PY105

- Hand-in Assignment 6 has been posted on WebCT. It's due on Oct. 18 (next Tuesday).

Uniform and Vertical Circular Motions

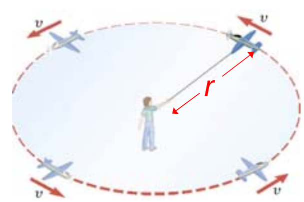
Modified from Prof. Skocpol's notes

Topics to Cover

- Uniform circular motion
  - velocity = ωr tangential
  - acceleration v^2/r or $\omega^2 r$ toward center
- $\Sigma F = ma$
- Inclined planes and banked turns
  - With and without friction!
- Vertical Circular Motion
- $\Sigma F = ma$

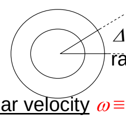
DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Uniform Circular Motion

- The path is a circle (radius r, circumference $2\pi r$).
- "Uniform" means constant speed $v = 2\pi r / T$, where the period T is the time to go around the circle once.
- Angle in "radians" $\equiv$ (arc length Δs) / (radius r)



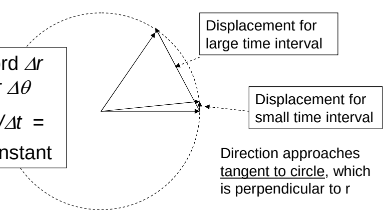
$\Delta\theta = \Delta s_1/r_1 = \Delta s_2/r_2$ is independent of the radius r of the circle, and is dimensionless

- Angular velocity $\omega \equiv \Delta\theta/\Delta t = 2\pi/T$ [rad/sec], is also independent of r
- Note that $v = r(2\pi/T) = r\,\omega$ [m/s], and therefore v is proportional to the radius of the circle.

Velocity on circular path

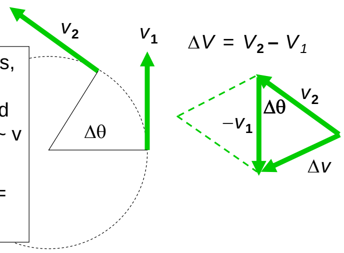
$v = \Delta r/\Delta t$ but chord Δr is almost arc $s = r\,\Delta\theta$

So again $v = (r\Delta\theta)/\Delta t = r(\Delta\theta/\Delta t) = \omega r$ = constant



For uniform circular motion, the velocity vector has magnitude $v = \omega r$, and direction is tangent to the circle at the position of the particle.

Magnitude of the acceleration



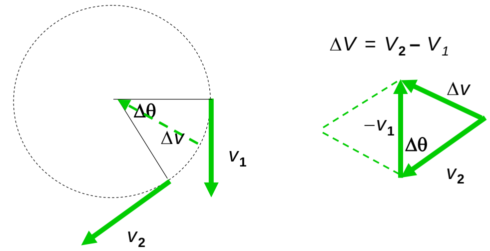
For small time intervals, the vector Δv points toward the center, and has the magnitude $\Delta v \sim v\,\Delta\theta$ so

$a = \Delta v / \Delta t = v\,(\Delta\theta/\Delta t) = v\,\omega = v^2/r$

For uniform circular motion, the magnitude of the acceleration is $\omega^2 r = v^2/r$, and the direction of the acceleration is toward the center of the circle.

Magnitude of the acceleration

In the last discussion, we have considered the case where the circular motion is counter-clockwise. Below, we show the vector diagram for when it is clockwise.



The above drawings show that the acceleration vector ***a*** (which is parallel to **Δv**) is still pointing towards the center of the circle.

Coins on a turntable

Two identical coins are placed on a flat turntable that is initially at rest. One coin is closer to the center than the other disk is. There is some friction between the coins and the turntable. We start spinning the turntable, steadily increasing the speed. Which coin starts sliding on the turntable first?

1. The coin closer to the center.
2. The coin farther from the center.
3. Neither, both coin start to slide at the same time.

A general method for solving circular motion problems

Follow the method for force problems!

- •Draw a diagram of the situation.
- •Draw one or more free-body diagrams showing all the forces acting on the object(s).
- •Choose a coordinate system. It is often most convenient to align one of your coordinate axes with the direction of the acceleration.
- •Break the forces up into their *x* and *y* components.
- •Apply Newton's Second Law in both directions.

•The key difference: use $a = \frac{v^2}{r}$ toward the center

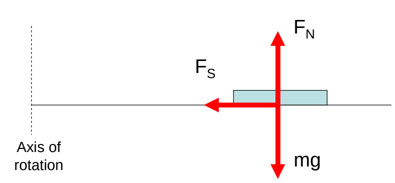
Coins on a turntable

Sketch a free-body diagram (side view) for one of the coins, assuming it is not sliding on the turntable.

