PY 452: Quantum Physics II Problem Set 4

Due date: Monday, October 4, by 5:00pm

Reading: This week will be devoted to the degenerate perturbation theory and its application to calculating the energy levels of atoms in which all corrections to the zeroth-other Coulomb potential are included. Please finish reaching chapter 6 of the text.

Note: I will be leaving for a scientific conference immediately after lecture this Thursday. Consequently, I've delayed the homework due date until the following Monday October 4. There will be no office hours this Friday; instead, I'll have an additional office hour on Monday from 2:15 until 3:30.

- 1. Calculate the shift in the energy of the n^{th} energy level of the one-dimensional harmonic oscillator in the presence of the perturbation $H' = \lambda x^4$. Perform this calculation to: (a) first-order and (b) to second-order in perturbation theory.
- 2. Text 6.6.
- 3. Text 6.7. Please note that part (c) is badly worded and the last sentence should read: "Show that with these states you obtain the first-order correction that you would have obtained by using Equation 6.9." Also in part (d), the "the theorem" refers to the one at the bottom of page 259.
- 4. Consider a more realistic model of the hydrogen atom in which the proton is treated as a uniformly charged sphere of radius R. The Coulomb potential experienced by the electron is then

$$V(r) = \begin{cases} -\frac{3e^2}{2R^3} \left(R^2 - \frac{r^2}{3} \right) & r < R \quad (\text{with } R \ll a_0) \\ -\frac{e^2}{r} & r > R \end{cases}$$

Calculate the energy shifts to the states $n = 1, \ell = 0$ and $n = 2, \ell = 0, 1$ that is caused by this perturbation.