Energy Conservation I

Tossing a ball

Let's apply the work - kinetic energy relation to a ball thrown straight up from an initial height of $y = 0$ that reaches a maximum height $y = h$ before falling back down to $y = 0$.

At the maximum height, the velocity is zero.

For the up part of the trip, $m\boldsymbol{g}$ and displacement point in opposite directions, so we get:

$$\Delta K = W_{net} = -mgh$$

So, K decreases by mgh on the way up.

On the way down, however, $m\boldsymbol{g}$ and displacement point in the same direction, so the force of gravity will do positive work = $+mgh$, which has the same magnitude as the negative work it did when the ball was on its way up. Therefore, K will increase back to its initial value.

Tossing a ball – Gravitational Potential Energy

From the above example, it looks as if the kinetic energy of the ball was transformed to another form of energy (with amount = mgh) as the ball was moving up. But on its way down, this energy (mgh) was released and transformed back to kinetic energy.

What kind of energy is mgh?

Gravitational potential energy. Kinetic energy is an energy associated with motion. Potential energy is an energy associated with position.

Gravitational potential energy provides us an alternative way of talking about the work done by gravity.

Energy Conservation involving K and U only

By the Work-Kinetic Energy Theorem,

$\Delta K = W = -mg\Delta h$ (choosing up to be positive)
$K_f - K_i = -mg(h_f - h_i)$
$K_f + mgh_f = K_i + mgh_i$
(Denote **gravitational potential energy** by $U = mgh$)
$K_f + U_f = K_i + U_i$

So, at every point along the trajectory, the **total mechanical energy** $(E = U + K)$ of the object remains constant, i.e. ***conserved ....*** *

• *caveat: only if we neglect air resistance*

Conserve and Non-conservative Forces

Gravitation is a sort of ***conservative force*** --- defined to be one where the associated energy, U, is determined solely by the position.

   When only conservative forces are involved, the total mechanical energy, E, is conserved (i.e., equal to a constant). But if non-conservative forces are involved, E is not conserved.

Forces that are not conservative: friction, air resistance, force exerted by you in moving a box on a horizontal surface, etc.…

Law of Conservation of Mechanical Energy

If there are non-conservative forces producing a net work, W_{nc}, on an object during its motion, we have

$$W_{nc} = \Delta K + \Delta U = \left(K_f - K_i\right) + \left(U_f - U_i\right)$$

$$\Rightarrow \boxed{U_i + K_i + W_{nc} = U_f + K_f}$$ Five-term energy conservation equation

or $W_{nc} = E_f - E_i$ (Recall: mechanical energy, $E = U + K$)

If the net work on an object by non-conservative forces is zero, then its total mechanical energy ***does not change***:

$$E_f = E_o$$

1

Work-KE Theorem vs. Law of Conservation of Mechanical Energy

Work-KE Theorem: $W_{net} = \Delta K$

Law of Conservation of Mechanical Energy:
$$W_{nc} = \Delta E$$

Here, you take any terms that contain mg out of the sum of forces for the net force

The Law of conservation of mechanical energy shifts our focus to the total mechanical energy of the system, which is a better representation of the energy state of the system.

Potential Energy – irrelevance of the choice of level zero for the coordinate system

Law of Conservation of Mechanical Energy:

$$W_{nc} = \Delta E$$
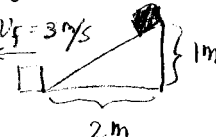$$= \Delta K + \Delta U$$
$$= \Delta K + mg\Delta h$$

From the above, we see that what matters in the Law of Conservation of Mechanical Energy is the change in altitude Δh of the object. That means it doesn't matter what we choose to be level zero.

Rolling a cart down an incline

A cart with mass 1 kg is rolled down from the top of an incline that measures 2 m horizontally and 1 m vertically. Suppose the cart is initially at rest. When it reaches the bottom of the incline, it acquires a speed of 2 m/s. Find the coefficient of kinetic friction. Use g = 10 m/s².

