What you can do with cold atoms on a honeycomb lattice that you cannot do with graphene !

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REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

The electronic properties of graphene

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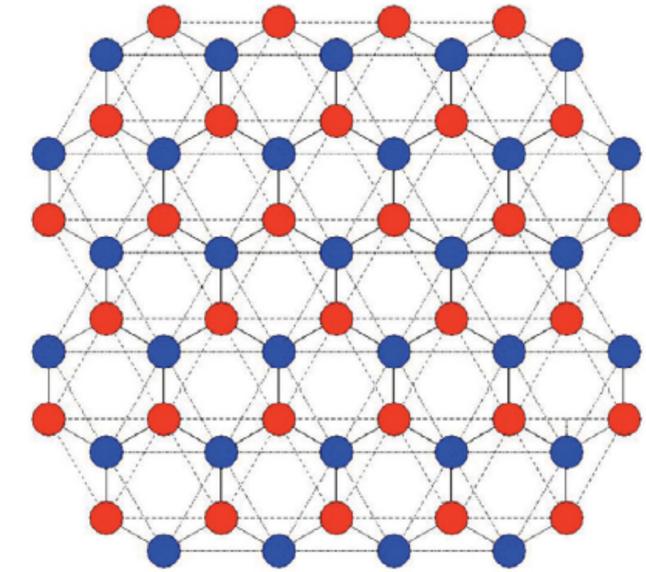
Outline

1. Graphene for beginners
2. Fermion-Fermion interactions
3. Fermi-Bose mixtures
4. Gauge fields and pseudo-magnetism
5. Bilayers
6. Non-equilibrium many-body physics
7. Conclusions

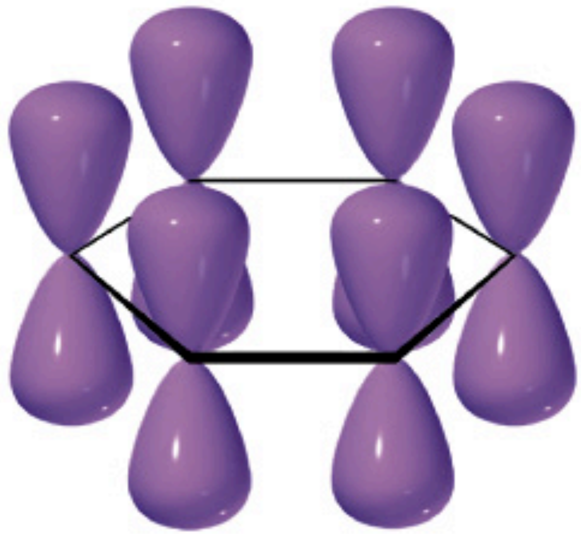
Origin of Dirac fermions in graphene

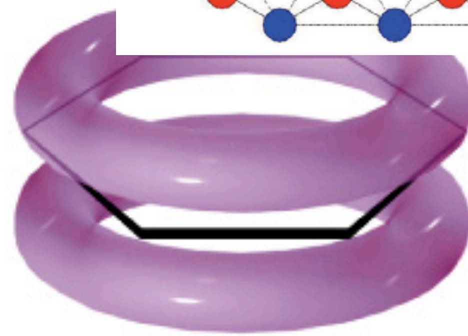
Benzene molec

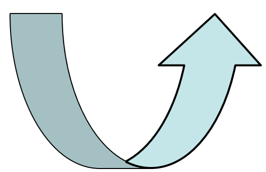


p_z states

π

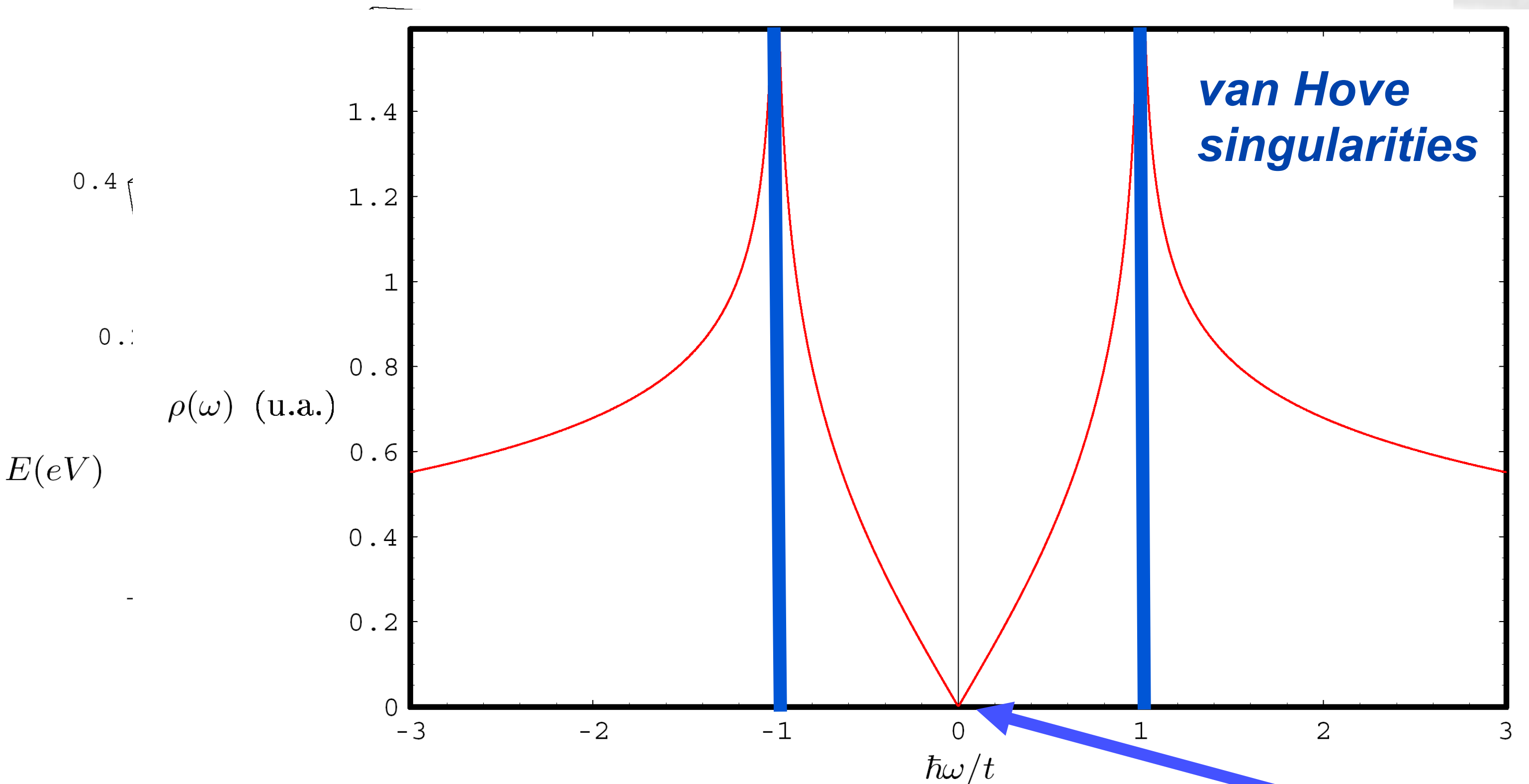






t ~ 2.7 eV

In momentum space



$$E_{\pm}(p) = \pm v_F \sqrt{p_x^2 + p_y^2} = \pm v_F p$$

$$E_{\pm}(p,m) = \pm\sqrt{m^2 v_F^4 + v_F^2 p_M^2} \quad \text{with} \quad m = 0$$

$$v_F = \frac{3ta}{2} \approx c/300$$

Vanishing density of states

Mathematically

$$H = v_F \begin{bmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{bmatrix}$$

