Pressure

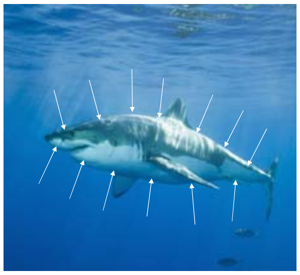
Pressure

Pressure is force per unit area: $P = \frac{F}{A}$

The direction of the force exerted on an object by a fluid is toward the object and perpendicular to its surface. At a microscopic level, the force is associated with the atoms and molecules in the fluid bouncing elastically from the surfaces of the object.

The SI unit for pressure is the pascal.
1 Pa = 1 N/m^2

The arrows below illustrate the directions of the force of pressure acting on a shark at various point of its body. They show that the force is always pointing towards and perpendicular to the surface of the shark.



Atmospheric pressure

1 atm = 1.01 x 10^5 Pa = 14.7 psi = 760 torr

(psi = pounds / square inch)

At atmospheric pressure, every square meter has a force of 100,000 N exerting on it, coming from air molecules bouncing off it!

Why don't we, and other things, collapse because of this pressure?

We, human, have an internal pressure of 1 atmosphere. General solid objects do not collapse because they possess an elastic modulus (i.e., rigidity) enabling them to deform only little when compressed by the atm. pressure.

Conceptual example 1: Blood pressure

Question A typical reading for blood pressure is 120 over 80. What do the two numbers represent? What units are they in?

Ans.

120 mm Hg (millimeters of mercury) is a typical systolic pressure, the pressure when the heart contracts.

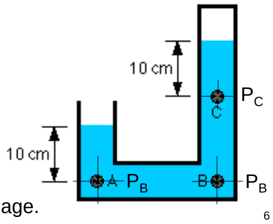
80 mm Hg is a typical diastolic pressure, the blood pressure when the heart relaxes after a contraction.

760 mm Hg is the typical atmospheric pressure. The blood pressure readings are not absolute – they tell us how much above atmospheric pressure the blood pressure is.

Conceptual example 2. Rank by pressure

A container, closed on the right side but open to the atmosphere on the left, is almost completely filled with water, as shown. Three points are marked in the container. Rank these according to the pressure at the points, from highest pressure to lowest.

[✓ 1.] A = B > C
2. B > A > C
3. B > A = C
4. C > B > A
5. C > A = B
6. some other order



See explanation on the next page.

Pressure in a static fluid

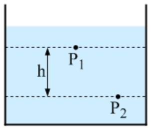
A static fluid is a fluid at rest. In a static fluid:

- Pressure increases with depth.
- Two points at the same vertical position experience the same pressure, no matter what the shape of the container.

If point 2 is a vertical distance *h* below point 1, and the pressure at point 1 is P_1, the pressure at point 2 is:

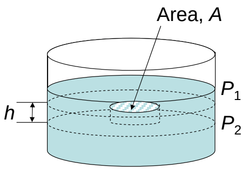
$$P_2 = P_1 + \rho gh$$

Point 2 does not have to be directly below point 1 - what matters is the vertical distance.



Pressure in a Static Fluid

To understand the equation, $P_2 = P_1 + \rho gh$, consider a column of fluid with area *A* and height *h* as shown. The weight of the fluid column is ρghA. This weight will cause an extra pressure of $\rho ghA/A = \rho gh$ at the bottom of the fluid column and explains the equation.

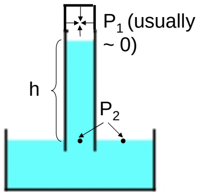


Measuring pressure

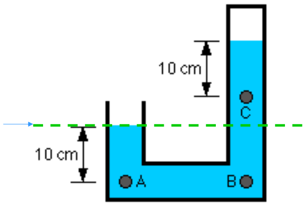
The relationship between pressure and depth is exploited in manometers (or barometers) that measure pressure.