$v_f = 3 \, m/s$

$$K_i + U_i + W_{nc} = K_f + U_f$$

$$\Rightarrow W_{nc} = K_f - \cancel{K_i} + \cancel{U_f} - U_i$$
$$= 0$$

choose bottom of incline to be $h = 0$

9

Rolling a cart down an incline

$$W_{nc} = -fkd$$
$$= -\mu_k (mg \cos\theta)d$$
$$= -\mu_k \, mg \, \underbrace{d \cos\theta}_{=2m}$$
$$\underbrace{= 10N}$$
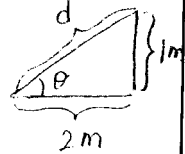
$$\therefore (-\mu_k)(20N) = \frac{1}{2}mv_f^2 - mgh_i$$
$$= \frac{1}{2}(1\,kg)(2\,m/s)^2 - (1\,kg)(10\,m/s^2)(1\,m)$$
$$= 2J - 10J = -8J$$

$$\Rightarrow \mu_k = -8J/-20J = 0.4$$

Roller Coaster

Suppose a coaster has a speed of 10 m/s at the bottom of a roller coaster hill. Because of losses, it loses 10% of its mechanical energy by the time it climbs to the top of the next hill, which is 3.25 m above the bottom of the first hill. Suppose the next hill can be approximated by a vertical circular arc with radius 10 m. Use g = 10 m/s².

(a) Find the speed of the coaster at the top of the next hill.
(b) Find the normal force divided by the force of gravity (or the g-force) experienced by the riders of the coaster at the top of the next hill.
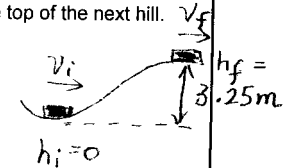
11

Roller Coaster

(a) Find the speed of the coaster at the top of the next hill.

$$K_i + U_i + W_{nc} = K_f + U_f$$
$$\underbrace{\quad}_{0.1 K_i}$$

$h_f = 3.25m$

$$K_f = 0.9 K_i - U_f$$

$h_i = 0$

$$\frac{1}{2}mv_f^2 = 0.9 \times \frac{1}{2}mv_i^2 - mgh_f$$

$$v_f^2 = 0.9 \times (10\,m/s)^2 - 2(10\,m/s^2)(3.25m)$$
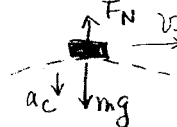
$$= (90 - 65)\,(m/s)^2$$

$$v_f = \sqrt{25}\,m/s = 5\,m/s$$

12

2

Roller Coaster

(b) Find the normal force divided by the force of gravity (or the g-force) experienced by the riders of the coaster at the top of the next hill.

$$mg - F_N = ma_c = \frac{mv_f^2}{r}$$

$$\Rightarrow F_N = mg - m\,v_f^2/r$$

$$\Rightarrow F_N/mg = 1 - v_f^2/rg$$

$$= 1 - (5\,m/s)^2/(10\,m \times 10\,m/s^2)$$
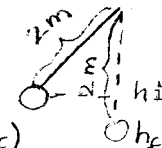
$$= 1 - 0.25 = 0.75$$

13

A pendulum

A ball with a mass of 1 kg is tied to a massless string that is 2 m long. The other end of the string is tied to a frictionless anchor at the ceiling. Suppose the ball is initially pulled back such that the string makes an angle θ with the vertical, where cosθ = 0.9. Then it is let go from rest there. Neglect any losses. Use g = 10 m/s².

At what height from where the ball is launched does it acquire the maximum speed? What's the speed of the ball there?

14

A pendulum

At what height from where the ball is launched does it acquire the maximum speed? What's the speed of the ball there?

$$\cancel{K_i} + U_i + \cancel{W_{nc}} = K_f + U_f$$

$$= 0$$

$$\Rightarrow K_f = U_i - U_f = mg\,(h_i - h_f)$$

∴ K_f and hence v_f is the max, when h_f is the min. This occurs at the bottom of the pendulum.

A pendulum

From the drawing, it is apparent that

$$h_i - h_f = 2m - 2m\cos\theta$$

$$= 0.2\,m \qquad = 0.9$$

∴ $\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}g \cdot 0.2m$

$$\Rightarrow v_f^2 = 2g \cdot 0.2\,m$$

$$\Rightarrow v_f = \sqrt{2 \times 10\,m/s^2 \cdot 0.2\,m} = 2\,m/s$$

3