## PY105 C1

1. Help for Final exam has been posted on WebAssign.
2. The Final exam will be on Wednesday December 15th from 6-8 pm.
3. You will take the exam in multiple rooms, divided as follows:
   SCI 107: Abbasi to Fasullo, as well as Khajah
   PHO 203: Flynn to Okuda, except for Khajah
   SCI B58: Ordonez to Zhang

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## First Law of Thermodynamics

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## Thermodynamics

Thermodynamics is the study of systems involving energy in the form of heat and work.

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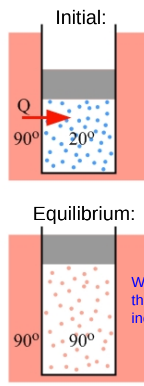
## Heat and Work done by a Gas

Consider a cylinder of ideal gas at room temperature. Suppose the piston on top of the cylinder is free to move vertically without friction. When the cylinder is placed in a container of hot water, heat is transferred into the cylinder. Where does the heat energy go?




Why does the volume increase?

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## The First Law of Thermodynamics

Some of the heat energy goes into raising the temperature of the gas (which is equivalent to raising the internal energy of the gas). The rest of it does work by raising the piston. Conservation of energy leads to:

$Q = \Delta E_{\text{int}} + W$ (the first law of thermodynamics)

***Q*** is the **heat added to a system** (or removed if it is negative)
$E_{\text{int}}$ is the internal energy of the system (the energy associated with the motion of the atoms and/or molecules).
$\Delta E_{\text{int}}$ is the change in the internal energy, and is proportional to the change in temperature.
***W*** is the **work done *by the system***.

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## The First Law of Thermodynamics

The First Law is often written as:

$$\Delta E_{\text{int}} = Q - W$$

This form of the First Law says that the change in internal energy of a system is equal to the heat supplied to the system minus the work done by the system (usually via expansion.) So, the First Law is a form of conservation of energy.

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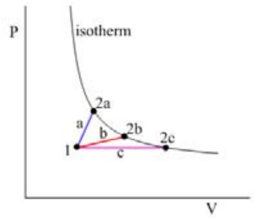
## A P-V diagram question

An ideal gas initially in state 1 progresses to a final state by one of three different processes (a, b, or c). Each of the possible final states has the same temperature.

For which process is the change in internal energy the largest?

Answer: Because the final temperature is the same, the change in temperature is the same as well. So, the change in internal energy is the same for all three processes.

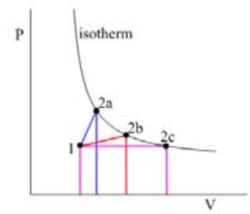





## Another P-V diagram question

An ideal gas initially in state 1 progresses to a final state by one of three different processes (a, b, or c). Each of the possible final states has the same temperature. For which process is more heat transferred into the ideal gas?

Answer: Q = ΔU + W. Since $T_f$ is the same for all three, ΔU is the same for all three. So whichever process involves more work requires more heat. Because W = the area under the P-V curve, it is clear that process c involves more work and thus it requires the most heat.






## Constant volume vs. constant pressure

We have two identical cylinders of ideal gas. Piston 1 is free to move. Piston 2 is fixed so cylinder 2 has a constant volume. We put both systems into a reservoir of hot water and let them come to equilibrium. Which statement is true?

1.Will the change in internal energy be the same for the two cylinders? If not, which will be bigger?

Ans. Since both systems undergo the same change in Temperature and they contain the same amount of gas, they have the same change in internal energy.

2.Will the change in heat be the same? If not, which will be bigger?

Ans. Work done by the gas W is nonzero in case 1 while W=0 in case 2. By the First Law of Thermodynamics, ΔQ is bigger in case 1.



## Constant volume (isochoric) process

No work is done by the gas: *W* = 0. The P-V diagram is a vertical line, going up if heat is added, and going down if heat is removed.

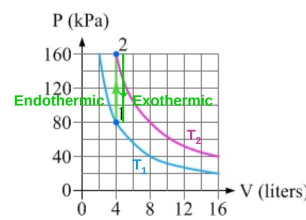
Applying the first law:

$$Q = \Delta E_{\text{int}}$$

For a monatomic ideal gas:

$$E_{\text{int}} = \frac{3}{2} nRT$$

$$Q = \Delta E_{\text{int}} = \frac{3}{2} nR\Delta T$$




Notice that $T_2 > T_1$. Why?