Apply Newton's Second Law, once for each direction.

Coins on a turntable

Sketch a free-body diagram (side view) for one of the coins, assuming it is not sliding on the turntable.



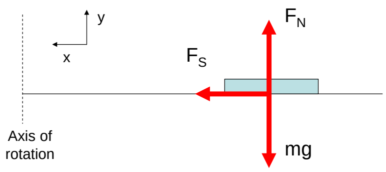
Can you tell whether the velocity is into or out of the screen?

Coins on a turntable

Apply Newton's Second Law, once for each direction.

y-direction: $F_N - mg = 0$ so that $F_N = mg$

x-direction: $F_S = ma_x = m(v^2/r)$ [both F_S and a are to left]



Can you tell whether the velocity is into or out of the screen? *

As you increase *r*, what happens to the force of friction needed to keep the coin on the circular path?

* It is the same diagram and result either way!

"Trick" question!

v has a "hidden" dependence on *r*, so that the "obvious" dependence on r is not the whole story. The two coins have different speeds.

Use angular velocity for the comparison, because the two coins rotate through the <u>same angle</u> in a particular time interval.

$$\omega = \frac{v}{r} \quad \text{so} \quad v = r\omega$$

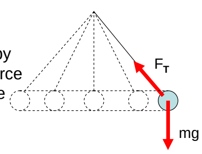
This gives: $F_S = \frac{mv^2}{r} = \frac{mr^2\omega^2}{r} = mr\omega^2$

As you increase *r*, what happens to the force of friction needed to keep the coin staying on the circular path?

The larger *r* is, the larger the force of static friction has to be. The outer one hits the limit first.

Conical pendulum

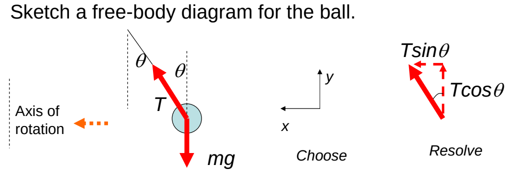
A ball is whirled in a horizontal circle by means of a string. In addition to the force of gravity and the tension, which of the following forces should appear on the ball's free-body diagram?



1. A normal force, directed vertically up.
2. A centripetal force, toward the center of the circle.
3. A centripetal force, away from the center of the circle.
4. Both 1 and 2.
5. Both 1 and 3.
6. None of the above.

Conical pendulum

Sketch a free-body diagram for the ball.



Apply Newton's Second Law, once for each direction.

y-direction: $T\cos\theta - mg = ma_y = 0$

x-direction: $T\sin\theta = ma_x = m(v^2/r)$

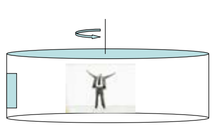
Solve: $(mg/\cos\theta)\sin\theta = mv^2/r$

$(rg\tan\theta)^{1/2} = v$

Gravitron (or The Rotor)

In a gravitron, riders are pressed against the vertical wall of the gravitron preventing them from falling under gravity. Which force acting on each rider is directed toward the center of the circle?

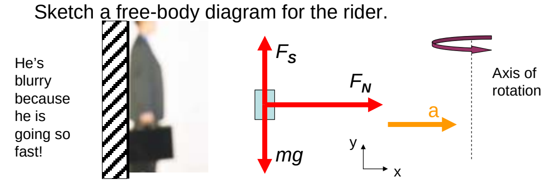
1. A normal force.
2. A force of gravity.
3. A force of static friction.
4. A force of kinetic friction.
5. None of the above.



http://www.youtube.com/watch?v=ewmdPNfyBzI&feature=related
Starship 2000

Gravitron

Sketch a free-body diagram for the rider.



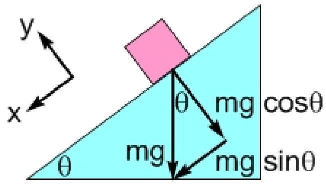
Apply Newton's Second Law, once for each direction.

y direction: $F_S - mg = ma_y = 0$ (he hopes)

x direction: $F_N = ma_x = m(v^2/r)$

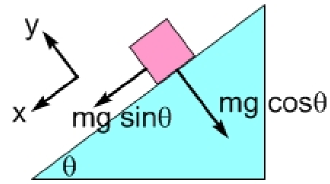
Inclined Plane – Brief Review

Breaking mg into x- and y- components



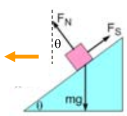
Inclined Plane – Brief Review

The end result – we replaced *mg* by its components.



Banked Turns

- The same picture can describe the motion of a car on a circular turn of radius R that has a sideways sloping road. Here the velocity is into the diagram, but the acceleration is v^2/R, horizontally to the left.