2 X 2 - Sub-latices

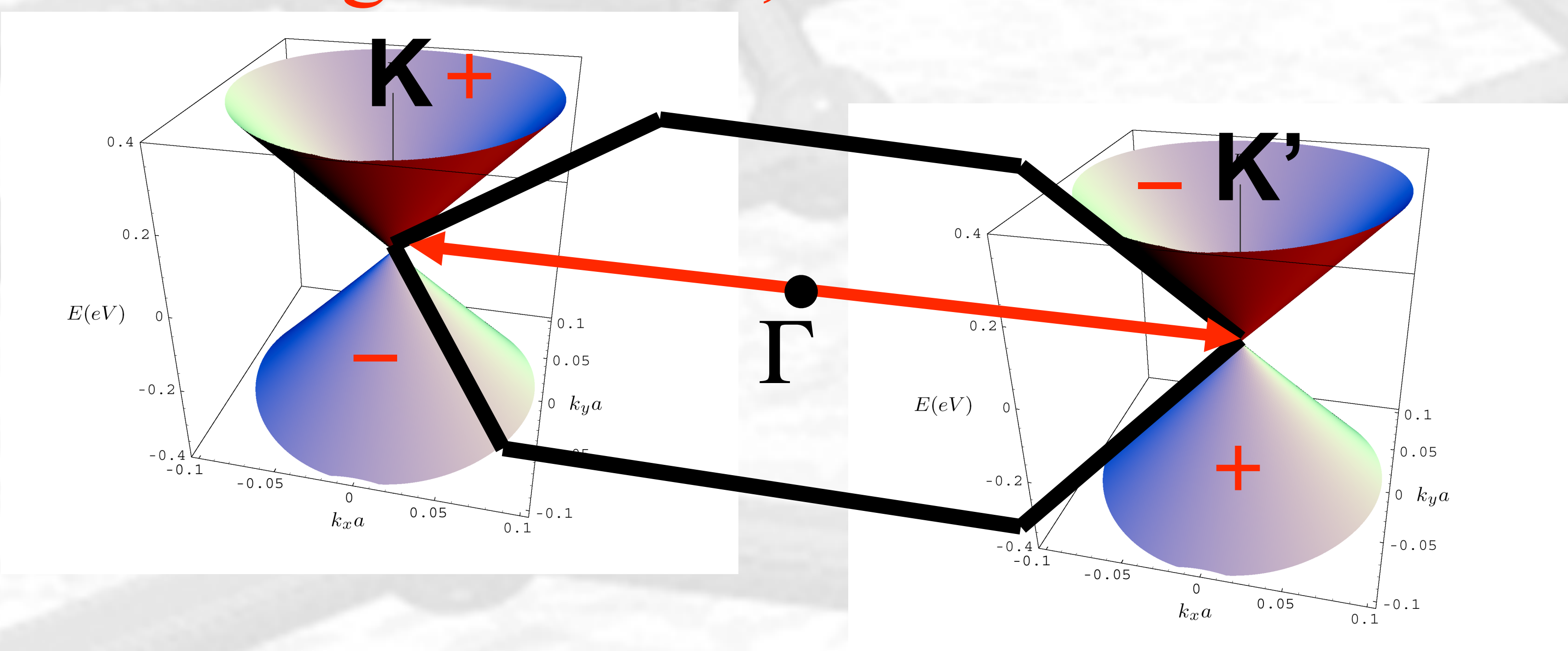
Px + i Py - Expansion around K point

$$H\Psi_{\pm}(\mathbf{p}) = \pm v_F p \Psi_{\pm}(\mathbf{p})$$

$$\sigma \cdot \mathbf{u_p} \Psi_{\pm}(\mathbf{p}) = \pm \Psi_{\pm}(\mathbf{p})$$

Chirality (helicity)

+ = right handed; - = left handed



Fermion-Fermion Interactions: μ = 0

$$\mathcal{H} = \hbar v_F \int d^2 r \bar{\Psi}(\vec{r})(i\sigma_x\partial_x + i\sigma_y\partial_y)\Psi(\vec{r})$$
$$+ \int d^2 r_1 \int d^2 r_2 \, U(\vec{r}_1 - \vec{r}_2)\bar{\Psi}(\vec{r}_1)\Psi(\vec{r}_1)\bar{\Psi}(\vec{r}_2)\Psi(\vec{r}_2)$$

In Graphene: $U(r) = e^2/(\varepsilon\, r)$ → $K/U = e^2/(\varepsilon\, v_F)$

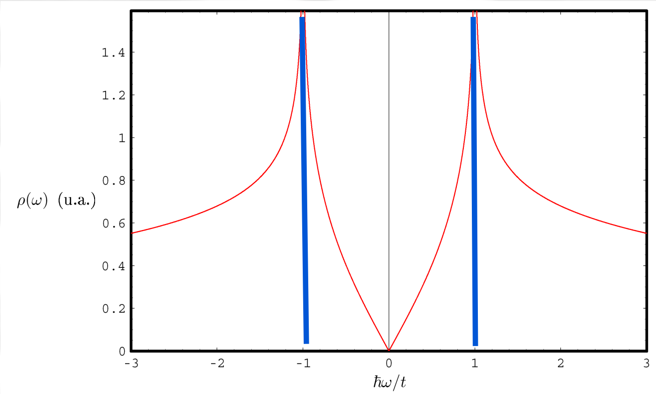
RG: marginally irrelevant

In AMO lattices: $U(r) = g\,\delta(r)$ → $K/U = v_F^2/(g\,E)$

RG: irrelevant

Phase transitions can occur at strong coupling requiring "exact" methods

Fermion-Fermion Interactions: $\mu = \pm t$



Condition hard to obtain in clean Graphene

PHYSICAL REVIEW VOLUME 89, NUMBER 6 MARCH 15, 1953

The Occurrence of Singularities in the Elastic Frequency Distribution of a Crystal

LÉON VAN HOVE
Institute for Advanced Study, Princeton, New Jersey
(Received December 5, 1952)

van Hove singularities

Collective Electron Specific Heat and Spin Paramagnetism in Metals

By EDMUND C. STONER, Ph.D. (Cambridge), Reader in Physics at the University of Leeds

(*Communicated by R. Whiddington, F.R.S.—Received December* 10, 1935)

Edmund C. Stoner

$$M = 2\mu^2 H \nu(\varepsilon_0)\left[1 + 2c_2(kT)^2\left\{\frac{1}{\nu}\frac{\partial^2\nu}{\partial\varepsilon^2} - \left(\frac{1}{\nu}\frac{\partial\nu}{\partial\varepsilon}\right)^2\right\}_{\varepsilon_0}\right]. \qquad (3.4)$$

Conditions for Ferromagnetism

$$\alpha_A = \tfrac{1}{2} z J_0 / N\mu^2$$

$$\alpha_A (\chi_A)_0 > 1,$$

$$\nu(\varepsilon_0) z J_0 / N > 1$$

New Many-Body States !

Fermion-Boson Mixtures

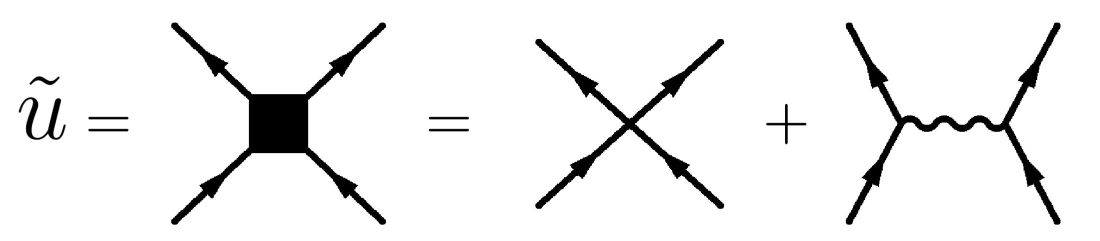
"Competing Orders in two-dimensional Bose-Fermi mixtures", by L. Mathey, S-W. Tsai, and A.H. Castro Neto, Phys. Rev. Lett. 97, 030601 (2006)

$$H = -t_f \sum_{\langle ij\rangle,s} f^\dagger_{i,s} f_{j,s} - t_b \sum_{\langle ij\rangle} b^\dagger_i b_j - \sum_i (\mu_f n_{f,i} + \mu_b n_{b,i}) + \sum_i \left[U_{ff} n_{f,i,\uparrow} n_{f,i,\downarrow} + \frac{U_{bb}}{2} n_{b,i} n_{b,i} + U_{bf} n_{b,i} n_{f,i} \right]$$

