

# Torque and Moments of Inertia - MBL

## I. Theory

In this experiment we will determine the moment of inertia  $I$  of a steel disk by measuring its angular acceleration  $\alpha$  as a function of applied torque  $\tau$ . The three are related by

$$\text{Newton's second law for rotation: } \tau = I \alpha. \quad (1)$$

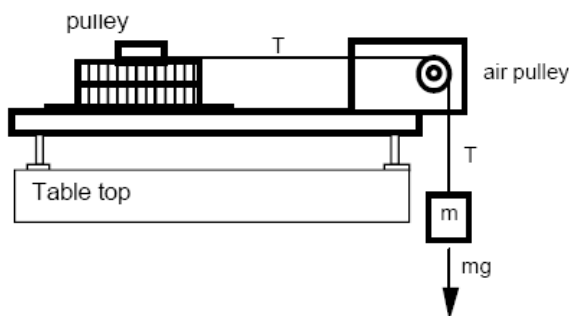


Figure 1: Side view of the Rotational Dynamics Apparatus

A diagram of the experimental apparatus is shown in Figure 1. To accelerate the system, a pulley of radius  $b$  is fastened to the top disk with a string wound around it. The string is guided over an air pulley and then allowed to hang vertically downward, held taut by a small mass  $m$  suspended from the free end of the string. The air pulley contains an air bearing that allows it to spin with very little friction. The tension in the string gives rise to a torque that causes the system to experience an angular acceleration. The torque applied by the string to the pulley is

$$\tau = T b \quad (2)$$

where  $T$  is the tension in the string caused by the mass hanging from its end. You will determine the steel disk's moment of inertia by measuring its angular acceleration for two values of pulley radius  $b$  and several values of the hanging mass  $m$ . We will neglect the small moments of inertia of the pulley on top of the steel disk and the air pulley.

The motion of the disk is measured by an optical reader that counts the number of alternating black and white stripes that pass by it in a particular time interval. The stripes are engraved on a tape glued to the outside of each disk. The reader sends the data to the computer, which graphs the angular position of the disk as a function of time. From this graph, you will determine  $\alpha$ .

Before coming to lab you should prove in your lab notebook that the disk's moment of inertia is:

$$I = \frac{mgb}{\alpha} - mb^2 \quad (3)$$

Hint – start with a free-body diagram of the weight when the system is accelerating, and combine the resulting equation you get from the free-body diagram with equations (1) and (2) above.

## II. Procedure

### Optimize the apparatus

In all experiments with this apparatus, both large disks must be used. The bottom disk can be identified by the red label on one side. This label must always face downward when the disk is slipped over the spindle. The top disk must be placed over the spindle with the hollow part of the air bearing on the bottom.

1. To minimize friction, the disk surfaces must be clean. Use the cleaning pad to do this.
2. The unit must be properly leveled; otherwise an off-center load on the disk will cause it to rotate non-uniformly. Place the bubble level on the base plate and adjust the leveling screws until the unit is level in both the x and y directions.
3. A more accurate leveling may be accomplished after the unit is set up. Turn on the air to the system and place a weight on the edge of the top disk (to produce an eccentric load). Give the disk a very small velocity, less than 0.1 revolutions per second. Watch as the disk slows down. It should continue in the same direction until it stops. If it reverses direction and oscillates back and forth, then the unit must be further leveled (it's too low at the point that the mass is oscillating about).
4. Attach one of the pulleys using an appropriate screw – **the end of the screw should never touch the bottom disk**. The thread should be attached to the pulley via the thread anchor washer. The length of the thread used should be such that the weight is close to, but not touching, the floor when the string is completely unwound.
5. Before taking any measurements you should make certain that frictional losses are not too large. To check this, do the following: first wind the thread up on the pulley until the top of the mass is level with the bottom of the air bearing bracket. Hold the disk stationary and then release it without imparting any initial velocity. The falling mass will accelerate the disk. When all the thread has unwound the rotation of the disk will wind the thread back up, reversing the direction of the mass. Ideally, with no frictional losses, the mass will return to its starting point. However, there is some friction and the mass will not come all the way back up. It should, however come within 3 to 5 centimeters of its starting point. If it does not, clean the disks with the cleaning pad, and repeat the procedure until the mass comes back to within 3-5 centimeters of its starting point.

### Part 1 – Determine the moment of inertia of the top steel disk

- (1) Measure the radii of the two pulleys to determine their values of  $b$ .
- (2) Initiate the Logger Pro program named Torque and Moments of Inertia in the Intro folder.

(3) Mount one of the pulleys on top of the steel disk. **Make sure that the smooth side of the pulley is facing down.** Wind the string on the pulley until the top of the weight is level with the bottom of the air bearing bracket. Hold the top disk stationary.

(4) Start the data collection process on the computer, and then let the disk go. You should see graphs being drawn on the computer screen. Don't worry if the bottom graph, showing angular acceleration, is not smooth – that's just because of the way the data is obtained.