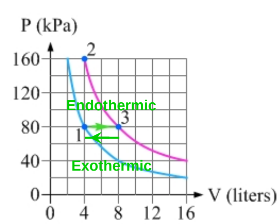


## Constant pressure (isobaric) process

In this case the region on the P-V diagram is rectangular, so its area is easy to find. $W = P\Delta V$

For a monatomic ideal gas:

$$Q = \Delta E_{\text{int}} + W$$
$$= \frac{3}{2} nR\Delta T + P\Delta V$$
$$= \frac{3}{2} nR\Delta T + nR\Delta T$$
$$= \frac{5}{2} nR\Delta T$$




$T_3 > T_1$.



## Constant-volume vs constant-pressure processes

From the above, we see that the amount of heat involved in a heating or cooling process (i.e., where the temperature of a system is changed) depends on the details of the process.

For constant-volume processes, the heat involved is the minimum since no work by the system ($P\Delta V = 0$) is involved and so *Q* = *U* + *W* = *U*.

For constant-pressure processes, the heat involved is bigger. If the system is an idea gas, we can calculate what the work done is by using $W = P\Delta V = nR\Delta T$. From that, we can determined the heat, *Q* by using *Q* = *U* + *W*.

## Heat capacity

For solids and liquids: $Q = mc\Delta T$, where $m$ is the mass of the specimen and $c$ is the specific heat per kg.

For gases: $Q = nC\Delta T$ , where $n$ is number of moles and $C$, the specific heat capacity per mole, depends on the process.



## Specific heat capacity <u>for monatomic ideal gas</u>

<u>For a ***monatomic*** ideal gas</u>

Constant volume: $Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$, so $C_V = \frac{3}{2}R$

Constant pressure: $Q = nC_P\Delta T = n\left(\frac{5}{2}R\right)\Delta T$, so $C_P = \frac{5}{2}R$

where $n$ is the number of moles of molecules contained in the gas.



## Specific heat capacity <u>for general ideal gas</u>

<u>For a ***general*** ideal gas</u>:

$\Delta \boldsymbol{E}_{int}$ per molecule = $(n_f + 3/2)kT$ or $\Delta \boldsymbol{E}_{int} = (n_f + 3/2)nRT$ per mole, where $n_f$ is the number of additional degrees of freedoms the gas has besides translational degrees of freedom.

So, $C_V = Q/n = \Delta \boldsymbol{E}_{int} / n$
Or, $C_V = (n_f + 3/2)RT$
and $C_P = C_V + R$

from $W = P\Delta V = nRT$



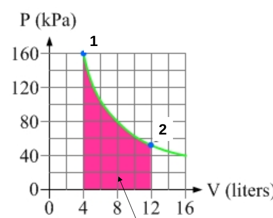
## Constant temperature (isothermal) process

No change in internal energy: $\Delta E_{int} = 0$
The P-V diagram follows the isotherm.

Applying the first law, and using a little calculus:

$$Q = W = nRT\ln\left(\frac{V_f}{V_i}\right)$$

From this equation, Q > 0 (endothermic) if $V_f > V_i$, but < 0 (exothermic) if $V_f < V_i$. What is the physical explanation for this?




Area under an isotherm = $nRT\ln(V_f/V_i)$



## Zero heat (adiabatic) process

When <u>$Q = 0$</u>, the P-V diagram is an interesting line, given by:

$PV^{\gamma} = \text{constant}$

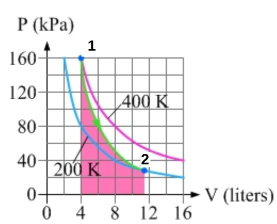
$\gamma = \frac{C_P}{C_V}$

For a monatomic ideal gas:

$$\gamma = \frac{C_P}{C_V} = \frac{5R/2}{3R/2} = \frac{5}{3}$$

Applying the first law:

$\Delta E_{int} = -W$




Note that W = area under the P-V curve is > 0 in the process 1→2, but < 0 in 2→1.



## An Ideal Gas with Fixed Number of Atoms

A container of monatomic ideal gas contains just the right number of moles so that nR = 20 J/K. The gas is initially in state 1 where

$P_1 = 20$ kPa, $V_1 = 100 \times 10^{-3}$ m$^3$

(a) What is the initial temperature $T_1$ of the gas?

If $Q = 2500$ J of heat is added to the gas, which expands at constant pressure and reaches a new equilibrium state.

(b) What is the final temperature $T_2$?
(c) How much work is done by the gas?
(d) What is the final volume $V_2$?

## An Ideal Gas with Fixed Number of Atoms

Solution

(a) Use the ideal gas law, $PV = nRT$, so:

(20 kPa)(100 x $10^{-3}$ m$^3$) = (20 J/K)$T_1 \Rightarrow T_1$ = 100 K

n.b. The factor of 1000 in the kPa cancels the factor of $10^{-3}$ in the volume.