Many roads are designed so that at the expected speed, no friction F_s is required to make the turn.

In that case (i.e., $F_s=0$) F_N has a horizontal component

$F_N \sin\theta = m v^2/R$

and a vertical component

$F_N \cos\theta - mg = 0.$

Putting these together gives

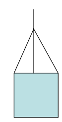
$mg \tan\theta = mv^2/R.$

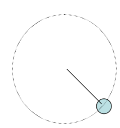
This gives the relationship between angle and speed for a curve of a certain radius R. Note that m cancels out, so any mass of car or truck needs the same speed on a given banked turn.

Vertical circular motion

Examples

- Ball on String
- Water buckets
- Cars on hilly roads
- Roller coasters





At the top or bottom of a circular arc, the apparent weight (F_N or T), as we have seen before, is:

$W_{app} = m(g+a)$ [for up ≡ +, a can be + or -]

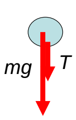
But now $a = \pm v^2/r$ [toward center]

Free-body diagram for the ball on string

Sketch a free-body diagram for the ball, at the top of the circle, and apply Newton's Second Law. Find the minimum speed of the ball at the top for it not to fall from there.

ma = *m*(v^2/r)

"Toward center" is down



$(mg + T) = m(v^2/r)$ {down is positive}

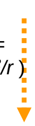
But critical speed is when $T = 0$

So $mg = mv^2_{min}/r$ or $v_{min} = (rg)^{1/2}$

Free-body diagram for the bucket and water

Sketch a free-body diagram for the bucket+water (m_b+m) = M), at the top of the circle, and apply Newton's Second Law.

Ma = *M*(v^2/r)



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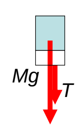


The string pulls down on the bucket

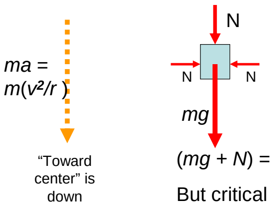
$(Mg + T) = M(v^2/r)$ {down is positive}

But critical speed is when $T = 0$

So $Mg = Mv^2_{min}/r$ or $v_{min} = (rg)^{1/2}$

Free-body diagram for the water

Sketch a free-body diagram for just the water, at the <u>top</u> of the circle, and apply Newton's Second Law.



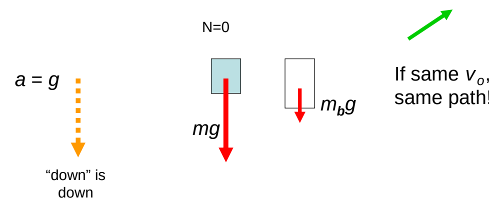
$(mg + N) = m(v^2/r)$ {down is positive}

But critical speed is when $N = 0$

So $mg = mv^2_{min}/r$ or $v_{min} = (rg)^{1/2}$

Free-body diagram for the water

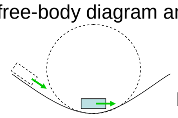
Sketch a free-body diagram for just the water, if the speed is <u>less than</u> the critical speed.

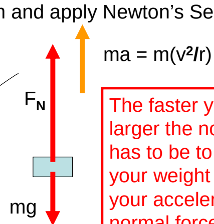


Roller coaster

On a roller coaster, when the coaster is traveling fast at the bottom of a circular loop, you feel much heavier than usual. Why?

Draw a free-body diagram and apply Newton's Second Law.





$F_N - mg = mv^2/r$ so

$F_N = m(g + v^2/r)$

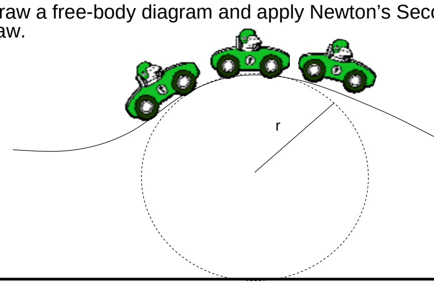
Check: [$F_N = m(g + a)$]

The faster you go, the larger the normal force has to be to support your weight <u>and</u> cause your acceleration. The normal force is your apparent weight.

Driving on a hilly road

As you drive at relatively high speed v over the top of a hill curved in an arc of radius r, you feel almost weightless and your car comes close to losing contact with the road. Why?

Draw a free-body diagram and apply Newton's Second Law.



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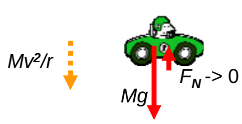

$F_N - Mg = M(-v^2/r)$

loses contact when

$F_N = 0$ at $v = (rg)^{1/2}$

Warning to drivers: Your braking is worst at the crest of a hill.