Applications of Newton's Second Law II and III

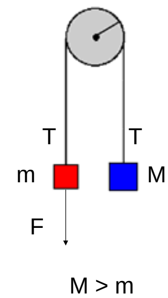
Atwood's Machine

If $a = 0$ (at equilibrium),

$F = (M - m)g.$
$T = (M - m)g$

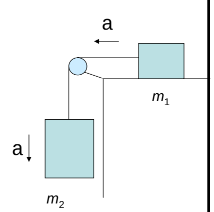
If $F = 0$ (M accelerates downward while m accelerates upward,

$a = (M - m)g/(M + m)$
$T = m(g + a)$



Motion of Two Boxes Connected by a String

Consider two boxes with masses m_1 and m_2 connected by a string as shown in the diagram at right. Suppose the coefficient of kinetic friction between the m_1 box and the table is μ_k. What is the acceleration of the two boxes? What is the tension in the string?

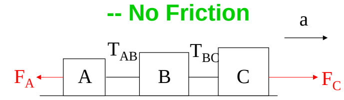


Ans.

$$a = \frac{m_2 g - \mu_k m_2 g}{m_2 + m_1} = \frac{(m_2 g) - \text{friction}}{m_2 + m_1}$$

$T = m_2(g - a)$

Motion of Three Boxes Connected by Two Strings -- No Friction

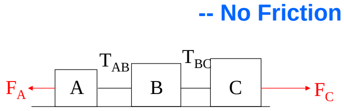


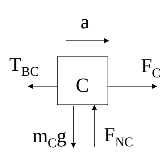
Suppose there's no friction between the boxes A, B, C and the table and $F_C > F_A$. What's the acceleration of the boxes and tensions T_{BC} and T_{AB}?

To solve the problem, the usual way is to write the Newton's equations: $T_{AB} - F_A = m_A a$ (for block A), $T_{BC} - T_{AB} = m_B a$ (for block B), and $F_C - T_{BC} = m_C a$ (for block C). Then solve these equations to find a, T_{AB} and T_{BC}. But a much simpler way is to **consider the net tangential force (i.e., tangential to the strings) acting on the system as a whole**. In this case, the tangential net force, $F_{net} = F_C - F_A$

So, $a = F_{net}/(\text{Total mass}) = (F_C - F_A)/(m_A+m_B+m_C)$

Motion of Three Boxes Connected by Two Strings -- No Friction



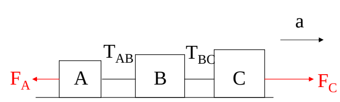


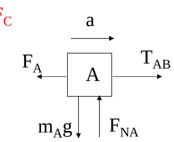
$a = F_{net}/(\text{Total mass}) = (F_C - F_A)/(m_A+m_B+m_C)$

Once you know a, you can find T_{BC} by using the Newton's equation for block C:

$F_C - T_{BC} = m_C a$
$\Rightarrow T_{BC} = F_C - m_C a$
$= F_C - m_C(F_C - F_A)/(m_A+m_B+m_C)$

Motion of Three Boxes Connected by Two Strings -- No Friction

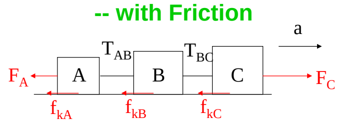




To find T_{AB}, use the Newton's equation for block A:

$T_{AB} - F_A = m_A a$
$\Rightarrow T_{AB} = F_A + m_A a$
$= F_A + m_A(F_C - F_A)/(m_A+m_B+m_C)$

Motion of Three Boxes Connected by Two Strings -- with Friction



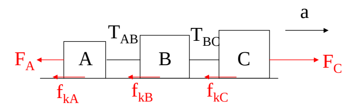
Suppose the coefficient of kinetic friction between the boxes A, B, C and the table is μ_K and $F_C > F_A$. What's the acceleration of the boxes and tensions T_{AB} and T_{BC}?

