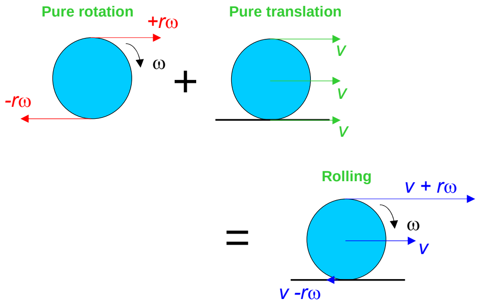
Rolling

Rolling

Rolling simulation

We can view rolling motion as a superposition of pure rotation and pure translation.



Rolling

Rolling simulation

The above picture shows that

(1) the velocity of a point at the center of mass of the disc is v, the translation velocity.

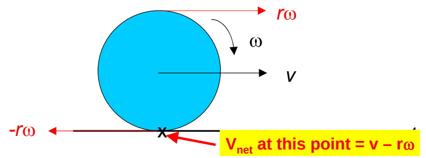
(2) the velocity of a point at the top of the disc is v + $r\omega$.

(3) the velocity of a point at the bottom of the disc (where the disc touches the ground) is v – $r\omega$.

Condition for Rolling Without Slipping

When a disc is rolling without slipping, the bottom of the disc is always at rest instantaneously. This leads to $\omega = v/r$ and $\alpha = a/r$

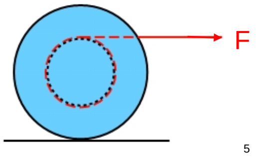
where *v* is the translational velocity and *a* is acceleration of the center of mass of the disc.



Big yo-yo

A large yo-yo stands on a table. A rope wrapped around the yo-yo's axle, which has a radius that's half that of the yo-yo, is pulled horizontally to the right, with the rope coming off the yo-yo above the axle. In which direction does the yo-yo move? There is friction between the table and the yo-yo. Suppose the yo-yo is pulled slowly enough that the yo-yo does not slip on the table as it rolls.

[✓ 1.] to the right
2. to the left
3. it won't move



Big yo-yo

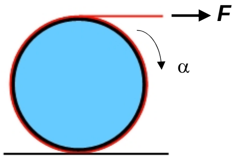
Since the yo-yo rolls without slipping, the center of mass velocity of the yo-yo must satisfy, $v_{cm} = r\omega$. With this, the direction of v_{cm} (= direction of motion) is determined by whether the yo-yo is rolling clockwise or counter-clockwise.

In this example, the tension on the rope produces a clockwise torque, which would cause the yo-yo to roll clockwise. So the yo-yo will move to the right.

An accelerating cylinder

A cylinder of mass *M* and radius *R* has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force *F*, what is the acceleration of the cylinder if the cylinder rolls without slipping? (Hint: Consider the direction of the frictional force and whether it will assist or resist *F*.)

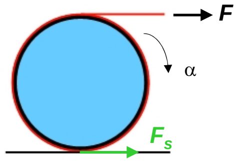
1. $a = \frac{F}{m}$
2. $a < \frac{F}{m}$
[✓ 3.] $a > \frac{F}{m}$



An accelerating cylinder

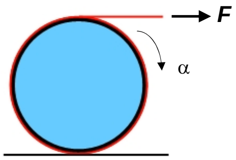
We would expect the frictional force to be pointing forward since the tensional force would produce a torque that rotates the cylinder clockwise, which would produce a tendency of the bottom of the cylinder to move backward relative to the ground. The net force on the cylinder, being the sum of the frictional force and the tensional force, would thus be bigger than the tensional force alone. Notice that the friction is static friction since there’s no slip or the cylinder is momentarily at rest relative to ground.

Simulation



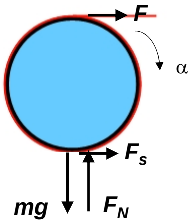
An accelerating cylinder – Finding a and F_s

A cylinder of mass *M* and radius *R* has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force *F*, what is the acceleration of the cylinder if the cylinder rolls without slipping? What is the frictional force acting on the cylinder? Use *I* = ½ MR^2.



An accelerating cylinder

First, draw a free-body diagram for the cylinder



An accelerating cylinder – Finding a and F_s

Take positive to be right, and clockwise positive for torque.

The normal force cancels *Mg* vertically. Apply Newton's Second Law for horizontal forces, and for torques:

Forces

$$\sum \vec{F}_x = M\vec{a}$$

$$+F + F_S = Ma$$

Torques

$$\sum \vec{\tau} = I\alpha$$

$$+RF - RF_S = I\alpha = \underbrace{\frac{1}{2}MR^2}_{I}\left(\frac{a}{R}\right)$$

$$F - F_S = \frac{1}{2}Ma$$

* Only for rolling without slipping can we use $\alpha = \frac{a}{R}$

An accelerating cylinder – Finding a and Fs

Forces

$$+F + F_S = Ma$$

Torques

$$F - F_S = \frac{1}{2}Ma$$

Adding these two equations gives $2F = \frac{3}{2}Ma$,

which leads to the result: $a = \frac{4F}{3M}$

We can make sense of this by solving for the force of static friction from the equation at the top left.

$$F_S = Ma - F = + \frac{1}{4}Ma = + \frac{1}{3}F$$

The positive sign means that our initial guess for the direction of the friction force being in the same direction as ***F*** is correct.