Bose condensation $b_0 \to \sqrt{N_0}$

$$H_{\text{eff.}} = \sum_{\mathbf{k}} \left\{ (\epsilon_{\mathbf{k}} - \mu_f) \sum_s f^\dagger_{\mathbf{k},s} f_{\mathbf{k},s} + \frac{U_{ff}}{V} \rho_{f,\mathbf{k},\uparrow} \rho_{f,-\mathbf{k},\downarrow} + \frac{1}{2V} V_{\text{ind.},\mathbf{k}}\ \rho_{f,\mathbf{k},} \rho_{f,-\mathbf{k}} \right\}, \qquad \tilde{V} = U_{bf}^2 / U_{bb}$$

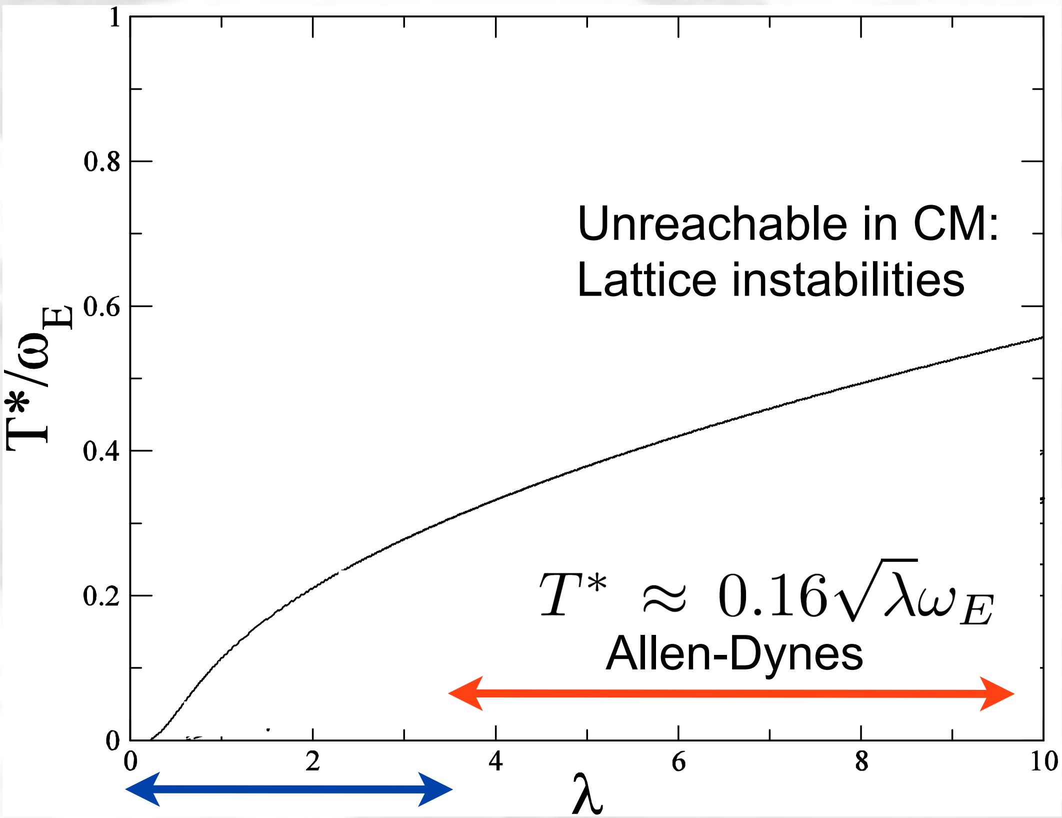
Retarded Interactions



Functional Renormalization Group Method: 2D Superfluidity

"Renormalization-group approach to superconductivity: from weak to strong electron-phonon coupling",
by S.-W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Philosophical Magazine 86, 2631 (2006).

$$\frac{d}{d\ell}\tilde{v}(\omega_1,\omega_3,\ell) = -\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\Lambda_\ell\ \tilde{v}(\omega_1,\omega,\ell)\tilde{v}(\omega,\omega_3,\ell)}{\Lambda_\ell^2 + Z_\ell^2(\omega)\omega^2}$$



$$\lambda = 2N(\mu)\tilde{V}^2/\omega_E$$

_T* is the temperature below which a superconducting gap appears.

_ Kosterlitz-Thouless transition: quasi-long range order driving by vortex unbinding.

$$T^* \approx 1.13\,\omega_E \exp\{-(1+\lambda)/(\lambda - \mu^*(1+\lambda))\}$$ McMillan regime

Nature or symmetry of the order parameter

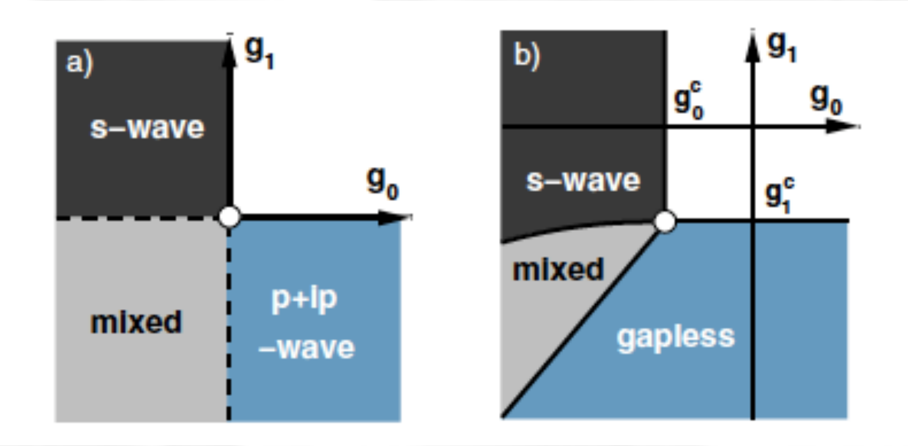
Superconducting States of Pure and Doped Graphene

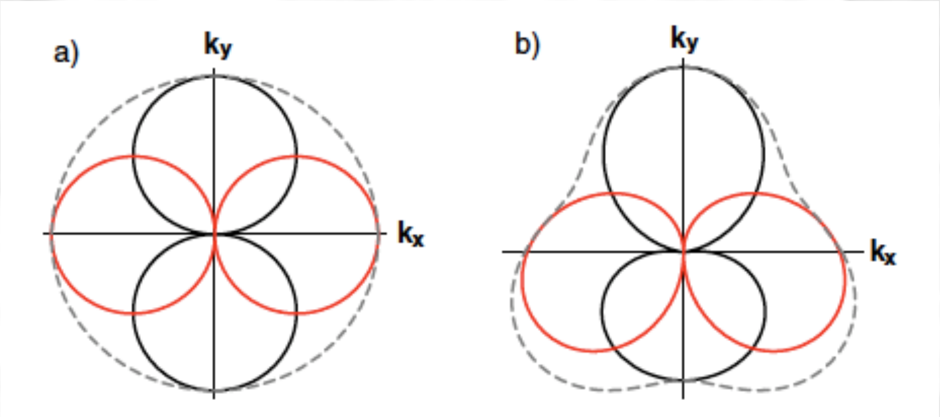
PRL 98, 146801 (2007)