(5) The best way to find the angular acceleration is to do a Quadratic Fit to the angular position graph. The equation giving the angular position as a function of time is:

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 \quad (5)$$

You must select the appropriate region for the fit, when the mass is either only moving down or only moving up. You can use the velocity graph as a guide in selecting an appropriate region.

**Question 1:** When you do a fit to the data, should you select a large time interval or a small time interval over which to do the fit? Experiment with different starting points and end points to see how sensitive the fit is to small changes in these points.

Do two fits for each trial, one when the mass is moving down and one when the mass is moving up. With each fit record the value of the coefficient corresponding to  $\frac{1}{2}\alpha$  (**don't worry about negative signs – just record the magnitude**).

**Question 2:** You should find that one of these values of  $\frac{1}{2}\alpha$  is always larger than the other. Which is larger, and why? Draw free-body diagrams of the top disk, including the frictional torque, when the mass is moving down and again when the mass is moving up, to help answer the question. By combining your two values of  $\frac{1}{2}\alpha$  appropriately you can obtain a value of  $\alpha$  that is not affected by the frictional torque. Explain how you should combine the two values, and how this compensates for the effect of the frictional torque.

(5) Use your value of  $\alpha$ , the angular acceleration, to calculate an experimental value of  $I$ , the moment of inertia of the disk.

(6) Repeat the procedure, finding  $\alpha$  from two fits to each graph and using  $\alpha$  to get  $I$ , for two different values of the pulley radius  $b$ . For each pulley radius use several different values of the mass on the string  $m$ .

(7) Measure the mass and radius of the steel disk and calculate its moment of inertia using the formula  $I = \frac{1}{2}MR^2$ . Compare this calculated value with those obtained from your measurements.

## Part 2 – Determine the moment of inertia of the ring

(1) Remove the pulley from the rotating steel disks assembly and replace it with the ring. You should find that the ring comes with a thin aluminum cover disk. Keep it on top of the ring. Stack one of the pulleys above the ring's cover disk then secure the whole assembly with a long screw. Adjust the position of the nut on the screw so that the screw is long enough to be able to reach the steel top disk but not too long to reach the steel bottom disk.

(2) Repeat the Part 2 procedure to find the angular acceleration  $\alpha$  of the system and hence the total moment of inertia  $I_{total}$  of the system. Remember that the system is now composed of the ring, the top disk of the ring and the rotating steel top disk. Use the fact that moments of inertia simply add, namely  $I_{total} = I_1 + I_2 + I_3 + \dots$ , where  $I_1$  is the moment of inertia of the object you want to measure and  $I_2, I_3, \dots$  are the moments of inertia of the other objects in the assembly. Measure the mass and radius of the cover disk of the ring and calculate its moment of inertia by using  $I = \frac{1}{2}MR^2$ . As with Part 2, do several trials with different values of  $m$ .

(3) Measure the mass and dimensions of the ring, and calculate its moment of inertia by using the equation  $I = \frac{1}{2}M(R_{outer}^2 + R_{inner}^2)$ . Compare this calculated value with those obtained from your measurements.

## Part 3 – Determine the moment of inertia of the bar

(1) Remove the pulley and the ring from the rotating assembly. Replace them with the bar and a pulley. The assembly should be in the order (from top to bottom): bar/pulley/steel top disk/steel bottom disk. Secure the assembly with the long screw. Adjust the position of the nut on the screw as needed.

(2) Repeat the Part 2 procedure to find the angular acceleration  $\alpha$  of the system and hence the total moment of inertia  $I_{total}$  of the system. Determine the moment of inertia of the bar by using  $I_{total} = I_1 + I_2 + I_3 + \dots$  as before. As with Part 2, do several trials with different values of  $m$ .

(3) Measure the mass and dimensions of the bar, and calculate its moment of inertia by using the equation  $I = ML^2/12$ . Compare this calculated value with those obtained from your measurements.

**Part 4 – Additional Questions** – Answers to all questions should be included in your lab report.

**Question 3** – For one of your trials in Part 2, determine the magnitude of the frictional torque on the disk, and compare it to the torque from the tension in the string.

**Question 4** – If, in Part 3, you had mounted the disk or bar upright on the disk instead of lying flat, would the moment of inertia have been the same, larger, or smaller? Explain.

**Question 5** – If, in Part 3, you had mounted the disk or bar off-center would the moment of inertia have been the same, larger, or smaller? Explain.

### **Part 5 – Sources of error**

There are several sources of error in the experiment. You should try to analyze these in detail to estimate how important each one is. If you think of sources of error not mentioned here, feel free to investigate those.

- Look carefully over your data to see whether you notice any trends in the values you obtained for the disk's moment of inertia. For instance, were the values always higher for one pulley versus the other? Do the values steadily increase or decrease as you use larger values of  $m$ , the mass on the string? If you notice any trends, can you come up with a possible explanation for them?
- We neglected the moment of inertia of the pulley and the air pulley when measuring the moment of inertia of the top disk. Can you estimate what the pulley's moment of inertia is? How does neglecting it affect your measured  $I$  for the top disk? What about the moment of inertia of the air pulley?