Simple Harmonic Motion - MBL

In this experiment you will use a pendulum to investigate different aspects of simple harmonic motion. You will first examine qualitatively the period of a pendulum, as well as the position, velocity, and acceleration of the pendulum as a function of time. You will then investigate different aspects of the energy of a pendulum. A computer will be used to collect, display, and help you analyze the data.

I. Theory

In any system where there is a restoring force (remember Hooke's law: **F** = -*k***x)** or a restoring torque that tends to move the system back towards an equilibrium position, the system will tend to oscillate. When the restoring force or torque is proportional to the displacement from equilibrium, the system undergoes simple harmonic motion, where the position or angle is given by:

$x = x_{max} \cos(\Omega t)$ ….. (1) or $\theta = \theta_{max} \cos(\Omega t)$ ….. (2)

where Ω is the angular frequency of the pendulum. Notice that Ω is different from $\omega \equiv d\theta/dt = -\theta_{max}\Omega\sin(\Omega t)$, which you will examine as a function of time below besides θ. The angular frequency, Ω, is related to the frequency, f, and the period, T, by the equation:

$\Omega = 2\pi f = 2\pi / T$ …. (3)

Two examples of systems experiencing simple harmonic motion are a mass on a spring, where the spring provides the restoring force that is proportional to the displacement, and a pendulum, where the restoring torque is provided by a component of the gravitational force. [Note that for a pendulum, the restoring torque is (*almost*) proportional to the displacement for small angles, but not for large angles.]

We will investigate pendulum motion in this experiment, focusing in particular on the energy. For a simple pendulum, like a mass on a string, the period of oscillation does not depend on mass and is proportional to the square root of the length. The pendulum in this experiment is either one mass on a rod, or two masses on a rod (see Fig. 1). Because the mass of the rod cannot be neglected so this pendulum is not a simple pendulum, but you should be able to observe the change in the period as the length changes.

Diagram of the apparatus

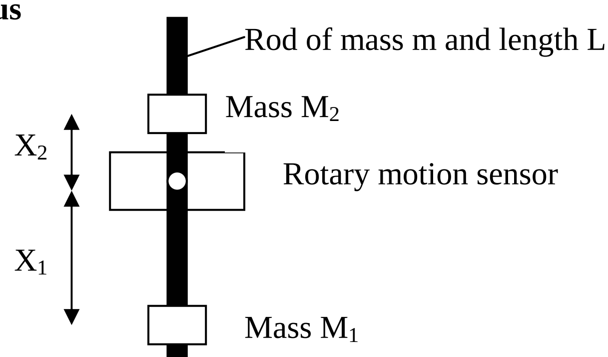


Fig. 1

PROCEDURE

Part I – Qualitative observations of the period of a pendulum.

1. Two different programs will be used in this experiment, one for parts I and II and the other for part III. For parts I and II, use the "SHM – Motion Graphs" program. This can be found in the "Intro I" folder on the desktop, and can be started by double-clicking on the "SHM – Motion Graphs" program in that folder.

NOTE: When collecting data, the computer takes the zero position to be the position the pendulum is in when the Collect button is pressed. *You should make zero the pendulum's equilibrium position.* To achieve this, follow the following steps in collecting data throughout this experiment:

- Make sure the pendulum is motionless at its equilibrium position.

- Hit the Collect button, and wait briefly until data collection begins.

- Move the pendulum away from equilibrium and release it.

- You may need to hit the STOP button after collecting data

  Feel free to try hitting the Collect button when the pendulum is already in motion, or displaced from equilibrium, to observe the problem we're trying to avoid with the steps above.

2. Measure the mass of the black rod without the screw, m, and the mass of each of the two brass masses, M_1 and M_2, including their screws.

3. The rod should be attached to the rotary motion sensor with the screw passing through the hole at the center of the rod. **Make sure that the screw is holding the rod securely in place, and that the rod does not slip as it oscillates**. Mount one of the brass masses on the rod and position it 8 cm from the center of the rod. The other mass will not be used in this part. When you stop the rod from moving, it should come to rest (or equilibrate) where the rod is vertical.

4. Measure the period, T, of the rod's oscillations. The period is the time it takes to go through one complete cycle. Any of the graphs can be used to measure the period. To increase the accuracy of your measurement you can find the time it takes for N oscillations and divide that by N. An easy way to measure period is to use the "X=?" button, which will give you the time for any point on the graph.

[Note that the program is set to take data for 10 seconds. You can adjust this if you like by going to the **Setup** menu, selecting **Data Collection**, and changing the experiment length in **Sampling**.]

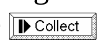
5. Find the period, T, when the mass is positioned 12 cm, and then 16 cm, from the center of the rod.

Question 1: As you increase the effective length, X_1, of the pendulum, does the period increase or decrease? Is there a linear relationship between period and $X_1^{1/2}$?

Part II – The angular position (θ), velocity (ω), and acceleration (α) of the pendulum as a function of time.

1. The rod should be attached to the rotary motion sensor with the screw passing through the center of the rod. Once again, make sure that the screw holds the rod securely in place, and that the rod does not slip as it oscillates. Mount one of the brass masses to the rod at one end. Slip the second mass on to the rod from the other end, and position it somewhere between the middle of the rod and the end. If you stop the rod from oscillating, it should equilibrate when the rod is vertical.