(b) With constant pressure for a monatomic ideal gas

$$Q = \Delta E_{int} + W = \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T$$

$$\Rightarrow \Delta T = \frac{2}{5}\frac{Q}{nR} = \frac{1000 \text{ J}}{20 \text{ J/K}} = 50\text{K}$$

$$T_2 = T_1 + \Delta T = 100 \text{ K} + 50 \text{ K} = 150 \text{ K}$$

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## An Ideal Gas with Fixed Number of Atoms

(c) With constant pressure,

$$W = P\Delta V = nR\Delta T = (20 \text{ J/K})(+50 \text{ K}) = +1000 \text{ J}$$

(d) By the ideal gas law,

$$V_2 = \frac{nRT_2}{P_2} = \frac{(20 \text{ J/K}) \times 150 \text{ K}}{20 \times 10^3 \text{ Pa}} = 150 \times 10^{-3} \text{ m}^3$$

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## A Three-Step Process

A thermodynamic system undergoes a three-step process. An adiabatic expansion takes it from state 1 to state 2; then heat is added at constant pressure to move the system to state 3; and finally, an isothermal compression returns the system to state 1. The system consists of a ***diatomic*** ideal gas with $C_V = 5R/2$.

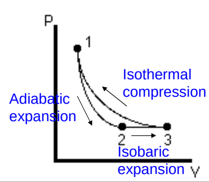
The number of moles is chosen so $nR$ = 100 J/K.

The following information is known about states 2 and 3.

Pressure: $P_2 = P_3$ = 100 kPa

Volume: $V_3$ = 0.5 m$^3$

(a) What is the temperature of the system in state 3?

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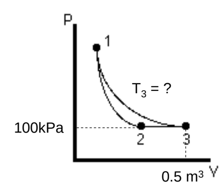
## A Three-Step Process

(a) $nR$ = 100 J/K; $P_2 = P_3$ = 100 kPa; $V_3$ = 0.5 m$^3$; $T_3$ = ?

Solution

By the Ideal Gas Law,

$$T_3 = \frac{P_3V_3}{nR} = \frac{(100 \times 10^3 \text{ Pa}) \times 0.5 \text{ m}^3}{100 \text{ J/K}} = 500 \text{ K}$$

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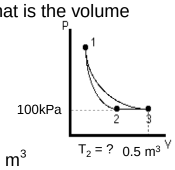
## A Three-Step Process

(b) The system does 20000 J of work in the constant pressure process that takes it from state 2 to state 3. What is the volume and temperature of the system in state 2?




Solution

For constant pressure, we can use:

$$W = P\Delta V = nR\Delta T$$

Finding volume: $\Delta V = \frac{W}{P} = \frac{+20000 \text{ J}}{100000 \text{ Pa}} = +0.2 \text{ m}^3$

$$\Delta V = V_3 - V_2 \quad \text{so} \quad V_2 = V_3 - \Delta V = +0.5 \text{ m}^3 - 0.2 \text{ m}^3 = 0.3 \text{ m}^3$$

Finding temperature: (use the ideal gas law, or …)

$$\Delta T = \frac{W}{nR} = \frac{+20000 \text{ J}}{100 \text{ J/K}} = +200\text{K}$$

$$\Delta T = T_3 - T_2 \quad \text{so} \quad T_2 = T_3 - \Delta T = 500\text{K} - 200\text{K} = 300\text{K}$$

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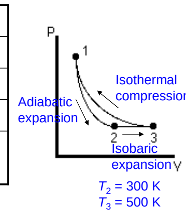
## A Three-Step Process

### Complete the table

Given that the total work done by the system in the cycle is –19400 J. Complete the table.

| Process | Q | ΔE_int | W |
|---|---|---|---|
| 1 to 2 | | | |
| 2 to 3 | | | +20000 J |
| 3 to 1 | | | |
| Entire cycle | | | -19400 J |




First fill in all the terms that are zero.

Each row satisfies the First Law of Thermodynamics.

Also remember that $C_V = 5nR/2$

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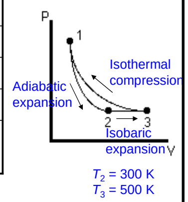
## A Three-Step Process

### Complete the table

For the same system, complete the table. The total work done by the system in the cycle is –19400 J.

| Process | *Q* | $\Delta E_{int}$ | *W* |
|---|---|---|---|
| 1 to 2 | 0 | | |
| 2 to 3 | | | +20000 J |
| 3 to 1 | | 0 | |
| Entire cycle | | 0 | -19400 J |




*Q* is zero for an adiabatic process.
The change in internal energy is zero for an isothermal process, and is always zero for a complete cycle.

