PY 452: Quantum Physics II Problem Set 8

Due date: **Monday**, November 8 by 5:00pm

Reading: Please read chapter 9 of the text on time-dependent perturbation theory. This topic will be the focus of the lectures for the coming week.

Note: For the next few weeks, I plan to continue posting the assignments on the weekend, with the due date on the Monday of the following week.

1. Consider the expression for the transition probability $P_{\ell\to m}(t)$ that will be derived in Tuesday's lecture. Suppose that the perturbed Hamiltonian can be written in the form

$$H'(\mathbf{r},t) = V'(\mathbf{r})\,\frac{e^{-t^2/2\tau^2}}{\sqrt{2\pi\tau^2}}$$

   Compute the transition probability $P_{\ell\to m}(t)$ for the perturbing Hamiltonian given above. Discuss the dependence of the transition probability on τ. What is the value of τ that maximizes the transition probability? What is the necessary condition for the perturbation to be regarded as small?

2. Text 9.18.

3. Consider a particle of mass m in an infinite square-well potential, with $V(x) = 0$ for $0 < x < a$ and $V(x) = \infty$ otherwise. Suppose that an additional time-dependent potential $V'(x) = \lambda\left(x - \frac{a}{2}\right)\sin\omega t$ is applied to the system.

   (a) Calculate the probability the a particle in the ground state of the square-well potential makes a transition to the first excited state.

   (b) What is the probability that the particle makes a transition from the ground state to the second excited state?

   (c) What happens to the above two results as $\omega \to 0$?

4. A charged particle with charge q also experiences a one-dimensional harmonic oscillator potential. The particle is initially in the ground state of this potential. At $t = -\infty$, an electric field of the form

$$E(t) = E_0 e^{-t^2/2\tau^2}$$

   is turned on. The direction of the field is parallel to the axis of the oscillator. What is the probability that the oscillator has undergone a transition to its first excited state by $t = \infty$ in the limits of: (a) $\omega\tau \gg 1$ and (b) $\omega\tau \approx 1$. For case (b), what other transitions (if any) can occur?