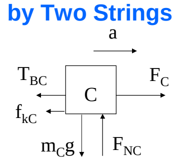
The net force F_{net} acting on the system as a whole is $F_C - F_A - f_{k,tot}$. Note that **the tensional forces do not contribute to the net force of the system because they come in pairs of equal magnitude and opposite direction and so cancel exactly among themselves.**

For the system as a whole, $f_{k,tot} = \mu_k(m_A+m_B+m_C)g$

So, $a = F_{net}/(\text{Total mass}) = (F_C - F_A)/(m_A+m_B+m_C) - \mu_k g$

Motion of Three Boxes Connected by Two Strings -- with Friction

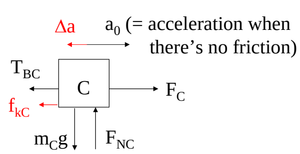




To find T_{BC}, write the Newton's equation for block C:

So, $F_C - T_{BC} - f_{kC} = m_Ca = m_C[(F_C-F_A)/(m_A+m_B+m_C) - \mu_k g]$
$\Rightarrow F_C - T_{BC} = m_C(F_C-F_A)/(m_A+m_B+m_C)$
$\Rightarrow T_{BC} = F_C - m_C(F_C-F_A)/(m_A+m_B+m_C)$

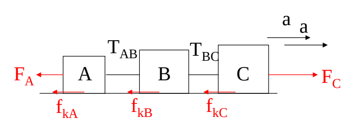
Motion of Three Boxes Connected by Two Strings -- with Friction

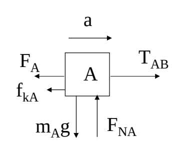


$T_{BC} = F_C - m_C(F_C-F_A)/(m_A+m_B+m_C)$

This is the same tension we found when there's no friction!
This is because the reduction Δa in a (= $\mu_k g$) is exactly accountable by $f_{kC} = \mu_k m_C g$ (= $m_C\Delta a$). It follows that $F_C - T_{BC}$ (and hence T_{BC}) must be the same as before when there was no friction.

Motion of Three Boxes Connected by Two Strings -- with Friction





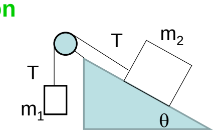
To find T_{AB}, write the Newton's equation for block A:

$T_{AB} - F_A - f_{kA} = m_Aa$
$\Rightarrow T_{AB} = F_A + m_Aa + f_{kC}$
$= F_A + [m_A(F_C-F_A)/(m_A+m_B+m_C) - \mu_k m_A g] + \mu_k m_C g$
$= F_A - m_A(F_C-F_A)/(m_A+m_B+m_C)$

Again, this is the same tension we found when there's no friction.

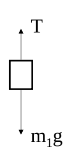
Motion of Two Boxes Connected by a String -- No friction

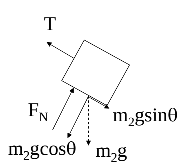
Consider the situation shown at right, where the pulley is frictionless and massless. Find the acceleration a and tension T in terms of m_1, m_2 and g, if there's no friction between m_2 and the incline.



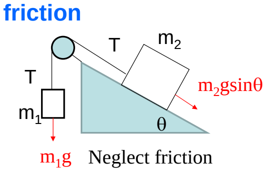
Neglect friction

First, draw the FBD of m_1 and m_2:





Motion of Two Boxes Connected by a String -- No friction



Neglect friction

From the FBD of m_2, we can see that the tangential force, besides tension T, that's acting on m_2 is $m_2g\sin\theta$. Similarly, from the FBD of m_1, the tangential force besides T, that's acting on m_1 is m_1g. We *arbitrarily* assume that m_1 weighs on the system more than m_2 does. (If this is a wrong guess, the sign of *a* we get will be negative, which will correct for the wrong direction.) With this, the net tangential force acting on the system as a whole can be written as:

$F_{net} = m_1g - m_2g\sin\theta$

So, $a = \dfrac{m_1 g - m_2 g\sin\theta}{m_2 + m_1}$

Motion of Two Boxes Connected by a String
-- No friction

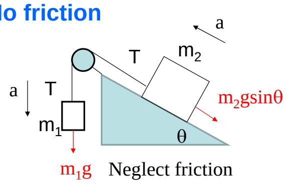
Next, use the FBD of m_1 to find T:

$T - m_1 g = -m_1 a$

$T = m_1(g - a)$

$$= m_1 g - m_1 \frac{m_1 g - m_2 g \sin\theta}{m_2 + m_1}$$

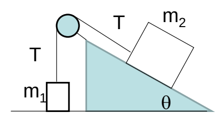
$$= \frac{m_1 m_2 g(1 + \sin\theta)}{m_1 + m_2}$$



Neglect friction

Motion of Two Boxes Connected by a String (2)
-- No friction

For the same problem considered above. What is the tension T if m_1 is resting on ground?