A standard barometer is a tube with one end sealed. The sealed end is close to zero pressure, while the other end is open to the atmosphere. The pressure difference between the two ends of the tube can maintain a column of fluid in the tube, with the height of the column being proportional to the pressure difference.

$$P_2 = P_1 + \rho gh$$



Conceptual example 2b. Rank by pressure



What is the pressure at this level?

1. Cannot be determined
2. $P_A - \rho_{fluid} g(0.1\text{m})$
3. $P_A + \rho_{fluid} g(0.1\text{m})$
4. Atmospheric pressure
[✓ 5. 2 and 4]
6. 3 and 4

Example 3. Water pressure

At the surface of a body of water, the pressure you experience is atmospheric pressure. Estimate how deep you have to dive to experience a pressure of 2 atmospheres. Given that density of water is 1000 kg/m^3 and g = 10 m/s^2 (or N/kg).

<u>Solution</u>:

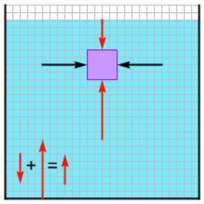
$$P_2 = P_1 + \rho gh$$

$$200000\text{ Pa} = 100000\text{ Pa} + (1000\text{ kg/m}^3)\times(10\text{ N/kg})\times h$$

h works out to 10 m. Every 10 m down in water increases the pressure by 1 atmosphere.

The origin of the buoyant force

$$P_2 = P_1 + \rho gh$$



The net upward buoyant force is the vector sum of the various forces from the fluid pressure.

Because the fluid pressure increases with depth, the upward force on the bottom surface is larger than the downward force on the upper surface of the immersed object.

$$F_{net} = \Delta P \times A = \rho_{fluid} gh \times A = \rho_{fluid} gV$$

This is for a fully immersed object. For a floating object, *h* is the height below the water level, so we get:

$$F_{net} = \rho_{fluid} gV_{disp}$$

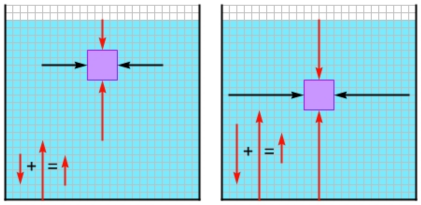
Conceptual example 4. When the object goes deeper …

Question: If we displace an object deeper into a fluid, what happens to the buoyant force acting on the object? You may assume that the fluid density is the same at all depths. The buoyant force …

1. increases
2. decreases
[✓ 3.] stays the same

Conceptual example 4. When the object goes deeper (cont'd)

Explanation: $P_2 = P_1 + \rho g h$



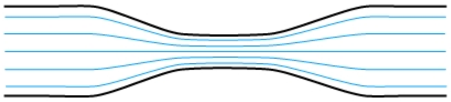
If the fluid density does not change with depth, all the forces increase by the same amount, leaving the buoyant force unchanged!

Fluid Dynamics

Flowing fluids – fluid dynamics

We'll start with an idealized fluid that:

1. Has streamline flow (no turbulence)
2. Is incompressible (constant density)
3. Has no viscosity (flows without resistance)
4. Is irrotational (no swirling eddies)

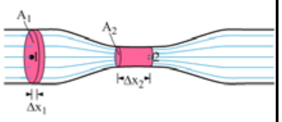


The continuity equation

We generally apply two equations to flowing fluids. One comes from the idea that the rate at which mass flowing past a point is constant, otherwise fluid builds up in regions of low flow rate.

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta x}{\Delta t} = \rho A v$$

The mass flow rate of a fluid must be continuous. Otherwise fluid accumulates at some points. So, we have:



$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Density of fluid doesn't change $\Rightarrow$ $A_1 v_1 = A_2 v_2$ (continuity equation)

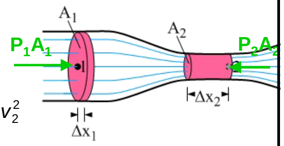
The fluid flows faster in narrow sections of the tube.

