Collisions

Conservation of Linear Momentum in Collisions involved in Isolated Systems

Recall that linear momentum is conserved in isolated systems. Almost all collisions we encounter in this course are isolated. So, we can almost always assume that the total linear momentum is conserved. Mathematically,

$$m_1v_{1,i} + m_2v_{2,i} + \ldots = m_1v_{1,f} + m_2v_{2,f} + \ldots$$

Four Kinds of Collisions

However, the kinetic energy is not necessarily conserved. There are four possible cases.

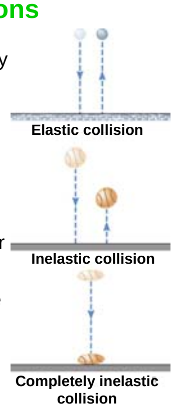
DEFINITIONS

Elastic collision -- One in which the total kinetic energy of the system (K) is the same before and after the collision.

Super elastic collision – One in which K after the collision is bigger than that before.

Inelastic collision -- One in which K after the collision is less than that before.

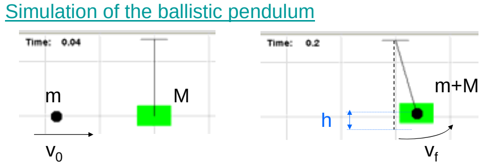
Completely inelastic collision – one where the objects stick together after colliding.



A ballistic pendulum

A ballistic pendulum is a device used to measure the speed of a bullet. A bullet of mass m = 50g is fired at a block of wood (mass M = 750 g) hanging from a string. The bullet embeds itself in the block, and causes the combined block plus bullet system to swing up a height h = 0.45 m. What is v_0, the speed of the bullet before it hits the block? (Ans. v_0 = 48 m/s) How much mechanical energy is lost? (Ans. 93.7%)

Simulation of the ballistic pendulum



A ballistic pendulum

(a) What is v_0, the speed of the bullet before it hits the block?

We will work backward. Apply the conservation of mechanical energy to the block-bullet system between right after the collision and the system's swinging up 0.45m from the point of collision:

$\frac{1}{2}(m+M)v^2 = (m+M)gh$
$\Rightarrow v = (2gh)^{1/2} = (2 \times 10 m/s^2 \times 0.45m)^{1/2} =$
3 m/s

Next, apply conservation of linear momentum to the velocities just before and just after the collision:

$mv_0 + 0 = (m+M)v$
$\Rightarrow v_0 = (m+M)v/m = (750g+50g)(3m/s)/(50g) =$
48 m/s

A ballistic pendulum

(b) How much mechanical energy is lost?

There is no change in gravitational potential energy in the collision. So, the change in mechanical energy is just ΔK:

$K_i = \frac{1}{2} mv_0^2 = \frac{1}{2} (50/1000 \text{ kg})(48 \text{ m/s})^2 = 57.6$ J

$K_f = \frac{1}{2} (m+M)v^2 = \frac{1}{2} ((50+750)/1000 \text{ kg})(3 \text{ m/s})^2 = 3.6$ J

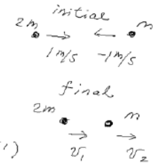
Loss in mechanical energy
= Loss in K
= 57.6 J – 3.6 J
= 54 J

An example of Elastic Collision

Ball 1 with mass 2m and velocity +1 m/s collides with Ball 2, with mass m, traveling with velocity -1 m/s. Find the final velocities of the two balls if the collision is elastic.

An example of Elastic Collision

Let v_1, v_2 be the velocity of ball 1 and 2, resp., after collision.



Conservation of momentum

$\Rightarrow (2m)(1) + m(-1) = (2m)v_1 + m v_2$

$\Rightarrow \cancel{m} = 2\cancel{m}v_1 + \cancel{m}v_2 \Rightarrow v_1 = \frac{1-v_2}{2} \quad \ldots(1)$

That the collision is elastic means that K is conserved.

