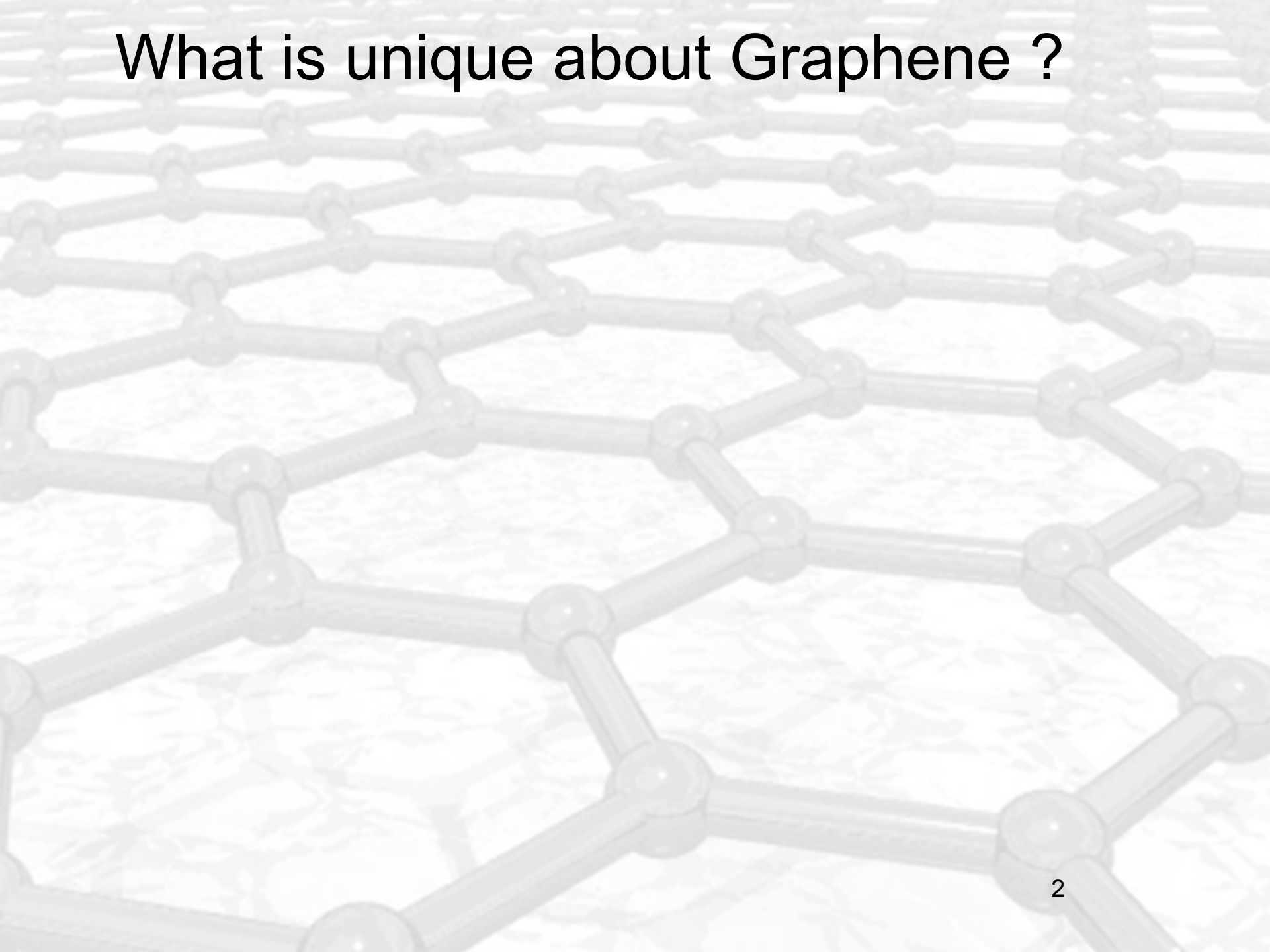


Graphene Wrinkles

Antonio H. Castro Neto



What is unique about Graphene ?