An application of continuity

The concept of continuity (a constant mass flow rate) is applied in our own circulatory systems. When blood flows from a large artery to small capillaries, the rate at which blood leaves the artery equals the sum of the rates at which blood flows through the various capillaries attached to the artery.

Applying energy conservation to fluids

Our second equation for flowing fluids comes from energy conservation.



$$U_1 + K_1 + W_{nc} = U_2 + K_2$$

$$mgy_1 + \frac{1}{2}mv_1^2 + W_{nc} = mgy_2 + \frac{1}{2}mv_2^2$$

$$W_{nc} = F_1\Delta x_1 - F_2\Delta x_2 = P_1A_1\Delta x_1 - P_2A_2\Delta x_2$$

$$mgy_1 + \frac{1}{2}mv_1^2 + P_1A_1\Delta x_1 = mgy_2 + \frac{1}{2}mv_2^2 + P_2A_2\Delta x_2$$

Dividing by volume:

$$\rho gy_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho gy_2 + \frac{1}{2}\rho v_2^2 + P_2$$

(Bernoulli's equation)

Bernoulli's equation

$$\boxed{\rho gy_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho gy_2 + \frac{1}{2}\rho v_2^2 + P_2}$$

Simple Interpretation of Bernoulli's equation

W_{nc} by the forces of pressure in moving a unit volume of the fluid from point 1 to point 2.

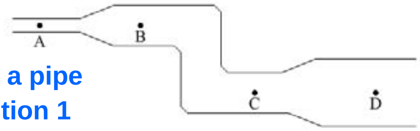
$$\rho gy_1 + \frac{1}{2}\rho v_1^2 + \overbrace{P_1 + (-P_2)} = \rho gy_2 + \frac{1}{2}\rho v_2^2$$

PE per unit volume of the fluid at point 1

KE per unit volume of the fluid at point 1

PE per unit volume of the fluid at point 2

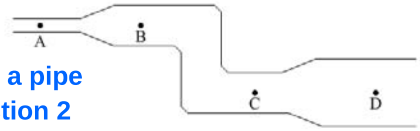
KE per unit volume of the fluid at point 2



Fluid in a pipe – question 1

The pipe is narrow where point A is located, widens out some where points B and C are located, and widens again where point D is located. The level of points C and D is lower than that of points A and B. If the fluid is at rest, rank the points based on their pressure.

1. Equal for all four
2. A>B=C>D
3. D>B=C>A
4. A=B>C=D
5. C=D>A=B
6. A>B>C>D
7. D>C>B>A
8. It's ambiguous - there are two possible answers



Fluid in a pipe – question 2

The pipe is narrow where point A is located, widens out some where points B and C are located, and widens again where point D is located. The level of points C and D is lower than that of points A and B. If the fluid is flowing from left to right, rank the points based on the fluid speed.

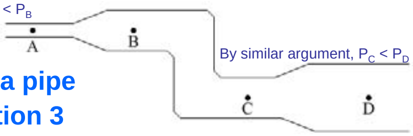
1. Equal for all four
2. A>B=C>D
3. D>B=C>A
4. A=B>C=D

[✓ 5. C=D>A=B]

6. A>B>C>D
7. D>C>B>A
8. It's ambiguous - there are two possible answers

Use the concept of continuity to figure out this problem.

$v_A > v_B \Rightarrow P_A < P_B$



By similar argument, $P_C < P_D$

Fluid in a pipe – question 3

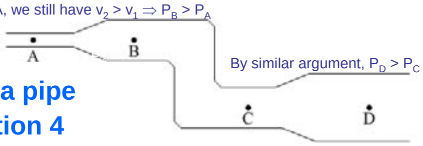
The pipe is narrow where point A is located, widens out some where points B and C are located, and widens again where point D is located. The level of points C and D is lower than that of points A and B. If the fluid is flowing from left to right, rank the points based on the pressure.

