture” of a fractional charge