Neglect friction

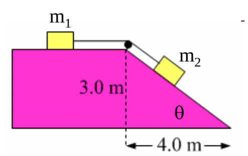
In this case, the system is at rest. A normal force acting on m_1 provides the support to counter-balance the different effects the two weights (i.e., $m_1 g$ and $m_2 g\sin\theta$) have on the system. To find T, let's consider the FBD of m_2, which remains the same as before, except that the r.h.s. is now zero because a = 0:

$- T + m_2 g\sin\theta = -m_2 a = 0$

$\Rightarrow T = m_2 g\sin\theta$

Motion of Two Boxes Connected by a String (3)
-- No Friction

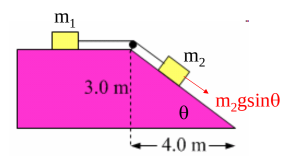
Consider the situation shown at right. Suppose you can neglect friction, and that the pulley between the two blocks is massless and frictionless. Find the tension in the string T and acceleration of the system a.



Neglect friction

Motion of Two Boxes Connected by a String (3)
-- No Friction

First, consider the net tangential force acting on the system. Because the only tangential force present, apart from tension, is $m_2 g\sin\theta$,



Neglect friction

$F_{net} = m_2 g\sin\theta$.

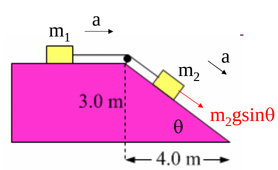
So, $a = F_{net}$ / (total mass)

$= m_2 g\sin\theta / (m_1 + m_2)$

Motion of Two Boxes Connected by a String (3)
-- No Friction

From the above,

$a = m_2 g\sin\theta / (m_1 + m_2)$

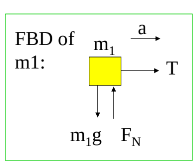


Neglect friction

To find tension, consider the FBD of m_1:

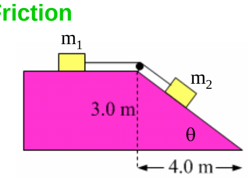
$T = m_1 a$

$= m_1 m_2 g\sin\theta / (m_1 + m_2)$



Motion of Two Boxes Connected by a String (4)
-- with Friction

Consider the same situation as the last example, except that there is friction acting on m_1 (coefficient of kinetic friction = μ_k) but not m_2. Find the tension in the string T and acceleration of the system a.



Friction on m_1 but none on m_2.

Motion of Two Boxes Connected by a String (4) -- with Friction

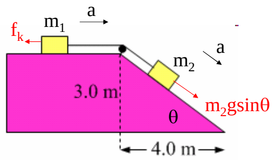
The net tangential force acting on the system as a whole is:

$F_{net} = m_2 g\sin\theta - f_k$

So, $a = F_{net}$ / (total mass)
$= (m_2 g\sin\theta - f_k)/(m_1+m_2)$

Next, substitute $f_k = \mu_k m_1 g$. We get:

$a = (m_2 g\sin\theta - \mu_k m_1 g)/(m_1+m_2)$



Motion of Two Boxes Connected by a String (4) -- with Friction

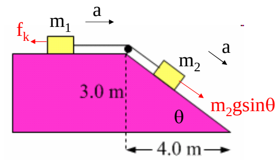
From the above,
$a = (m_2 g\sin\theta - \mu_k m_1 g)/(m_1+m_2)$

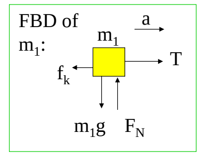
We write it as:
$a = a_0 - \mu_k m_1 m_2 g/(m_1+m_2)$,
where a_0 is the acceleration when there's no friction.

To find tension, consider the FBD of m_1:

$T - f_k = m_1 a$
$= m_1 a_0 - \mu_k m_1 m_2 g/(m_1+m_2)$

Next, substitute $f_k = \mu_k m_1 g$. We get:

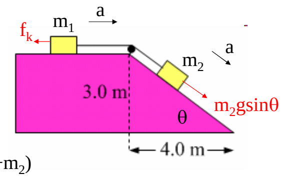




Motion of Two Boxes Connected by a String (4) -- with Friction

From last page,
$T - f_k$
$= m_1 a_0 - \mu_k m_1 m_2 g/(m_1+m_2)$

Next, substitute $f_k = \mu_k m_1 g$:

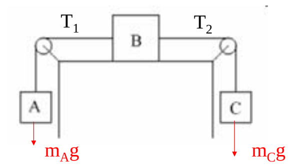


$T = m_1 a_0 + \mu_k m_1 g - \mu_k m_1 m_2 g/(m_1+m_2)$
$= \underbrace{m_1 a_0} + \mu_k m_1 m_2 g/(m_1+m_2)$

= value of tension when friction is zero. This result shows that tension is increased when friction is turned on. This is because the amount of acceleration reduction is less than accountable by the frictional force, i.e., $m_1 \Delta a < f_k$. So, tension has to work harder than before to overcome friction while providing the correct acceleration consistent with $a = F_{net}$/(total mass).