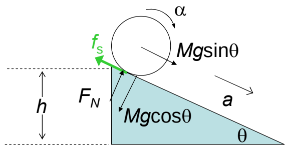
Racing Shapes

We have three objects, a solid disk, a ring, and a solid sphere, all with the same mass, *M* and radius, *R*. If we release them from rest at the top of an incline, which object will win the race? Assume the objects roll down the ramp without slipping.

[✓ 1.] The sphere
2. The ring
3. The disk
4. It's a three-way tie
5. Can't tell - it depends on mass and/or radius.

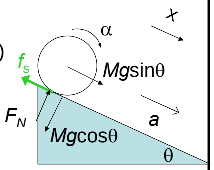
Racing Shapes

<u>Question:</u> For the situation considered in the previous question, find the frictional force for various shapes. Use your result to explain why the sphere should win the race.



Racing Shapes

Forces (in x)	**Torques**
$Mg\sin\theta - f_s = Ma$	(1): $f_sR = cMR^2\underbrace{(a/R)}_{\alpha}$
(2): $g\sin\theta - f_s/M = a$	(3): $f_s/cM = a$



(2) and (3) $\Rightarrow g\sin\theta - f_s/M = f_s/cM$

$\Rightarrow Mg\sin\theta = f_s(1/c+1)$

$\Rightarrow f_s = Mg\sin\theta/(1/c+1)$

Equation (2) shows that f_s reduces the linear acceleration a. At the same time, equation (3) shows that the shape with the biggest moment of inertia requires the biggest f_s for a given α or a (= rα). So the sphere, with the smallest moment of inertia, accelerates the fastest down the incline.

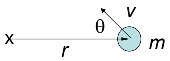
A race

If we take the winner of the rolling race (the sphere) and race it against a frictionless block, which object wins the race? Assume the sphere rolls without slipping.

1. The sphere
[✓ 2.] The block
3. It's a tie
4. Can't tell

Angular momentum

The angular momentum of a spinning object is represented by *L*.



1. $\boxed{\vec{L} = I\vec{\omega}}$.

If the object is a point mass, $I = mr^2$ and $L = mr^2\omega = mrv\sin\theta$, where θ is the angle between *v* and *r* so that $v\sin\theta$ is the tangential velocity.

2. Angular momentum is a vector, pointing in the direction of the angular velocity. So, it's clockwise or counter-clockwise.

3. If there's no net torque acting on a system, the system's angular momentum is conserved.

4. A net torque produces a change in angular momentum that is equal to the torque multiplied by the time interval over which the torque was applied.

A Figure Skater

A spinning figure skater is an excellent example of angular momentum conservation. The skater starts spinning with her arms outstretched, and has a rotational inertia of I_i and an initial angular velocity of ω_i. When she moves her arms close to her body, she spins faster. Her moment of inertia decreases, so her angular velocity must increase to keep the angular momentum constant.

Conserving angular momentum:

$$\vec{L}_i = \vec{L}_f$$

$$I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

http://www.youtube.com/watch?v=9921LChDlBc&feature=related at 0:28, 1:04
http://www.youtube.com/watch?v=O9B4CBcFlYw at 2:17

A Bicycle Wheel

Question: A person standing on a turntable while holding a *spinning* bicycle wheel is an excellent place to observe angular momentum conservation in action. Initially, the bicycle wheel is spinning about a horizontal axis, and the person is at rest. Can you predict what happens when the person flips the wheel to bring the rotational axis vertical?

Solution: The initial angular momentum about a horizontal axis contributes no angular momentum to the turntable, which can rotate about a vertical axis only. If the person re-positions the bicycle wheel so its rotation axis becomes vertical, the person must spin (on the turntable) in the opposite direction to maintain the total angular momentum (about the turntable's rotational axis) zero at all times.

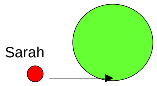
Flipping the bike wheel over makes the person spin in the opposite direction.

Jumping on a Merry-go-round

This is an example involving a rotational collision:

Question: Sarah, with mass m and velocity v, runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah and the merry-go-round (mass M, radius R, and $I = cMR^2$) then spin together with a constant angular velocity ω_f. If Sarah's initial velocity is tangent to the circular merry-go-round, what is ω_f?

Simulation



Solution:

What concept should we use to attack this problem?

Conservation of angular momentum.

Jumping on a Merry-go-round

The system clearly has angular momentum after the completely inelastic collision, but where is the angular momentum beforehand?

It's with Sarah. Sarah's linear momentum can be converted to an angular momentum relative to an axis through the center of the merry-go-around. Here, we can treat Sarah as a point mass with mass m and initial velocity v_i.

$L_i = rmv_i\sin\theta$, where θ is the angle between r and v_i.

In this case, $\theta = 90^o$ and the angular momentum is directed counter-clockwise.

$L_i = Rmv_i\sin(90^o) = Rmv_i$

Jumping on a Merry-go-round

Conserving angular momentum: $\vec{L}_i = \vec{L}_f$

Let's define counterclockwise to be positive.

$$+Rmv_i = +I_{tot}\omega_f$$

$$+Rmv_i = +(cMR^2 + mR^2)\omega_f$$

Solving for the final angular speed: $\omega_f = \dfrac{mv}{cMR + mR}$