Bruno Uchoa and A. H. Castro Neto

$$H_t = -\mu \sum_i \hat{n}_{g,i} - t \sum_{\langle ij \rangle} \sum_{s=\uparrow\downarrow} (a^\dagger_{i,s} b_{j,s} + \text{H.c.}),$$

$$H_P = \frac{g_0}{2} \sum_{is} [a^\dagger_{is} a_{is} a^\dagger_{i-s} a_{i-s} + b^\dagger_{is} b_{is} b^\dagger_{i-s} b_{i-s}] + g_1 \sum_{\langle ij \rangle} \sum_{s,s'} a^\dagger_{is} a_{is} b^\dagger_{js'} b_{js'},$$



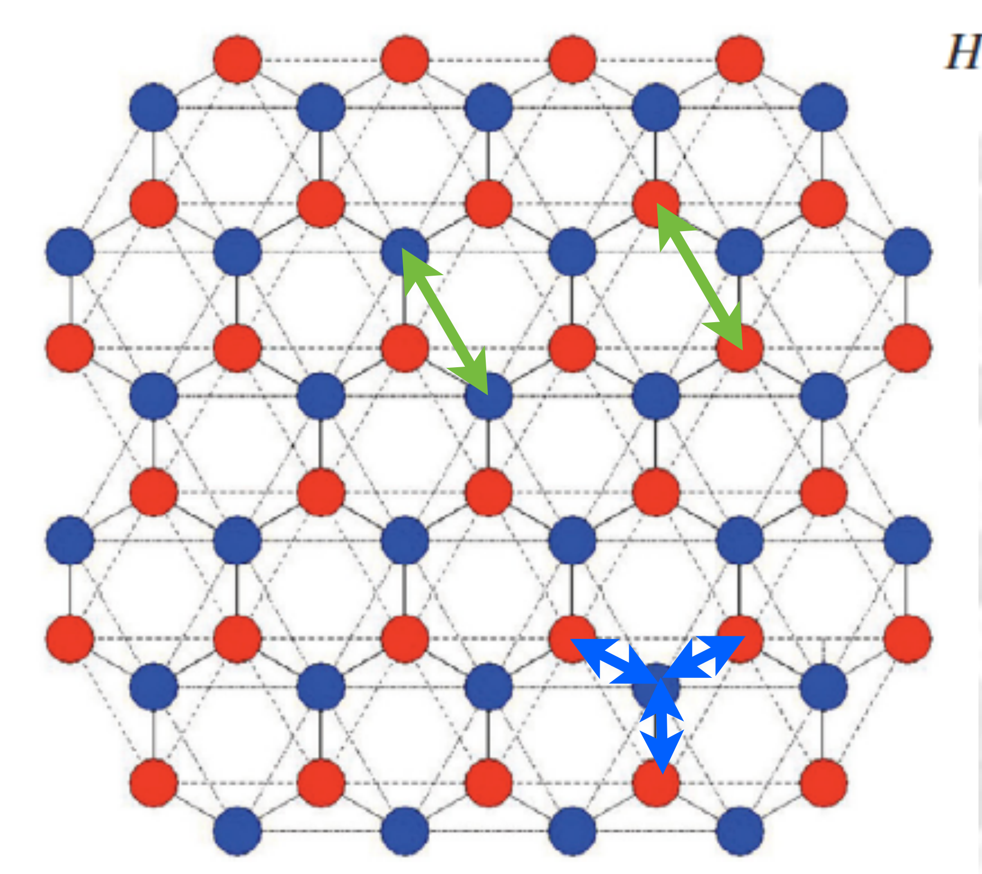


p+ i p

_ The vortices of a p + i p order parameter are Majorana fermions

_ Have been proposed for quantum computation - **D. A. Ivanov, Phys. Rev. Lett. 86, 268 (2001).**

Gauge Fields in the honeycomb lattice



$$H_{\rm od} = \sum_{i,j} \{\delta t_{ij}^{(ab)}(a_i^\dagger b_j + {\rm H.c.}) + \delta t_{ij}^{(aa)}(a_i^\dagger a_j + b_i^\dagger b_j)\},$$

$$H_{\rm od} = \int d^2 r \{\mathcal{A}(\mathbf{r}) a_1^\dagger(\mathbf{r}) b_1(\mathbf{r}) + {\rm H.c.} + \phi(\mathbf{r})[a_1^\dagger(\mathbf{r}) a_1(\mathbf{r}) + b_1^\dagger(\mathbf{r}) b_1(\mathbf{r})]\},$$

$$\mathcal{A}(\mathbf{r}) = \sum_{\vec{\delta}_{ab}} \delta t^{(ab)}(\mathbf{r}) e^{-i\vec{\delta}_{ab}\cdot\mathbf{K}},$$

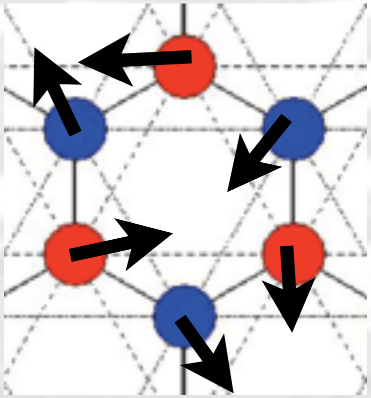
$$\mathcal{A}(\mathbf{r}) = \mathcal{A}_x(\mathbf{r}) + i\mathcal{A}_y(\mathbf{r}).$$

$$\phi(\mathbf{r}) = \sum_{\vec{\delta}_{aa}} \delta t^{(aa)}(\mathbf{r}) e^{-i\vec{\delta}_{aa}\cdot\mathbf{K}}.$$

$$H_0 = i v_F \sigma \cdot \nabla \quad \Rightarrow \quad H_M = \sigma \cdot (i v_F \nabla + \mathcal{A}(\mathbf{r})) + \Phi(\mathbf{r})$$

At long wavelengths the effect of local changes in the hopping parameters is equivalent to the addition to vector and scalar potentials to the Dirac equation

Changing the hopping parameters by changing the lattice



$\vec{u}(\vec{r})$

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Strain Tensor

$$\phi_{\rm dp}(\mathbf{r}) = g(u_{xx} + u_{yy}),$$

Compression and Dilation

$$\mathcal{A}_x^{(s)} = \tfrac{3}{4}\beta\kappa(u_{xx} - u_{yy}),$$

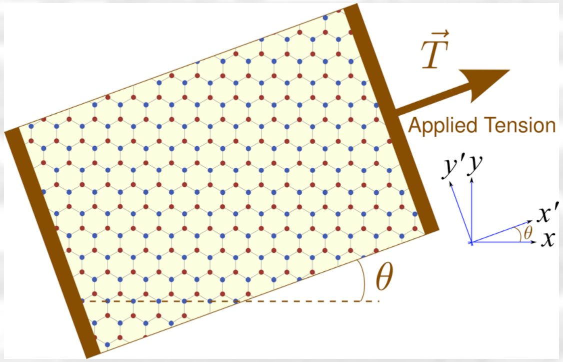
$$\mathcal{A}_y^{(s)} = \tfrac{3}{2}\beta\kappa u_{xy}.$$

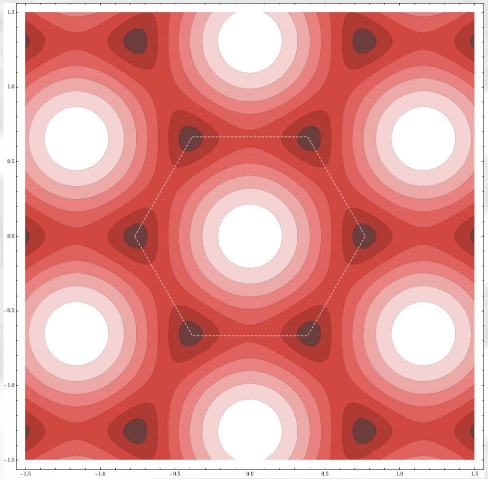
Strain and Shear

$$\beta = \frac{\partial t^{(ab)}}{\partial \ln(a)}.$$

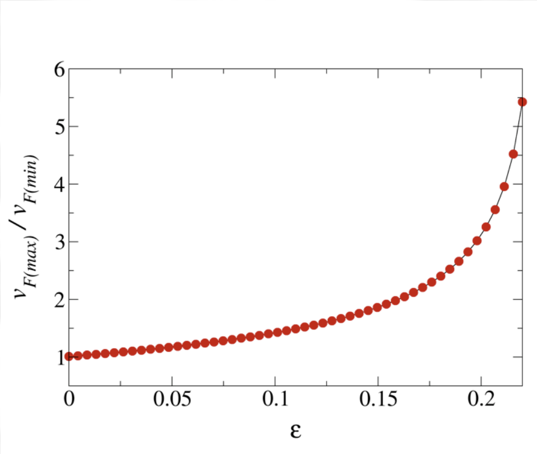
Uniform changes do not produce gauge fields but ....

