PY 452 Midterm

Date: Thursday October 15, 2009, 9:30am–11:00am

This is a CLOSED book exam; no notes, calculators, or any electronic devices allowed! PLEASE EXPLAIN YOUR WORK CLEARLY to maximize the possibility of partial credit.

1. Three non-interacting particles of the same mass m are all in a one-dimensional simple harmonic oscillator potential, with total energy $E = 13\hbar\omega/2$.

   (a) If the particles are distinguishable, determine the possible occupation-number configurations (*i.e.*, give the possible sets of numbers $(n_1, n_2, \ldots)$, where n_k is the number of particles in the k^{th} energy state). Also, enumerate all the distinct three-particle states for each allowable configuration.

   (b) Repeat for the case of identical spinless Fermions.

   (c) Repeat for the case of identical spinless Bosons.

2. This problem involves first- and second-order perturbation theory.

   (a) Consider a Schrödinger Hamiltonian of the form $H_0 + \lambda H'$, in which the states $\{\psi_n\}$, $n = 1, 2, 3, \ldots$ are a complete set that are exact solutions of H_0 with corresponding energies E_n^0. Derive, to first-order in perturbation theory, the correction to the n^{th} energy eigenvalue.

   (b) Consider a free electron in the spin-up state $|\uparrow>$ (with magnetic moment $\vec{\mu} = -e\mathbf{S}/(mc)$) in the presence of a strong magnetic field $\mathbf{B} = B_z\hat{z}$ that points along the z direction. The Hamiltonian due to the field is $H_0 = -\mu_z B_z$. Suppose that a small additional magnetic field $B_x\hat{x}$ is imposed. What is the change in the electron energy to: (i) first order in the perturbation and (ii) second order in the perturbation?

   For your reference, the Pauli matrices are:

   $$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

   and the formal expression for the second-order energy correction is:

   $$E_n^2 = \sum_{m \neq n} \frac{|<\psi_m^0|H'|\psi_n^0>|^2}{E_n^0 - E_m^0}.$$

3. Consider a particle that moves in the two-dimensional simple harmonic oscillator potential with Hamiltonian

   $$H_0 = \frac{(p_x^2 + p_y^2)}{2m} + \frac{m\omega_0^2(x^2 + y^2)}{2} \, ,$$

   and with energy levels $E(n_x, n_y) = (n_x + n_y + 1)\hbar\omega$. The perturbation $H' = V\,xy$ is now imposed. Compute, to first order in the perturbation $V\,xy$:

   (a) The shift in the ground-state energy.

   (b) The energy shift of the doubly-degenerate first excited state by applying degenerate perturbation theory.

   (c) The energy shift of the triply-degenerate second excited state, following the same approach as part (b).

   Hint: The coordinate and momentum operators are related to the raising and lowering operators for the x- and y-coordinates, $a^\dagger, a$ and $b^\dagger, b$, by:

   $$x = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a) \quad y = \sqrt{\frac{\hbar}{2m\omega}}(b^\dagger + b) \qquad p_x = i\sqrt{\frac{2}{\hbar m\omega}}(a^\dagger - a) \quad p_y = i\sqrt{\frac{2}{\hbar m\omega}}(b^\dagger - b).$$

   For unperturbed oscillator states in one dimension: $a^\dagger|n>= \sqrt{n+1}\ |n+1>$ and $a|n>= \sqrt{n}\ |n-1>$.