Question 2: The masses are mounted this way to give you control over the period of the oscillations. Suppose the bottom mass is mounted at the end of the rod, how should you position the top mass to make the pendulum oscillate relatively quickly? How should you position the top mass so the pendulum oscillates very slowly? Why? [Hint: Think about the torque acting on the pendulum and its moment of inertia]

2. Position the bottom mass 16 cm from the center of the rod. Adjust the position of the top mass so the pendulum oscillates with a period of about 1.5 seconds, which should give you nice smooth graphs. Collect data with the pendulum undergoing small-amplitude oscillations (i.e., don't displace it too far from equilibrium after hitting the 

button) and compare the three graphs.

Question 3: How do the periods of the three graphs compare? How would you describe the shape of each of the graphs?

Question 4: Compare the graph of angular position to the graph of angular velocity. Where is the pendulum when the angular velocity reaches a peak? Where is the pendulum when the angular velocity is zero?

Question 5: Compare the graph of angular position to the graph of angular acceleration. Where is the pendulum when the angular acceleration reaches a peak? Where is the pendulum when the angular acceleration is zero? When the angular position is positive, what is the sign of the angular acceleration? Why?

3. Obtain motion graphs when the pendulum is undergoing a large-amplitude oscillation.

Question 6: How do the graphs for the large-amplitude oscillations compare to those for small-amplitude oscillations?

Part III – Energy in simple harmonic motion.

1. For part III you will need a new set of graphs. Shut down the "SHM – Motion Graphs" program by clicking on the Close button in the top right corner of the screen, and double click on the "SHM – Energy" program in the Intro I folder on the desktop.

2. Kinetic energy for a rotating system is given by KE = ½ $I\omega^2$, where I is the total moment of inertia of the system and ω is the angular velocity. The angular velocity is obtained directly from the rotary motion sensor, so you don't need to worry about that. But the moment of inertia depends on how you position the two masses on the rod. The total moment of inertia of the pendulum is the sum of the moment of inertia for the rod, which has a mass m, plus that of the two masses M_1 and M_2:

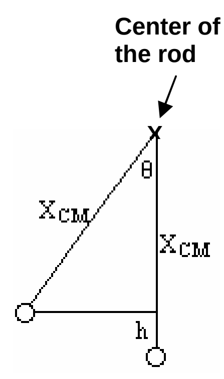
$$I = mL^2/12 + M_1X_1^2 + M_2 X_2^2$$

where L is the length of the rod, X_1 is the distance from M_1 to the center of the rod, and X_2 is the distance from M_2 to the center of the rod (see Fig. 1).

Calculate the moment of inertia for your pendulum, showing the work in your lab write-up. A typical value is about 0.00375 $kg{\cdot}m^2$. You need to insert your value into the equation the program uses to calculate the kinetic energy from the sensor measurements. To do that, go to the **Data** menu and select **Modify Column**. Choose **Kinetic Energy**, and in the box where the equation is type your value of I in place of the number 0.00375 that is there.

3. You will also have to modify the equation the program uses to find the potential energy. This time the number you need to change in the equation is the distance from the support screw to the center of mass of the system. Taking x = 0 to be the center of the rod, where the screw is, the distance from there to the center of mass is given by:

$$X_{CM} = \frac{M_1 X_1 + M_2 X_2}{m + M_1 + M_2}$$



Take the positions below the center of the rod to be positive. With this, X_1 is positive and X_2 is negative (Fig. 1). A typical value of X_{CM} is 0.0277 m.

The potential energy is given by $(m+M_1+M_2)gh$, where h is the height the center of mass of the system rising above its equilibrium position. From the drawing at right, one may see that $h = X_{CM} – X_{CM}\cos\theta$, so the potential energy is given by $PE = (m+M_1+M_2)gX_{CM}(1- \cos\theta)$.

To insert your values of the total mass and X_{CM} into the equation the program uses to calculate the potential energy, go to the **Data** menu and select **Modify Column**. Choose **Potential Energy**, and in the box where the equation is type your value of the total mass in place of 0.176 that is there, and type your value of X_{CM} in place of 0.0277 in the equation

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4. You should now be ready to investigate the energy of the pendulum. Collect some data to get graphs of the kinetic, potential, and total energy as a function of time.

Question 7: What do you expect the total energy graph to look like? Note that if it looks too unusual you may want to check your calculations in steps 2 and 3 above! Compare what you obtain for the shape of the total energy graph to what you expect to get.

Question 8: Compare the kinetic energy graph to the graph of the angular velocity. When the kinetic energy is a maximum, where is the pendulum? How do the periods compare? Explain these observations.

Question 9: Compare the potential energy graph to the graph of the angular position. When the potential energy is a maximum, where is the pendulum? How do the periods of the graphs compare? How do you explain these observations?

Question 10: How does the peak value of the potential energy compare to the peak value of the kinetic energy? What is the expected relationship between these peak values? Can you come up with any possible explanations for any difference you may observe?

5. Finally, record the data for a long time. You can adjust the time by going to the **Setup** menu, selecting **Data Collection**, and changing the experiment length in **Sampling.**

Question 11: What happens to the total energy? Does it stay constant, increase, or decrease as the pendulum oscillates over a long time interval? How can you explain this?