V. Pereira, AHCN, N. Peres, Phys. Rev. B 80, 045401 (2009)

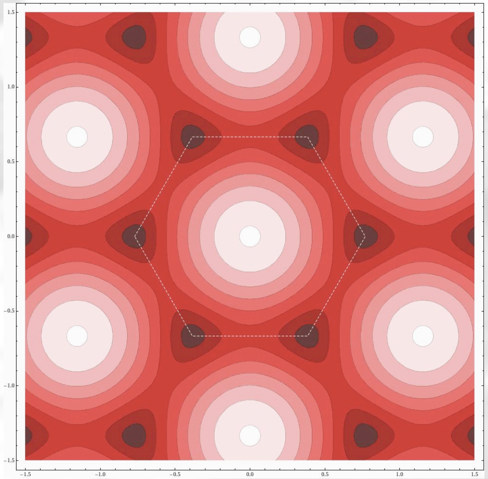




Armchair direction

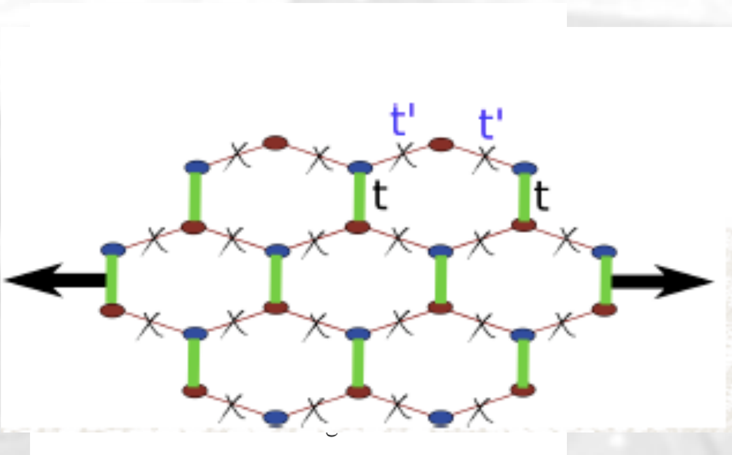


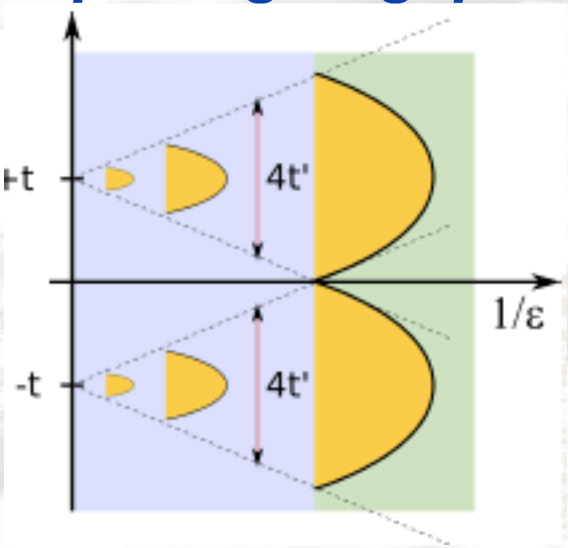
Spatial Anisotropy: 1D system



Zigzag direction

Opening of gaps



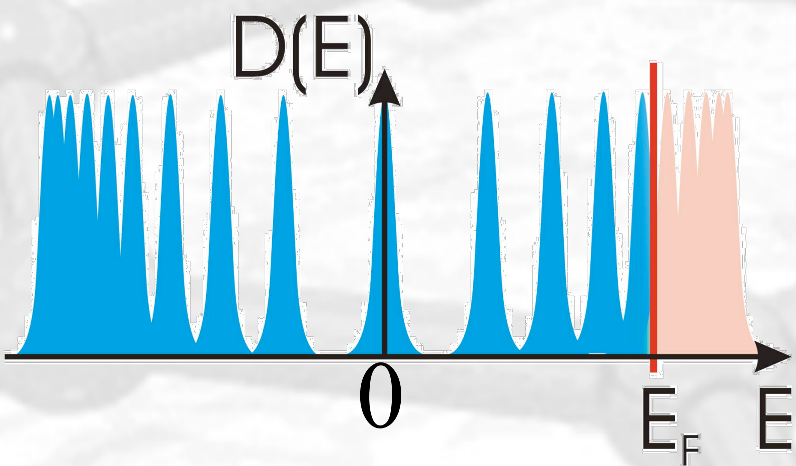


Dirac fermions in a uniform magnetic field

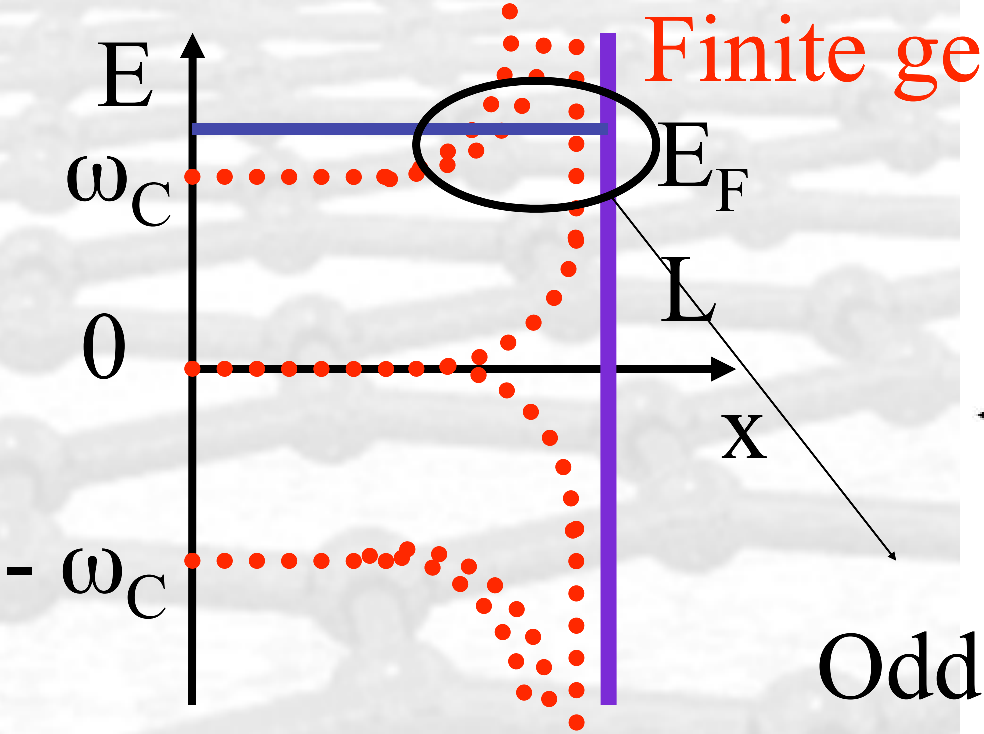
$$H = \sigma \cdot \left(i v_F \nabla + \frac{e}{c} \mathbf{A}(\mathbf{r}) \right)$$
$$\mathbf{A}(\mathbf{r}) = -By\mathbf{x}$$