$\Rightarrow \frac{1}{2}(2\cancel{m})(1)^2 + \frac{1}{2}\cancel{m}(-1)^2 = \frac{1}{2}(2\cancel{m})v_1^2 + \frac{1}{2}\cancel{m}v_2^2$

$\Rightarrow 3 = 2v_1^2 + v_2^2$

Sub. eqn. (1) in the above:

$3 = 2\left(\frac{v_2^2 - 2v_2 + 1}{4}\right) + v_2^2 \Rightarrow 6 = v_2^2 - 2v_2 + 1 + 2v_2^2$

$\Rightarrow 3v_2^2 - 2v_2 - 5 = 0$

$\Rightarrow v_2 = \frac{2 \pm \sqrt{4 - 4(3)(-5)}}{6} = \frac{2 \pm \sqrt{64}}{6} =$ +5/3 m/s or -1 m/s

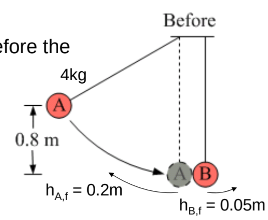
Subs. in (1) $\Rightarrow v_1 =$ -1/3 m/s or +1 m/s resp.

This set of solution req. ball 1 & 2 cross. ∴ neglect.

Two pendulums

Two balls hang from strings of the same length. Ball A, with a mass of 4 kg, is swung back to a point 0.8 m above its equilibrium position. Ball A is released from rest and swings down and hits ball B. After the collision, ball A rebounds to a height of 0.2 m above its equilibrium position, and ball B swings up to a height of 0.05 m.

(a) How fast is ball A going just before the collision? Use g = 10 m/s^2.
(b) Find the mass of ball B.
(c) What kind of collision is this?



Two pendulums: Speed of ball A, before

(a) How fast is ball A going, just before the collision? Use g = 10 m/s^2.

<u>Solution:</u>

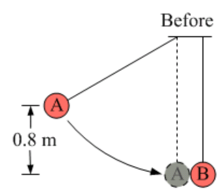
Apply energy conservation.

$U_i + \cancel{K_i} + \cancel{W_{nc}} = \cancel{U_f} + K_f$

$U_i = K_f$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh} = \sqrt{16 \text{ m}^2/\text{s}^2} = 4.0 \text{ m/s}$



Two pendulums: Speed of the balls, after the collision

(b) Find the mass of ball B.

First, find the velocities of the balls after the collision. One can use the same equation for the situation just after the collision.

For ball A afterwards:

$v = \sqrt{2gh} = \sqrt{4.0 \text{ m}^2/\text{s}^2} = 2.0 \text{ m/s} \qquad (h = 0.2 \text{ m})$

For ball B afterwards:

$v = \sqrt{2gh} = \sqrt{1.0 \text{ m}^2/\text{s}^2} = 1.0 \text{ m/s} \qquad (h = 0.05 \text{ m})$

Two pendulums: Find the mass of ball B

Apply momentum conservation.

$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$

$m_A v_{Ai} + 0 = m_A v_{Af} + m_B v_{Bf}$

How do we account for the fact that momentum is a vector?

Choose a positive direction (say, to the right), so the velocity of ball A after the collision is negative.

$m_B = \frac{m_A v_{Ai} - m_A v_{Af}}{v_{Bf}} = \frac{(4 \text{ kg})\times(+4 \text{ m/s}) - (4 \text{ kg})\times(-2 \text{ m/s})}{+1 \text{ m/s}}$

$\boxed{m_B = 16 \text{ kg} + 8 \text{ kg} = 24 \text{ kg}}$

Two pendulums: What kind of collision?

(c) What kind of collisions?

K_i before the collision = ½ (4kg)(4 m/s)^2 = 32 J

K_f after the collision
= ½ (4kg)(2 m/s)^2 + ½ (24kg)(1 m/s)^2
= 8 J + 12 J = 20 J

K_f / K_i = 5/8

This is less than 1 so the collision is inelastic. It is not completely inelastic because the two balls do not stick together after the collision.