Motion of Three Boxes Connected by Two Strings (1) – no friction

Suppose there's <u>no friction</u> between box B and the table. Find the acceleration of the system and tensions T_1 and T_2. You may also assume that the pulleys are frictionless and massless.



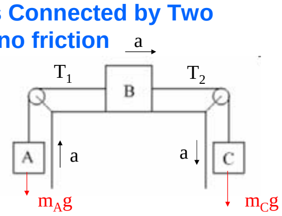
Let's arbitrarily assume that block A weighs more than block C. Then the net force acting on the system is $(m_C - m_A)g$

So, $a = F_{net}$/(Total mass)
$= (m_C - m_A)g / (m_A + m_B + m_C)$

Motion of Three Boxes Connected by Two Strings (1) – no friction

From the above,
$a = (m_C - m_A)g / (m_A + m_B + m_C)$

To find T_1, write the Newton's equation for block A:



$T_1 - m_A g = m_A a$
$\Rightarrow T_1 = m_A(g + a)$

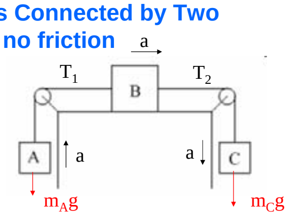
Substitute $a = (m_C - m_A)g / (m_A + m_B + m_C)$, we get:

$T_1 = m_A(m_B + 2m_C)g/(m_A + m_B + m_C)$

Motion of Three Boxes Connected by Two Strings (1) – no friction

From the above,
$a = (m_C - m_A)g / (m_A + m_B + m_C)$

To find T_2, write the Newton's equation for block C:



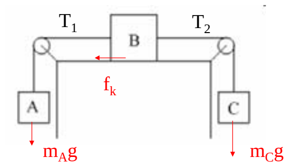
$T_2 - m_C g = -m_C a$
$\Rightarrow T_2 = m_C(g - a)$

Substitute $a = (m_C - m_A)g / (m_A + m_B + m_C)$, we get:

$T_2 = m_C(2m_A + m_B)g/(m_A + m_B + m_C)$

Motion of Three Boxes Connected by Two Strings (2) – with friction

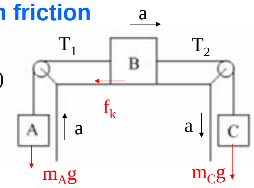
For the system considered in the last example, suppose there's friction between box B and the table. Find a, T_1 and T_2. Assume that the coefficient of kinetic friction is μ_k.



Let's still assume that block A weighs more than block C. Then the net force acting on the system is $(m_C - m_A)g - f_k = (m_C - m_A)g - \mu_k m_B g$

So, $a = F_{net}/(\text{Total mass})$
$= (m_C - m_A - \mu_k m_B)g / (m_A + m_B + m_C)$

Motion of Three Boxes Connected by Two Strings (2) – with friction



From the above,
$a = (m_C - m_A - \mu_k m_B)g / (m_A + m_B + m_C)$

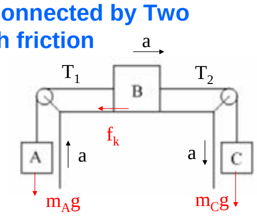
To find T_1, write the Newton's equation for block A:

$T_1 - m_A g = m_A a$
$\Rightarrow T_1 = m_A(g + a)$

Note that the value of T_1 without tension is $T_{1,0} = m_A(g + a_0)$, where a_0 is the acceleration of the system when there's no friction. Clearly, $a = a_0 - \mu_k m_B g/(\Sigma m)$. So,

$T_1 = T_{1,0} - \mu_k m_A m_B g / (m_A + m_B + m_C)$
$= m_A g[2m_C + m_B(1-\mu_k)] / (m_A + m_B + m_C)$

Motion of Three Boxes Connected by Two Strings (2) – with friction



To find T_2, write the Newton's equation for block C:

$T_2 - m_C g = -m_C a$
$\Rightarrow T_2 = m_C(g - a)$
$= T_{2,0} - m_C(-\mu_k m_B g)/(\Sigma m)$
$= T_{2,0} + \mu_k m_B m_C g / (\Sigma m)$

Substitute $T_{2,0} = m_C(2m_A + m_B)g / (\Sigma m)$, we get:

$T_2 = m_C g[2m_A + m_B(1+\mu_k)] / (m_A + m_B + m_C)$