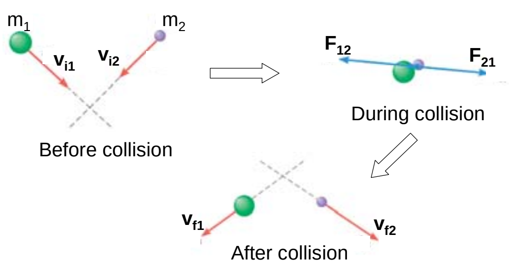
PY105 (A1)

1. Next Assignment is a hand-in, it will be due on next Tuesday.
2. Test 2 will be on Nov 9 (in Rm. 101, Kenmore Classroom Building, 565 Comm. Ave.) and the final exam. will be on Dec. 16 from 6-8pm. If you have a schedule conflict, you must notify me at least one week in advance. For the final exam., if you are scheduled for three or more exams. in one day, you are entitled for a reschedule of the PY105 exam. date.

Impulse & Momentum

A mid-air collision

For a mid-air collision between two objects with masses m_1 and m_2, determine the relation between the initial velocities, v_{i1} and v_{i2} and final velocities, v_{f1} and v_{f2}.



A mid-air collision

$$\begin{cases} \vec{F}_{12} = m_1\vec{a}_1 = m_1\dfrac{\Delta\vec{v}_1}{\Delta t} = m_1\dfrac{\vec{v}_{f1}-\vec{v}_{i1}}{\Delta t} \quad \cdots\cdots\cdots \ (1) \\ \vec{F}_{21} = m_2\vec{a}_2 = m_2\dfrac{\Delta\vec{v}_2}{\Delta t} = m_2\dfrac{\vec{v}_{f2}-\vec{v}_{i2}}{\Delta t} \quad \cdots\cdots\cdots \ (2) \end{cases}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0 \Rightarrow m_1\frac{\vec{v}_{f1}-\vec{v}_{i1}}{\Delta t} + m_2\frac{\vec{v}_{f2}-\vec{v}_{i2}}{\Delta t} = 0$$

$$\Rightarrow m_1(\vec{v}_{f1}-\vec{v}_{i1}) + m_2(\vec{v}_{f2}-\vec{v}_{i2}) = 0$$

$$\Rightarrow \underbrace{m_1\vec{v}_{f1}} + \underbrace{m_2\vec{v}_{f2}} = \underbrace{m_1\vec{v}_{i1}} + \underbrace{m_2\vec{v}_{i2}}$$

$$\Rightarrow \underbrace{\boldsymbol{p}_{f1} + \boldsymbol{p}_{f2}} = \underbrace{\boldsymbol{p}_{i1} + \boldsymbol{p}_{i2}}$$

$$\Rightarrow \boldsymbol{p}_{f,total} = \boldsymbol{p}_{i,total}$$

Define **linear momentum**: $\boldsymbol{p} = mv$ (Unit: kgm/s)

Conservation of Linear Momentum

From the above example, we see that

$$\boldsymbol{p}_{total} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = \text{constant}$$

$$\Delta\boldsymbol{p}_{total} = \Delta\boldsymbol{p}_1 + \Delta\boldsymbol{p}_2 = 0$$

Note that the system we considered is an **isolated system**, i.e., one in which the net external force is zero. The above law of conservation of linear momentum applies to isolated systems only.

Relation between Force and Momentum

Copy eqn. (1) from above:

$$\vec{F}_{12} = m_1\frac{\Delta\vec{v}_1}{\Delta t} = \frac{\Delta\vec{p}_1}{\Delta t}$$

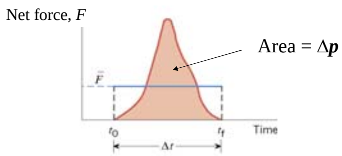
Drop the subscripts:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \quad \cdots\cdots\cdots \ (3)$$

This shows that **force is the rate of change of linear momentum.** Notice that if m = constant, eqn. (3) gives $\boldsymbol{F} = m\boldsymbol{a}$. So, Newton's second law is a special case of the more general relation given by eqn. (3), which applies even to cases where the mass is not a constant.

Impulse and Momentum

With $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$, $\Delta\boldsymbol{p}$ is the area under a *F*-*t* graph.



$$\text{Area under } F\text{-}t \text{ graph} = \underbrace{\int \vec{F}\, dt}_{\text{Impulse, J (Unit: Ns)}} = \underbrace{\Delta\vec{p}}_{\text{Change in momentum}}$$

Defined as: **Impulse, J (Unit: Ns)** Change in momentum

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