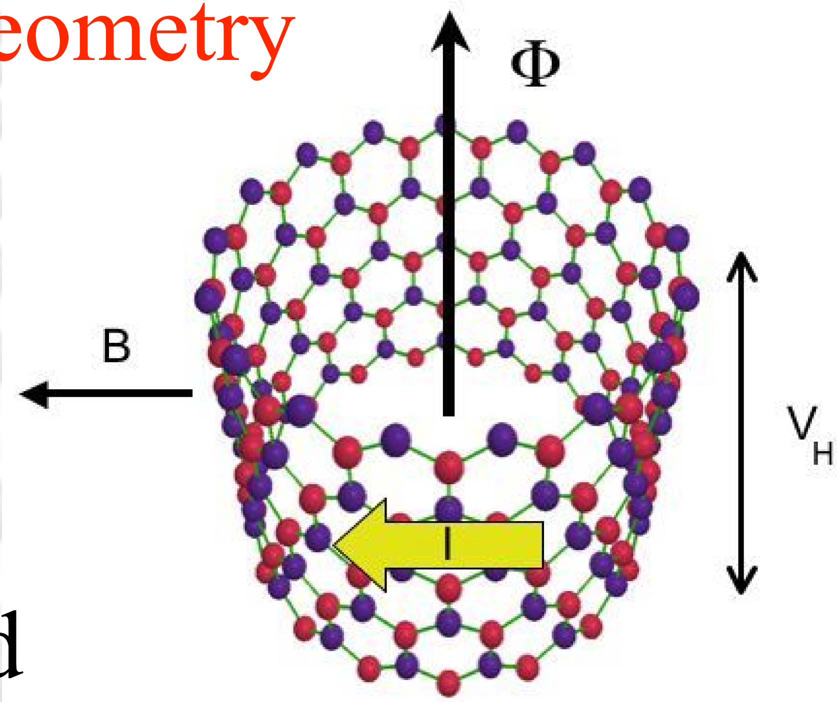
$$H\Psi_{n,\pm} = \pm E(n)\Psi_{n,\pm}$$
$$E(n) = \omega_C \sqrt{n}$$

$$\omega_c = \sqrt{2ehv_F^2 B}$$



Finite geometry





$$I = c\frac{\delta E}{\delta \Phi} \qquad \delta\Phi = \Phi_0 = \frac{hc}{e}$$

$$\delta E = \pm 2(2N+1)eV_H$$

IQHE

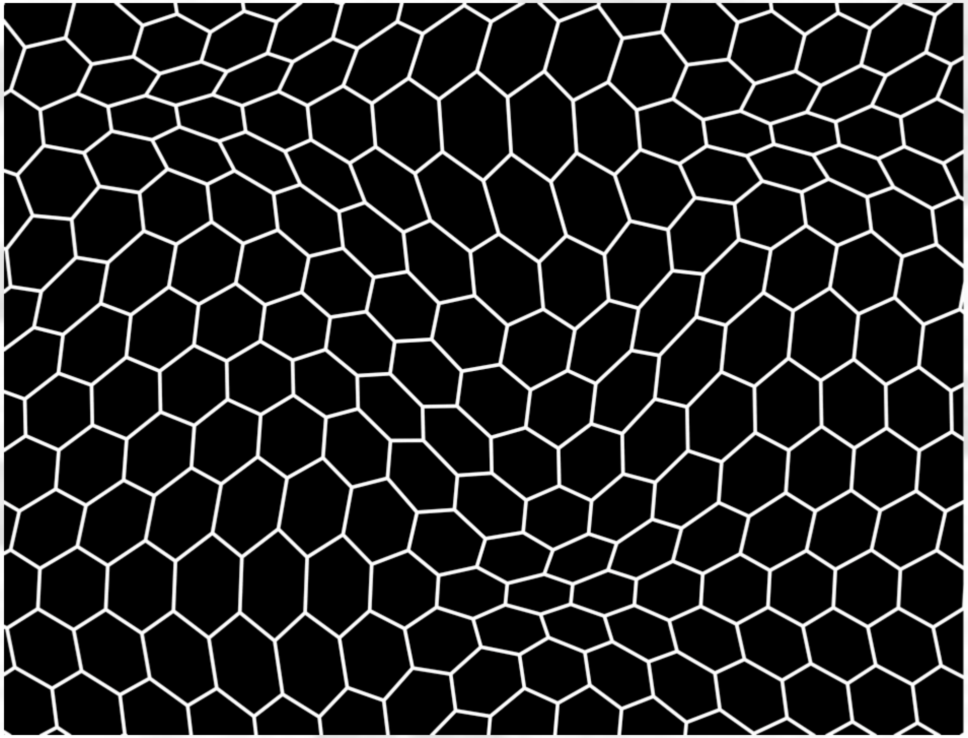
$$\sigma_{XY} = \frac{I}{V_H} = \frac{c}{V_H}\frac{\delta E}{\delta \Phi} = \pm 2(2N+1)\frac{e^2}{h}$$

Inverse engineering: from the magnetic field obtain the distortion

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathcal{A}_x = \frac{3}{4}\beta(u_{x,x} - u_{y,y})$$
$$\mathcal{A}_y = \frac{3}{2}\beta u_{x,y}$$

$$B = \frac{3}{4}\beta(u_{x,xy} + u_{y,yx})$$

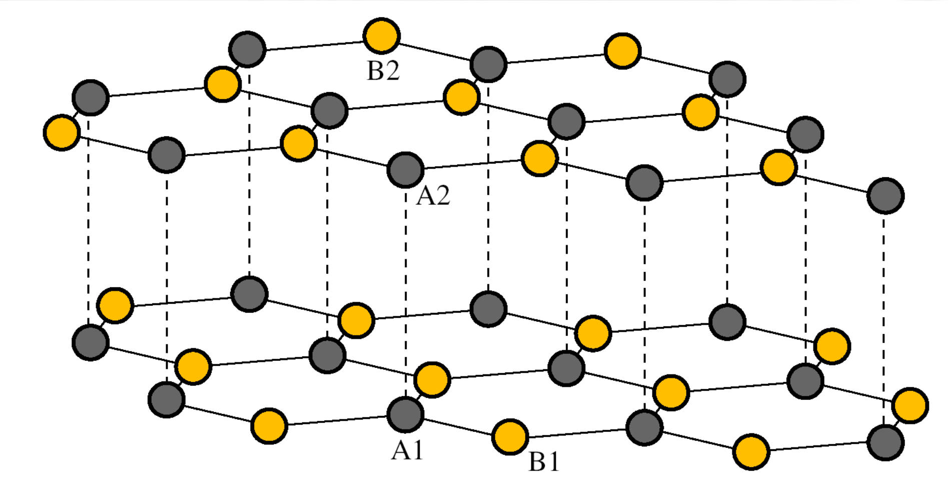


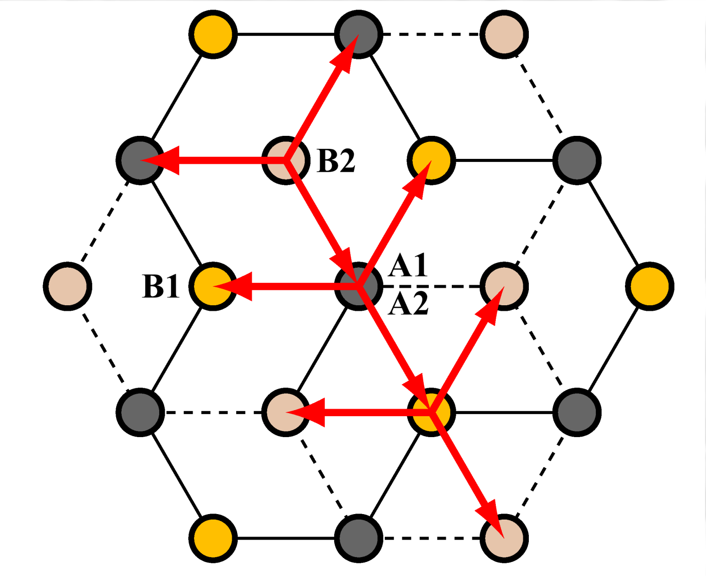
_ Quantum Hall Effect (Integer and Fractional) without a magnetic field

