PY452 - Problem Set 10

1

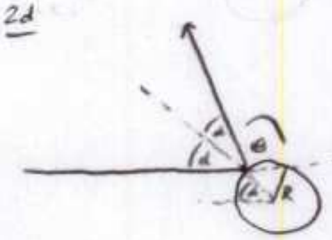
1d

$\rightleftharpoons \cdot \quad \frac{\sigma_b}{\sigma_f} = \frac{1}{0} = \infty$

In 1d. everything goes back, nothing to calculate

✓

2d



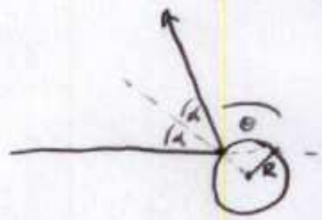
$\theta > \pi/2 \Rightarrow \pi - 2\alpha > \pi/2 \Rightarrow \alpha < \frac{\pi}{4}$

$b = R\sin(\alpha) < \frac{R}{\sqrt{2}} \Rightarrow b < R/\sqrt{2}$

$\hookrightarrow \Phi_b \sim 2\int_0^{R/\sqrt{2}} db = \frac{2R}{\sqrt{2}}$

$\Phi_f = \Phi_{tot} - \Phi_b = 2R - \frac{2R}{\sqrt{2}} \Rightarrow \frac{\Phi_b}{\Phi_f} = \frac{2R\left(\frac{1}{\sqrt{2}}\right)}{2R\left(1-\frac{1}{\sqrt{2}}\right)}$ ✓

3d



$\theta > \frac{\pi}{2} \Rightarrow \alpha < \frac{\pi}{4}, \; b = R\sin(\alpha) < \frac{R}{\sqrt{2}} \Rightarrow b < R/\sqrt{2}$

$\Phi_b \sim \int_0^{2\pi}\int_0^{R/\sqrt{2}} b\,db\,d\theta = \pi R^2\left(\frac{1}{\sqrt{2}}\right)^2$

$\hookrightarrow \Phi_f = \Phi_{tot} - \Phi_b = \pi R^2\left(1-\left(\frac{1}{\sqrt{2}}\right)^2\right)$

$\hookrightarrow \frac{\Phi_b}{\Phi_f} = \frac{\pi R^2\left(\frac{1}{\sqrt{2}}\right)^2}{\pi R^2\left(1-\left(\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{1-\left(\frac{1}{\sqrt{2}}\right)^2}$ ✓

general d

$\Phi_b \sim 2\int_0^{R/\sqrt{2}} b^{d-2}\,db \sim \left|\frac{R}{\sqrt{2}}\right)^{d-1}, \; \Phi_{tot} \sim R^{d-1}, \; \Phi_f = \Phi_{tot} - \Phi_b \sim R^{d-1}\left(1-\left(\frac{1}{\sqrt{2}}\right)^{d-1}\right)$

$\hookrightarrow \frac{\Phi_b}{\Phi_f} = \frac{R^{d-1}\left(\frac{1}{\sqrt{2}}\right)^{d-1}}{R^{d-1}\left(1-\left(\frac{1}{\sqrt{2}}\right)^{d-1}\right)} = \boxed{\frac{\left(\frac{1}{\sqrt{2}}\right)^{d-1}}{1-\left(\frac{1}{\sqrt{2}}\right)^{d-1}}}$

✓

2

a) As griffiths hints, this is non-trivial and I follow his hint of referencing Marion + Thornton:

Our potential is $V(r) = \frac{k}{r}$ with $k = \frac{q_1 q_2}{4\pi\epsilon_0}$.

Rather than copy the details of the solution, I refer you to Marion + Thornton Ch. 9. (or any other decent mechanics textbook)

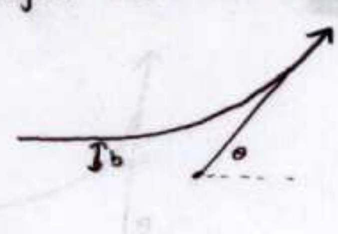
$$\hookrightarrow \boxed{b = \frac{q_1 q_2}{8\pi\epsilon_0 E}\cot(\theta/2)}$$

b) $$D(\theta) = \frac{b}{\sin(\theta)}\left|\frac{db}{d\theta}\right| = \left[\frac{q_1 q_2}{16\pi\epsilon_0 E \sin^2(\theta/2)}\right]^2 \qquad \text{(Eq. 11.4)}$$

c) $$\sigma = \int D(\theta)\sin\theta \, d\theta \, d\varphi \sim \int_0^{\pi} \frac{\sin\theta}{\sin^4(\theta/2)} d\theta$$

Near ϕ, this integrand is $\propto \frac{1}{\theta^3}$ whose integral does not converge.

d) $$N_{\text{Scattered}}(\theta) = N_{\text{Incident}}(\theta)\, n \, d\sigma(\theta)$$



$$= N_{\text{Incident}}(\theta)\, n \, D(\theta)\, d\Omega$$

$$= N_{\text{Incident}}(\theta)\, n \, D(\theta)\, 2\pi\sin(\theta)\, d\theta$$

$$\hookrightarrow \boxed{\text{fraction}(\theta) = n\, D(\theta)\cdot 2\pi \sin(\theta)\, d\theta}$$

e) $q^{\alpha} = 2e$, $q^{\text{gold}} = 79e$, $m_\alpha = 3.727 \text{ GeV}/c^2$, $\rho_{\text{gold}} = 19.3 \text{ g/cm}^3$

$$n = 19.32 \frac{\text{g}}{\text{cm}^3} \cdot 10^{-6}\text{m} \cdot \left(\frac{100\text{cm}}{1\text{m}}\right)^3 \cdot \frac{1 \text{ amu}}{1.66\cdot 10^{-24}\text{g}} \cdot \frac{1 \text{ gold}}{197 \text{ amu}} = 5.9\cdot 10^{22} \frac{\text{gold}}{\text{m}^2}$$

$$D\left(\tfrac{\pi}{2}\right) = 5.2\cdot 10^{-28}\text{ m}^2 \Rightarrow \text{Fraction}\left(\tfrac{\pi}{2}\right) = 1.93\cdot 10^{-4} \approx \boxed{1/5200}$$