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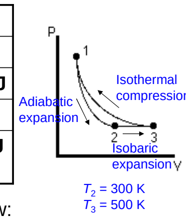
## A Three-Step Process

### Complete the table

For the same system, complete the table. The total work done by the system in the cycle is –19400 J.

| Process | *Q* | $\Delta E_{int}$ | *W* |
|---|---|---|---|
| 1 to 2 | 0 | | |
| 2 to 3 | | | +20000 J |
| 3 to 1 | | 0 | |
| Entire cycle | -19400 J | 0 | -19400 J |




Even the last row has to satisfy the first law:

$$Q = \Delta E_{\rm int} + W$$

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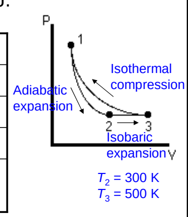
## A Three-Step Process

### Complete the table

For the same system, complete the table. The total work done by the system in the cycle is –19400 J.

| Process | *Q* | $\Delta E_{int}$ | *W* |
|---|---|---|---|
| 1 to 2 | 0 | | |
| 2 to 3 | | +50000 J | +20000 J |
| 3 to 1 | | 0 | |
| Entire cycle | -19400 J | 0 | -19400 J |




Find the change in internal energy for the 2 → 3 process.

$$\Delta E_{\rm int} = nC_V\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}(100\text{ J/K})(+200\text{K}) = +50000\text{ J}$$

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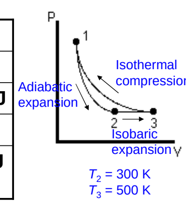
## A Three-Step Process

### Complete the table

For the same system, complete the table. The total work done by the system in the cycle is –19400 J.

| Process | *Q* | $\Delta E_{int}$ | *W* |
|---|---|---|---|
| 1 to 2 | 0 | -50000 J | |
| 2 to 3 | +70000 J | +50000 J | +20000 J |
| 3 to 1 | | 0 | |
| Entire cycle | -19400 J | 0 | -19400 J |




Rows have to obey the first law.
Columns have to sum to the value for the entire cycle.

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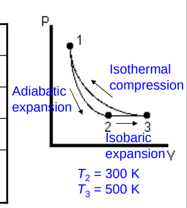
## A Three-Step Process

### Complete the table

For the same system, complete the table. The total work done by the system in the cycle is –19400 J.

| Process | *Q* | $\Delta E_{int}$ | *W* |
|---|---|---|---|
| 1 to 2 | 0 | -50000 J | +50000 J |
| 2 to 3 | +70000 J | +50000 J | +20000 J |
| 3 to 1 | -89400 J | 0 | -89400 J |
| Entire cycle | -19400 J | 0 | -19400 J |




Rows have to obey the first law.
Columns have to sum to the value for the entire cycle.

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## Three Ways of Adding The Same Heat, Q

An ideal gas is contained in a cylinder, which is sealed at the top by a piston that can move up or down, or can be fixed in place to keep the volume constant. The weight of the piston can be adjusted to adjust the pressure as necessary. Starting with the same initial conditions, you do three experiments, each involving adding the same amount of heat, *Q*.

A – Add the heat at constant pressure.

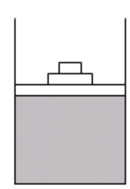
B – Add the heat at constant temperature.

C – Add the heat at constant volume.

(a)Rank the processes by the final temperature.

(b)Rank the processes by the work.

(c)Rank the processes by the final pressure.



## Three Ways of Adding The Same Heat, Q

Solution:

(a) Rank by final temperature:

$\Delta U = nC_V(T_f - T_i) = Q - W$. Hence the bigger $Q - W$ is, the bigger $T_f$ would be.

In process B (constant $T$), there is no change in $T$.

In process A (constant $P$), some of the heat added goes to doing work.

In process C (constant $V$), the gas does no work and all the heat added goes to increasing the temperature.

C > A > B

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## Three Ways of Adding The Same Heat, Q

(b) Rank by work:

In process C (constant $V$), no work is done.

In process A (constant $P$), $W = Q - \Delta E_{int} \Rightarrow$ only some of the heat added goes to doing work.

In process B (constant $T$), $W = Q - \Delta E_{int} = Q - C_V\Delta T = Q \Rightarrow$ all the heat added goes to doing work.

B > A > C

(c) Rank by final pressure:

$P_f = nRT_f/V_f$

In process A (constant $P$), the pressure stays constant.

In process B (constant $T$), $W = Q > 0 \Rightarrow \Delta V > 0$. With $T$ constant, the pressure must decrease.

In process C (constant $V$), $\Delta E_{int} = Q > 0 \Rightarrow \Delta T > 0$. With $V$ constant, the pressure must increase.

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C > A > B