Center of Mass

1

A guy in a canoe

Consider a man with mass m_1 = 90 kg sitting at one end of a canoe with mass m_2 = 30 kg and length 2.4 m. (a) If the man moves by distance Δx_1 towards the opposite end of the canoe, find the distance Δx_2 moved by the canoe. (b) What is the distance moved by the canoe when the man reaches the opposite end of the canoe?

Canoe simulation

Solution

(a) By conservation of linear momentum,

$m_1v_1 + m_2v_2 = 0$

$m_1\Delta x_1/\Delta t + m_2\Delta x_2/\Delta t = 0$

$\Delta x_2 = -m_1\Delta x_1/m_2 = -3\Delta x_1$

2

A guy in a canoe

(b) What is the distance moved by the canoe when the man reaches the opposite end of the canoe?

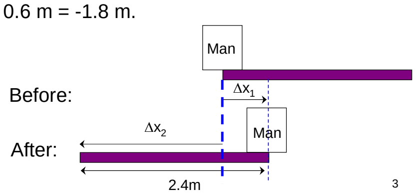
Solution

From the draw below, it is clear that

$\Delta x_1 - \Delta x_2 = 2.4$ m …. (1)

Subs. $\Delta x_2 = -3\Delta x_1$ in eqn. (1), we get $\Delta x_1 = 0.6$ m.

So, $\Delta x_2 = -3 \times 0.6$ m = -1.8 m.



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A guy in a canoe – Center of Mass

Consider the guy in a canoe to be a system of two masses, with one (the guy) located at where the guy is and mass equals to 90 kg and one located at the center of of the boat, and mass equal to 30 kg. Define the **center of mass**

$x_{cm} = (m_1x_1(t) + m_2x_2(t))/(m_1+m_2)$

Before:

x_{cm} = (90kg x 0 + 30kg x 1.2m) / (90kg + 30 kg) = 0.3 m

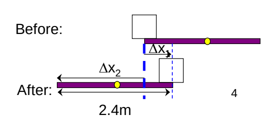
After the guy moved to the opposite end of the canoe:

x_{cm} = (90kg x 0.6m + 30kg x -0.6m) / (120kg)

= (60kg x 0.6m)/(120kg)

= 0.3 m

This is the same as before!



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A guy in a canoe – Center of Mass

That the **center of mass** of the guy-canoe system remains unchanged is not a coincidence. Consider the following:

$x_{cm} = [m_1x_1(t) + m_2x_2(t)]/(m_1+m_2)$

$\Rightarrow \Delta x_{cm} = [m_1\Delta x_1(t) + m_2\Delta x_2(t)]/(m_1+m_2)$

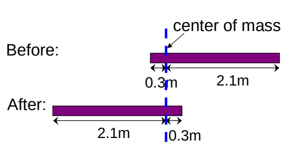
$= \Delta t\,[m_1\Delta x_1(t)/\Delta t + m_2\Delta x_2(t)/\Delta t]/(m_1+m_2)$

$= \Delta t\, P_{tot}/(m_1+m_2)$

$\Rightarrow \Delta x_{cm}/\Delta t = P_{tot}/(m_1+m_2)$

$\Rightarrow$ $v_{cm} = P_{tot}/(m_1+m_2)$

Since P_{tot} = 0 in the guy-canoe system, v_{cm} = 0 and x_{cm} doesn't change with time.



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Center of Mass – in 1D

For a general system containing *N* masses,

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + ... + m_Nx_N}{m_1 + m_2 + ... + m_N}$$

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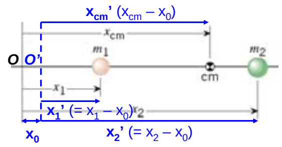
Changing the Origin



The <u>position of the center of mass is independent of the choice of the origin.</u> You can show that the value of x_{cm}' found in a new coordinate system with origin *O'* displaced by $+x_0$ from the old origin *O* is given by $x_{cm} - x_0$. This gives the same position as that determined by using the old coordinate system (see the figure above).

Center of Mass – in 2D

In the case where the component masses are distributed in a 2 dimensional plane (say, the x-y plane), the position of the center of mass (x_{cm}, y_{cm}) can be determined for x_{cm} and y_{cm} independently by applying the 1D formula to the x and y coordinates, respectively, of the component masses.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_N x_N}{m_1 + m_2 + \dots + m_N}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots + m_N y_N}{m_1 + m_2 + \dots + m_N}$$

Velocity of Center of Mass

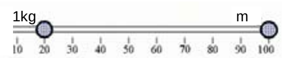
For a general system containing *N* masses,

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2 + \dots + m_N v_N}{m_1 + m_2 + \dots + m_N}$$

= total momentum / total mass

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

A massless rod with two balls



Two balls, one with mass 1kg and the other with mass m, are placed on a massless rod as shown above. Suppose we adjust the mass of the ball at the 100-cm position from zero to infinity, over what range will the position of the center of mass of the broken rod vary?

<u>Solution</u>

Let's consider the two extreme cases.
(1) When m = 0, the center of mass will lie on the 20-cm mark. (2) When the mass is infinity, the center of mass will lie on the 100-cm mark. So, the position of the center of mass will range from the 20-cm mark to the 100-cm mark as the mass at the 100-cm position varies from zero to infinity.

Center of mass in 2D - Example

For the system shown above, suppose a third ball with unknown mass, when placed at some distance directly above the mass at the 20-cm mark, causes the center of mass of the system to be shifted to (x_{cm},y_{cm}) = (40cm, 20cm). Find the location of the third ball and its mass.

Center of mass in 2D - Example

<u>Solution</u>:

That the third ball is directly above the 20-cm mark means that its x-coordinate, x, is 20 cm.

$$x_{cm} = 40\text{ cm} = \frac{m \times 20\text{ cm} + 1\text{ kg} \times 20\text{ cm} + 1\text{ kg} \times 100\text{ cm}}{m + 1\text{ kg} + 1\text{ kg}}$$

This gives *m* = 2 kg.

To find the y coordinate of the third ball, consider y_{cm}:

$$y_{cm} = 20\text{ cm} = \frac{2\text{ kg} \times y + 1\text{ kg} \times 0\text{ cm} + 1\text{ kg} \times 0\text{ cm}}{2\text{ kg} + 1\text{ kg} + 1\text{ kg}}$$

This gives *y* = 40 cm.