PY 452: Quantum Physics II Problem Set 6

Due date: **Monday**, October 25, by 5:00pm

Reading: Please complete reading chapter 7 of the text on the variational principle.

Note: I will be travelling immediately after my class this coming Thursday 10/21 until Sunday evening 10/24. Consequently, the due date of assignment 6 is delayed until Monday 10/25 at 5pm. My office hours on Friday are cancelled, but I'll have office hours on Monday 10/25 from 2:15 until 3:30pm. If you want to see me at another time on Monday, please let me know. I apologize for any inconvenience.

I will also be away on Wednesday 10/27 for a one-day conference. I will announce this absence again in the next assignment sheet.

1. Text 7.1. In addition, generalize slightly and use the Gaussian trial wave function to estimate the ground-state energy of the n^{th}-order potential $V(x) = \alpha x^n$, with n even and $n \to \infty$. What is the limiting form of the potential in this limit?

2. Use the variational method to compute the ground state energy of the hydrogen atom using the trial wavefunction
$$\psi(r) = A\, e^{-(Br)^\nu}.$$
In class, I showed how to compute the normalization constant A. Complete the calculation for the energy with this trial wave function (I will provide a few more hints in the Tuesday lecture). Minimize this energy with respect to the free parameter B to obtain an optimal value for the ground-state energy. You should obtain
$$E_{\text{gs}} = -\frac{e^2}{a_0}\,\frac{2}{\nu+1}\,\frac{[\Gamma(2/\nu)]^2}{\Gamma(3/\nu)\Gamma(1/\nu)}\,,$$
where a_0 is the Bohr radius and $\Gamma(z)$ is the Euler gamma function. Plot E_{gs} versus ν (you may use any graphical software that you like) and show that you recover the exact ground-state energy when $\nu = 1$.

3. Text 7.9.

4. Text 7.15.