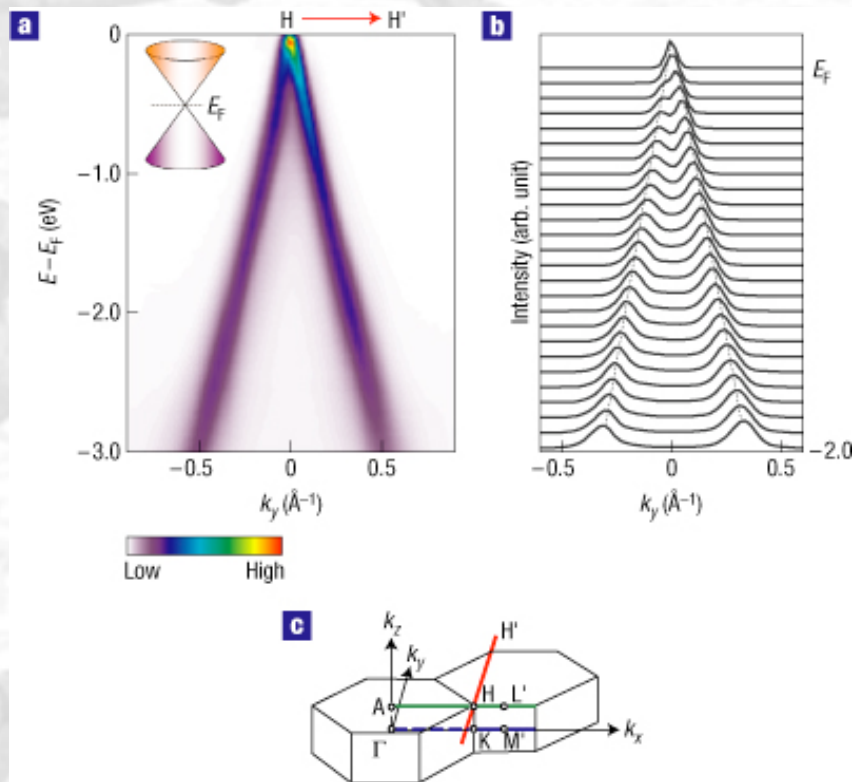


What is unique about Graphene ?

Is Diracness unique to Graphene ?

What is unique about Graphene ?

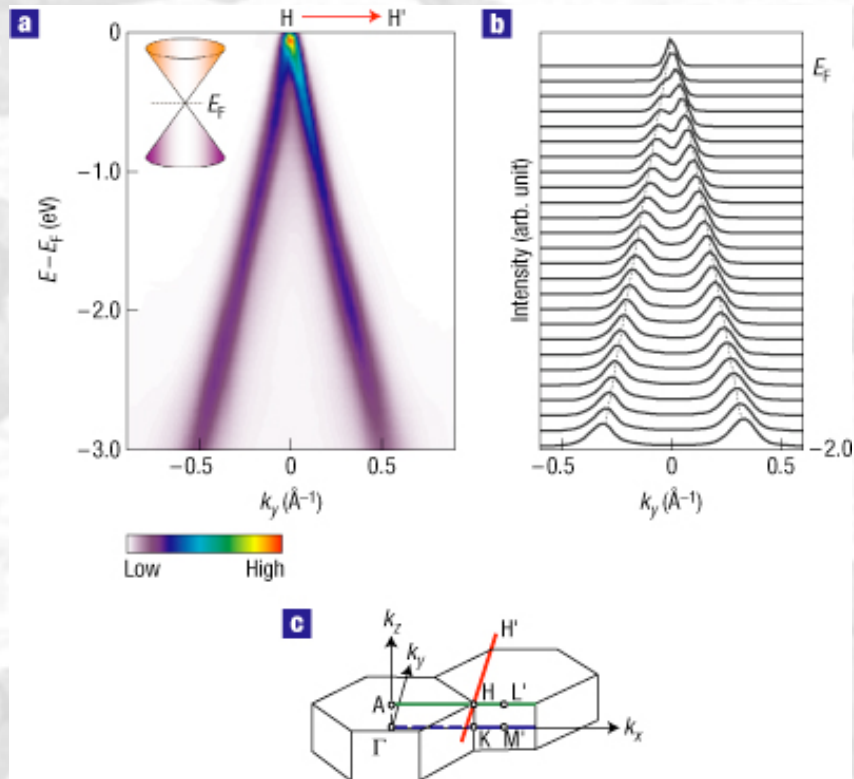
Is Diracness unique to Graphene ?



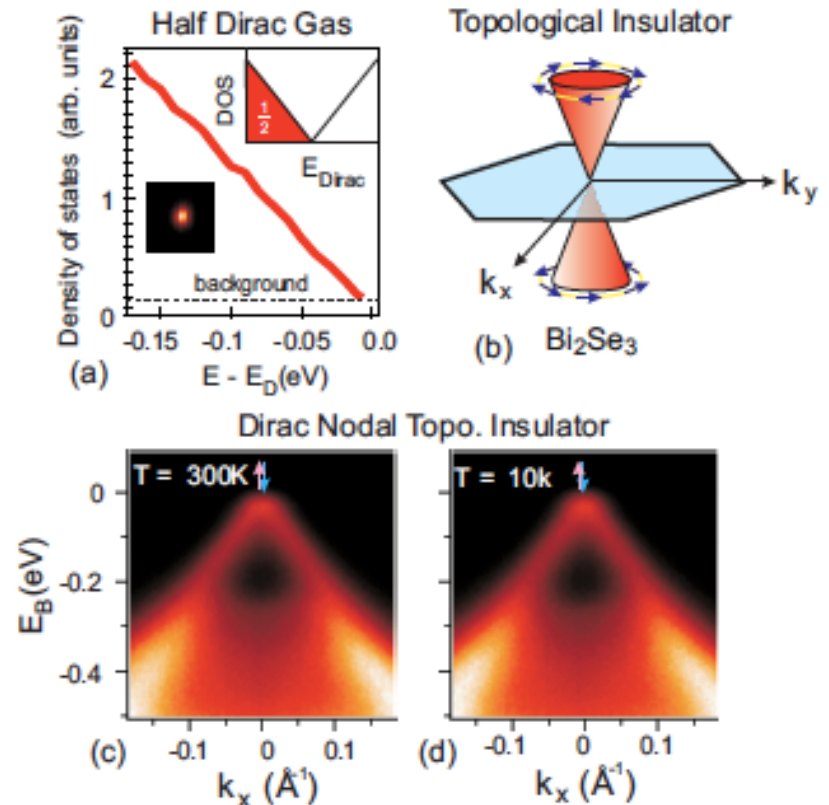
Zhou et al, Nature Physics 2, 595 - 599 (2006)