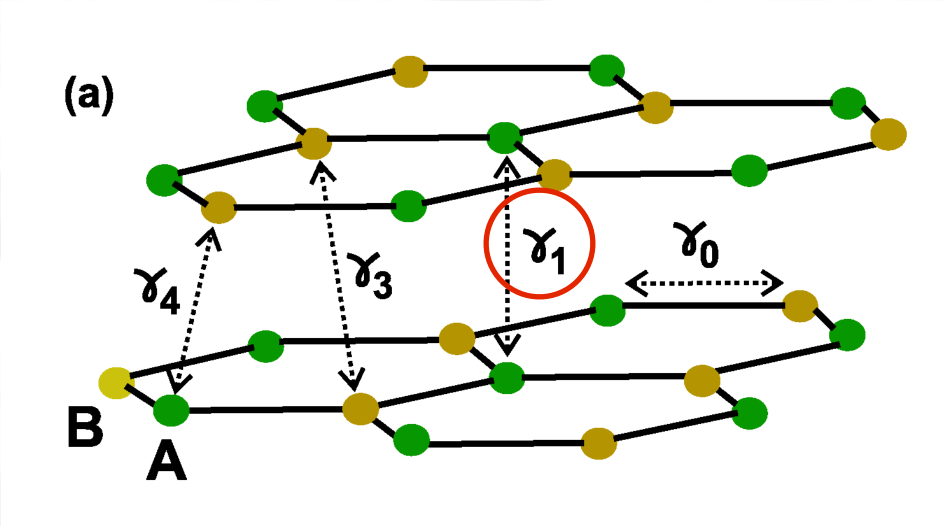
_ Mimic the effect of enormous magnetic fields (neutron stars, etc)

Uniform Magnetic Field: Guinea, F., *et al., Nature Physics* (2009) **6**, 30.

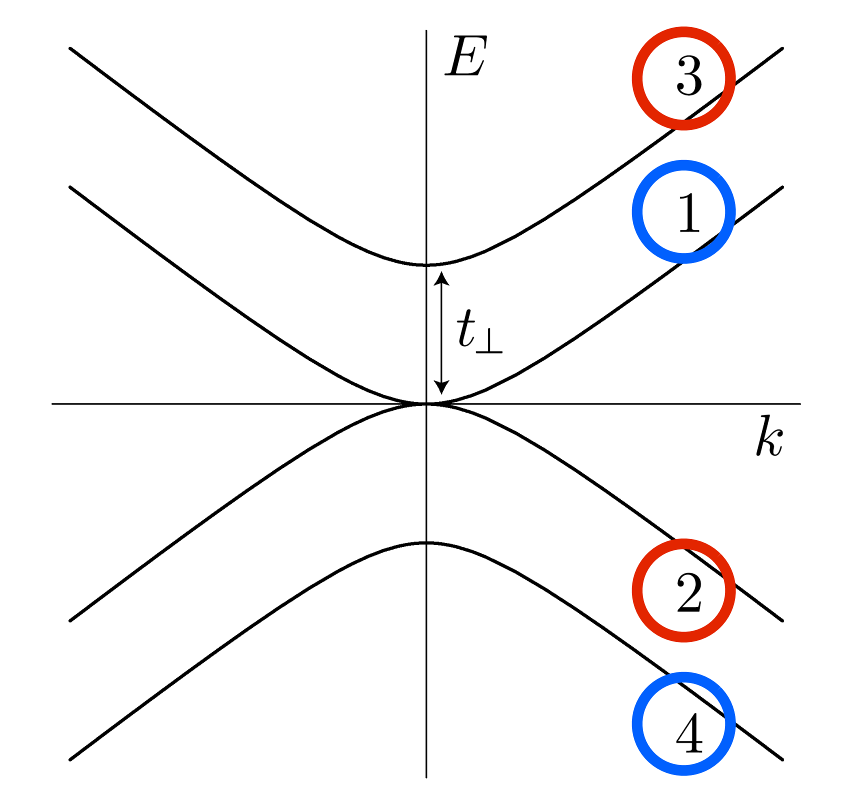
Bilayers







$$H = \begin{pmatrix} 0 & v_F p e^{i\varphi_p} & \gamma_1 & 0 \\ v_F p e^{-i\varphi_p} & 0 & 0 & 0 \\ \gamma_1 & 0 & 0 & v_F p e^{-i\varphi_p} \\ 0 & 0 & v_F p e^{i\varphi_p} & 0 \end{pmatrix}$$

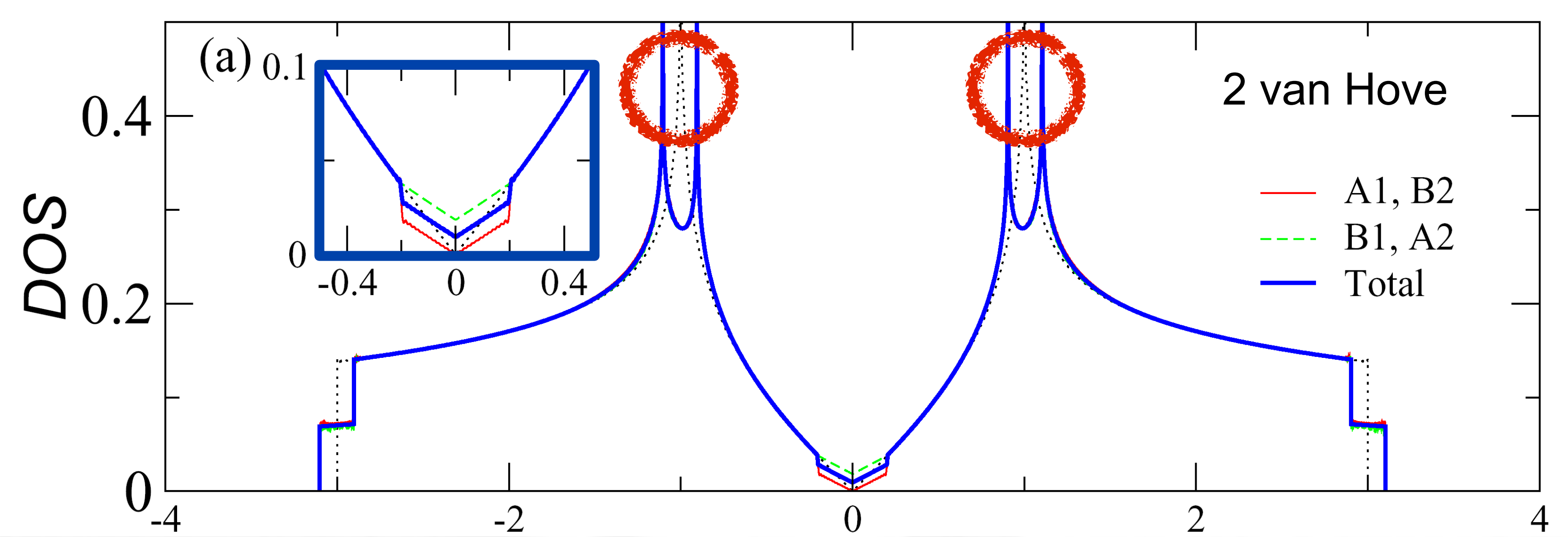


Minimal Model

$$E(P) = \pm\left(- m v_F^2 + \sqrt{m^2 v_F^4 + v_F^2 P^2} \right)$$

$$m v_F^2 = \gamma_1 = t_\perp$$

2 Sets of Massive Dirac Particles



Enhanced Density of States -> Enhanced Interactions

"Electron-electron interactions and the phase diagram of a graphene bilayer" by Johan Nilsson, A. H. Castro Neto, N. M. R. Peres, and F. Guinea, Phys. Rev. B 73, 214418 (2006).