Collisions in two dimensions

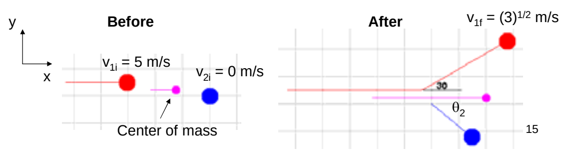
The Law of Conservation of Momentum applies in two and three dimensions, too. To apply it in 2-D, split the momentum into x and y components and keep them separate. Write out two conservation of momentum equations, one for the x direction and one for the y direction. That is,

$$m_1 v_{1,ix} + m_2 v_{2,ix} + \ldots = m_1 v_{1,fx} + m_2 v_{2,fx} + \ldots$$

$$m_1 v_{1,iy} + m_2 v_{2,iy} + \ldots = m_1 v_{1,fy} + m_2 v_{2,fy} + \ldots$$

Collisions in 2D – Example1

A billiard ball with initial speed 5 m/s collides with another billiard ball with identical mass that's initially at rest. After the collision, the first ball bounces off with speed $(3)^{1/2}$ m/s in a direction that makes an angle +30^o with the original direction. The second ball bounces in a direction that makes an angle θ_2 on the other side of the first ball's original direction (see figure). Neglect friction. Use the x-y coordinate system shown. (a) What're the y- and x-components of ball 2's velocity after collision? (b) What's the value of θ_2? (Ans. $v_{2f,y} = -(3)^{1/2}/2$ m/s, $v_{2f,x} = +0.5$ m/s, $\theta_2 = 60^o$.)



Collisions in 2D – Example1

Denote the x and y component of the velocity of the second ball after the collision by v_x and v_y, respectively.

Consider $P_{tot,y}$:

$$m(5\sin 0^\circ) + m(0) = m(\sqrt{3}\sin 30^\circ) + m v_y$$

$$0 = \sqrt{3}/2 + v_y$$

$$\Rightarrow v_y = -\sqrt{3}/2 \text{ m/s}$$

Consider $P_{tot,x}$:

$$m(5\cos 0^\circ) + m(0) = m(\sqrt{3}\underbrace{\cos 30^\circ}_{=\sqrt{3}/2}) + m v_x$$

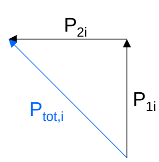
$$5 = 3/2 + v_x$$

$$\Rightarrow v_x = 3.5 \text{ m/s}$$

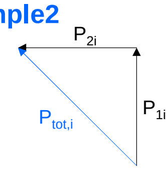
$$\theta_2 = \tan^{-1}\left|\frac{v_y}{v_x}\right| = \tan^{-1}\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{3.5}\right) = 14^\circ$$

Collisions in 2D – Example2

A car driving due north at 25 m/s collides with another car driving due west at 20 m/s. Suppose the two car stick together after the collision and the second car has a mass that's 125% that of the second car. Find the velocity of the two cars immediately after the collision.



Collisions in 2D – Example2



<u>Solution</u>

Magnitude of $P_{1i} = m_1 v_{1i} = m_1$(25 m/s)

Magnitude of $P_{2i} = m_2 v_{2i}$ = (5/4)m_1(20 m/s)
= m_1(25 m/s)

So, $P_{1i} = P_{2i}$

From the vector diagram,
The magnitude of $P_{tot,i} = (2)^{1/2} P_{1i} = (2)^{1/2} m_1$(25 m/s) … (1)

After the collision, $P_{tot,f} = m_{tot} v_f = (m_1+m_2) v_f = (9/4m_1) v_f$ … (2)

Since $P_{tot,f} = P_{tot,i}$, by (1) and (2) we have

$(2)^{1/2} m_1$(25 m/s) = $(9/4m_1) v_f \Rightarrow v_f$ = 140/9 m/s ≈ 15.6 m/s

For the direction of $\mathbf{v_f}$, use the fact that it's in the same direction as $\mathbf{P_{tot,f}}$, which in turn is the same as that of $\mathbf{P_{tot,i}}$. From the vector diagram, it's obviously N45^{o}W.