1. Equal for all four
2. A>B=C>D
3. D>B=C>A
4. A=B>C=D
5. C=D>A=B
6. A>B>C>D

[✓ 7. D>C>B>A]

8. It's ambiguous - there are two possible answers

In going from B to A, we still have $v_2 > v_1 \Rightarrow P_B > P_A$



By similar argument, $P_D > P_C$

Fluid in a pipe – question 4

The pipe is narrow where point A is located, widens out some where points B and C are located, and widens again where point D is located. The level of points C and D is lower than that of points A and B. If the fluid is flowing from right to left, rank the points based on the pressure.

1. Equal for all four
2. A>B=C>D
3. D>B=C>A
4. A=B>C=D
5. C=D>A=B
6. A>B>C>D
7. [✓ D>C>B>A]
8. It's ambiguous - there are two possible answers

The answer here being the same as that to question 3 is because Bernoulli's equation, depending on v_1^2 and v_2^2, does not depend on the sign of the velocities.

Three holes in a cylinder

A cylinder, open to the atmosphere at the top, is filled with water. It stands upright on a table. There are three holes on the side of the cylinder, but they are covered to start with. One hole is 1/4 of the way down from the top, while the other two are 1/2 and 3/4 of the way down.

When the holes are uncovered, water shoots out.

Which hole shoots the water farthest horizontally on the table?

1. The hole closest to the top.
2. The hole halfway down.
3. [✓ The hole closest to the bottom.]
4. It's a three-way tie.

Three holes in a cylinder

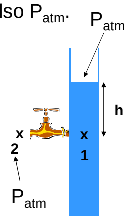
The pressure at the top of the cylinder is the atmospheric pressure (P_{atm}) since the cylinder is open to the atmosphere and normally the fluid speed there is zero. Similarly, the pressure at point 2 is also P_{atm}.

At point 1: $P_1 = P_{atm} + \rho hg$ $v_1 = 0$

At point 2: $P_2 = P_{atm}$

$$\cancel{\rho g y_1} + \frac{1}{2}\cancel{\rho v_1^2} + P_1 = \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2 + P_2$$

$(y_1 = y_2)$



Hence, $v_2 = (2hg)^{1/2}$

It shows that the deeper the hole is, the higher the speed of the fluid will be when it emerges from the hole.

Example A siphone

A flexible tube can be used as a simple siphon to transfer fluid from one container to a lower container. The fluid has a density of 1000. kg/m^3. See the dimensions given in the figure, and take atmospheric pressure to be 100 kPa and g = 10 m/s^2.

If the tube has a cross-sectional area that is much smaller than the cross sectional area of the higher container, what is the absolute pressure at:

(a) Point P? (b) Point Z

100 kPa 100 kPa

What is the speed of the fluid at:

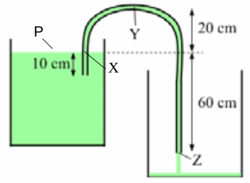
(c) Point Z? (d) Point Y? (e) Point X?

All three are the same equal 3.46 m/s

What is the absolute pressure at:

(e) Point Y? (f) Point X?

92 kPa 94 kPa



Example A siphone

(c) Solution for the speed of the fluid at Z (v):

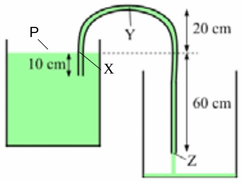
First, note that the pressures at P and Z are the same and equal 1 atm.

Apply the Bernoulli's equation to points P and Z and take h=0 at point P:

1atm = 1atm + ρg(-0.6m) + (1/2)ρv^2

$\Rightarrow$ (1/2)ρv^2 = ρg(0.6m)

$\Rightarrow$ v = (2g(0.6))$^{1/2}$ = 3.46 m/s



Example A siphone

(e) Solution for pressure at Y (P_Y):

Apply the Bernoulli's equation to points P and Y, and take h=0 at point P:

1atm = P_Y + ρg(0.2m) + (1/2)ρv^2

$\Rightarrow$100 kPa = P_Y + (1000)(10)(0.2) + (1000)(10)(0.6)

$\Rightarrow$ P_Y = (100 – 2 – 6) kPa = 92 kPa