$$[\chi_{\rm AFM}(\mathbf{q},\omega)]^{-1} = [\chi^0_{\rm AFM}(\mathbf{q},\omega)]^{-1} - U$$

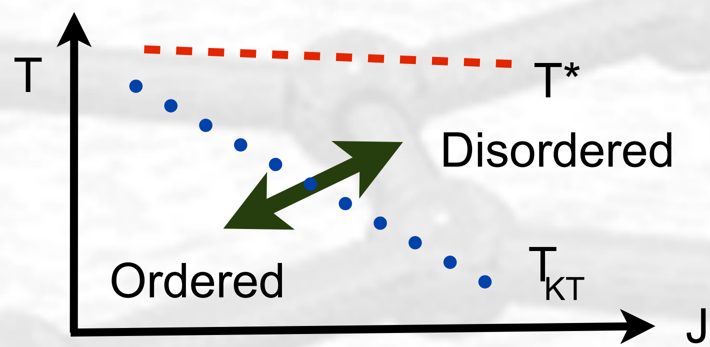
$$\chi^0_{\rm AFM}(\boldsymbol{q},\omega) \propto \log\left(\frac{\Lambda^2/2t}{\omega - |\mathbf{q}|^2/4t}\right) - \frac{|\boldsymbol{q}|^2}{4t\omega}\log\left(\frac{|\boldsymbol{q}|^2/2t}{\omega - |\mathbf{q}|^2/4t}\right)$$

A bilayer experiment not possible in CM: Many-Body Non-Equilibrium Problem

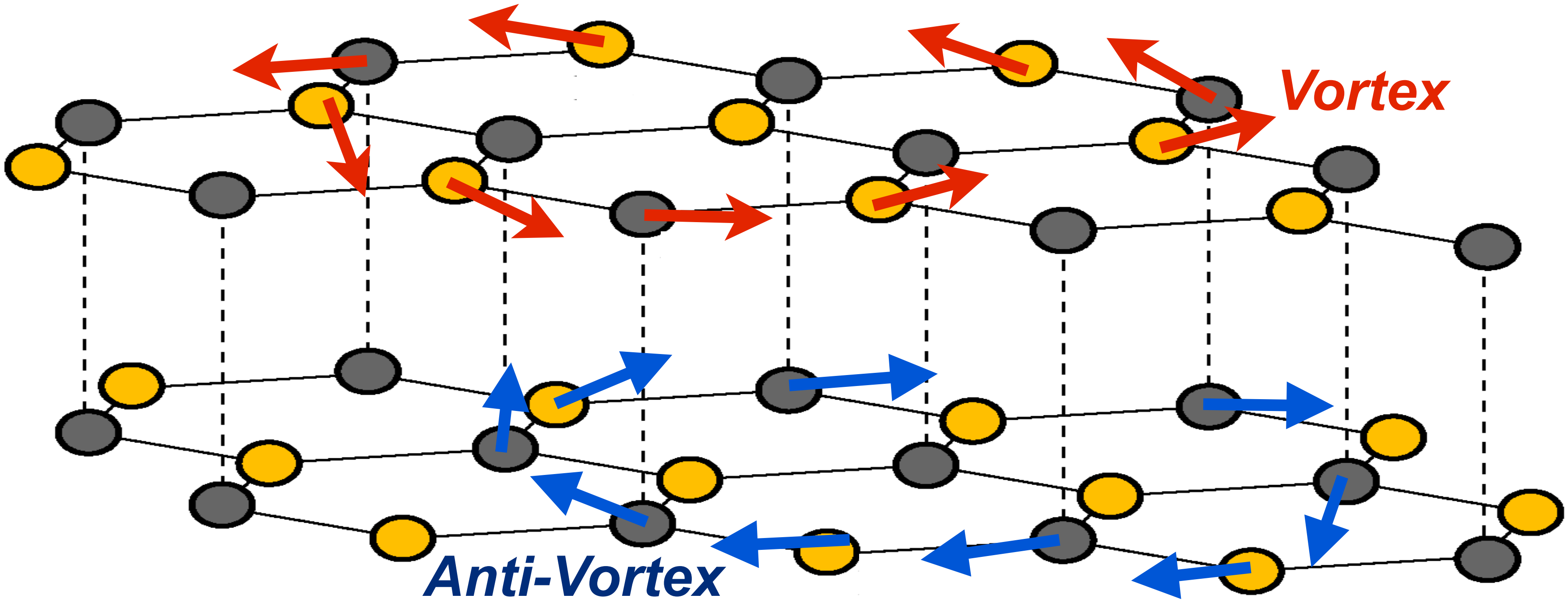
t=0 $\gamma_1 = 0$ $T^* > T > T_{KT}$ → Decoupled incoherent superfluids

Vortex-anti-vortex pairs destroy order

t>0 $\gamma_1 \neq 0$ → **?**



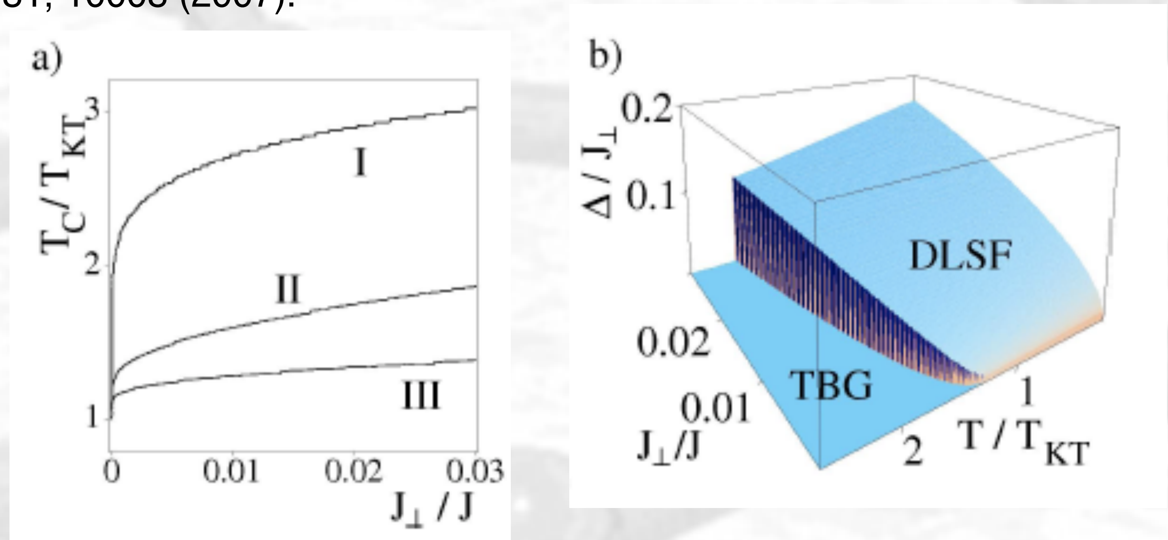
A system that is fast driven by a phase transition can generate topological defects at a density with is related to the rate at which the transition is crossed. This **Kibble-Zurek** (KZ) effect is believed to have occurred just after the Big Bang, when the emerging phase of the vacuum became disordered, giving rise to a large number of topological defects such as cosmic strings, which may be the primal ingredient for the formation of galaxies.



”Phase locking transition of coupled low dimensional superfluids” by Ludwig Mathey, A. Polkovnikov, and A. H. Castro Neto, Europhysics Letters 81, 10008 (2007).

$$J = 2T_{KT}/\pi$$

$$A_1 \sim J\exp(-J/T)$$



Conclusions:

_ *In cold atom systems it is possible to reach situations which are hard, or even impossible in real graphene.*

_ *This "unconventional" situations can lead to new many-body states that are rather unique and unusual in condensed matter.*

_ *The control over the lattice allows for study pseudo-magnetic field phenomena without the presence of a real magnetic field.*

_ *Permits to put the system into "extreme" situations which can only be obtained in certain astronomical environments.*

_ *Non-equilibrium many-body physics can be studied in a controlled "environment".*