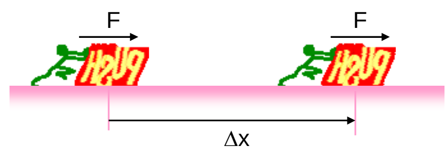
Work and Energy

Work Done by a Constant Force

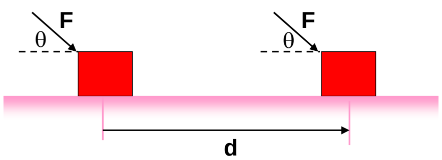


The work done, W, by a constant net force, F, causing an object to move by a displacement, d, is:

$$W = F \cdot d$$

SI unit: 1 N·m = 1 joule (J)

Work Done by a Constant Force

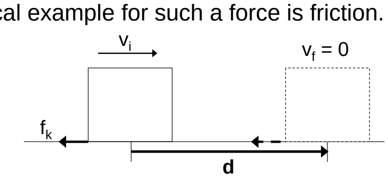


In general, if the net force, **F**, makes an angle θ, with the displacement vector, **d**, the work done W by **F** is:

$$W = F \cdot d\cos\theta$$

Negative Work Done

Negative work done occurs when the force and displacement vectors are in opposite directions. A typical example for such a force is friction.



Net Work Done = $(-f_k)d = \underbrace{-f_k d}_{\text{Negative work done}}$

Work Done and Kinetic Energy

Consider the work done by a constant force, **F**, on an object with mass, m.

$W = F \cdot d = ma \cdot d = m(a \cdot d)$ ……… (1)

The acceleration ***a*** brought about by the force would cause the velocity of the object to change from v_0 to v_f, following the relation, $v_f^2 = v_0^2 + 2a \cdot d$, implying $ad = ½ (v_f^2 - v_0^2)$. Substitute this in eqn. (1), we get:

$W = ½\, m(v_f^2 - v_0^2)$ ……. (2)

The RHS represents a change in the quantity, $½\, mv^2$, between the initial (o) and final state (f). What is $½\, mv^2$? Notice that it must be an energy since the LHS is an energy.

Kinetic Energy

The kinetic energy K of an object with mass *m* and velocity *v* is defined as:

$$K = ½\, mv^2$$

Work-Kinetic Energy Theorem

Given the definition of K, we can rewrite eqn. (2) as:

$W = ½ m(v_f^2 – v_0^2) = \Delta K$

Or, $\Delta K = W = Fd\cos\theta$

$\Delta K = W$

This is the work-kinetic energy theorem. The RHS is the net work. It is:

-zero when the net force is perpendicular to the displacement.

- positive when the net force has a component in the direction of the displacement (or velocity).

- negative when the net force has a component opposite to the direction of the displacement (or velocity).

Net Work done by multiple forces Acting on an Object

The work done, W_i by a particular force, $\mathbf{F_i}$, on an object is given by:

$W_i = (F_i\cos\theta_i)d$

If there are multiple forces, $\mathbf{F_1}$, $\mathbf{F_2}$, … acting on an object, the net work done on the object is the sum of the work done by all the forces $\mathbf{F_1}$, $\mathbf{F_2}$, ….

$W_{net} = \Sigma[(F_i\cos\theta_i)d] = (F_{net}\cos\theta)d,$

where θ is the angle $\mathbf{F_{net}}$ makes with **d**. This expression of W_{net} is the same as what we had before. This shows that the net work done is just that by the net force acting on the object.

The net force vs. position graph

Given the relation:

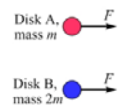
$\Delta K = W = F_{net,//} \cdot d \quad (F_{net,//} = F\cos\theta)$

Work-KE Theorem

Change in KE is the area under the net force vs. position graph.

This should be contrasted with the use of the net force vs. time graph, where change in momentum is the area under the net force vs. time graph.

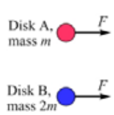
Two disks



Two disks are initially at rest. The mass of disk B is two times larger than that of disk A. The two disks then experience equal net forces *F*. These net forces are applied for the same amount of time. After the net forces are removed:

1. The disks have the same momentum and kinetic energy.
2. The disks have equal momentum; disk A has more kinetic energy.
3. The disks have equal momentum; disk B has more kinetic energy.
4. The disks have equal kinetic energy; disk A has more momentum.
5. The disks have equal kinetic energy; disk B has more momentum.

Two disks -- scenario 2



Two disks are initially at rest. The mass of disk B is two times larger than that of disk A. The two disks then experience equal net forces *F*. These net forces are applied over equal displacements. After the net forces are removed:

1. The disks have the same momentum and kinetic energy.
2. The disks have equal momentum; disk A has more kinetic energy.
3. The disks have equal momentum; disk B has more kinetic energy.
4. The disks have equal kinetic energy; disk A has more momentum.
5. The disks have equal kinetic energy; disk B has more momentum.

Momentum and Kinetic Energy

Momentum	Kinetic Energy
SI unit: kgm/s	SI unit: J
vector	scalar
$p = m\vec{v}$	$KE = mv^2/2 = p^2/(2m)$

$p^2 = 2m(KE)$

If two objects have the same momentum, the object with smaller mass will have bigger kinetic energy.

If two objects have the same kinetic energy, the object with smaller mass will have smaller momentum.

Work -- example 1

You hold an object that weighs 10 N at a fixed, elevated height for 15 minutes. How much work do you do to the object?

1. Zero.
2. Positive.
3. Negative.

The displacement is zero. So W = Fs = 0.

Work -- example 2

You raise a 10 N object up by a vertical distance of 0.5 m. You maintain a constant acceleration of 1 m/s^2 throughout the process. The net work you do to the object is …

1. 5 J
2. More than 5 J
3. Less than 5 J

The net force, F required to move the 10 N object vertically up at a constant acceleration of 1 m/s^2 is +11 N (up). So W = Fd = (11N)(0.5m) = 5.5 J

Work -- example 3

You lower a 10 N object at a constant speed by a vertical distance of 0.5 m. The net work done on the object is …

1. 5 J
2. More than 5 J
3. Less than 5 J

The net force, F required to move the 10 N object vertically down at a constant speed is 10 N (up). So W = Fd = (10N)(-0.5m) = -5 J