What is unique about Graphene ?

Is Diracness unique to Graphene ?



Zhou et al, Nature Physics 2, 595 - 599 (2006)



Hsieh et al, Nature 460, 1101 (2009).

What is unique about Graphene ?

Is Diracness unique to Graphene ?

Lenoir et al. Transport properties of Bi-rich Bi-Sb alloys

91

J. Phys. Chem. Solids Vol. 57, No. 1, pp. 89-99, 1996

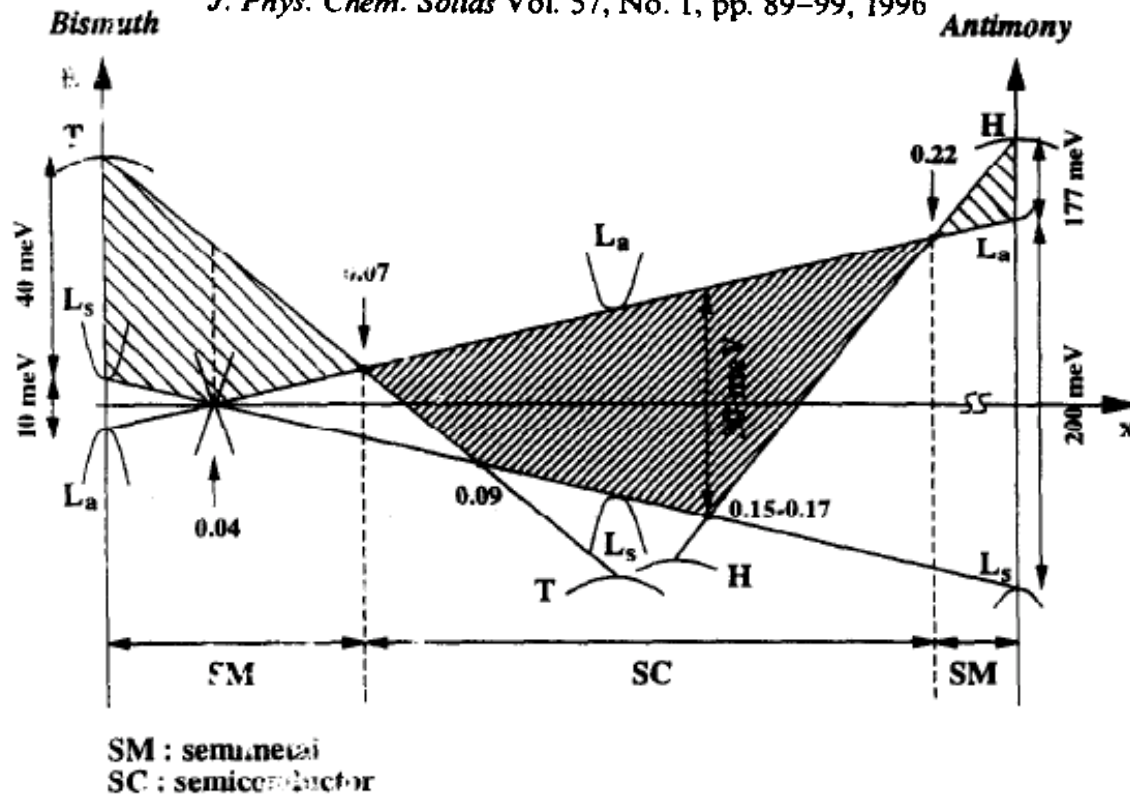


Fig. 1. Schematic representation of the energy bands near the Fermi level for $\text{Bi}_{1-x}\text{Sb}_x$ alloys as a function of x at 0 K. For simplicity, the L-, T- and H-point bands are drawn one on top of the others. Note that Mendez [4] has observed a strong temperature dependence of the L band parameters.

What is unique about Graphene ?

Is Diracness unique to Graphene ?

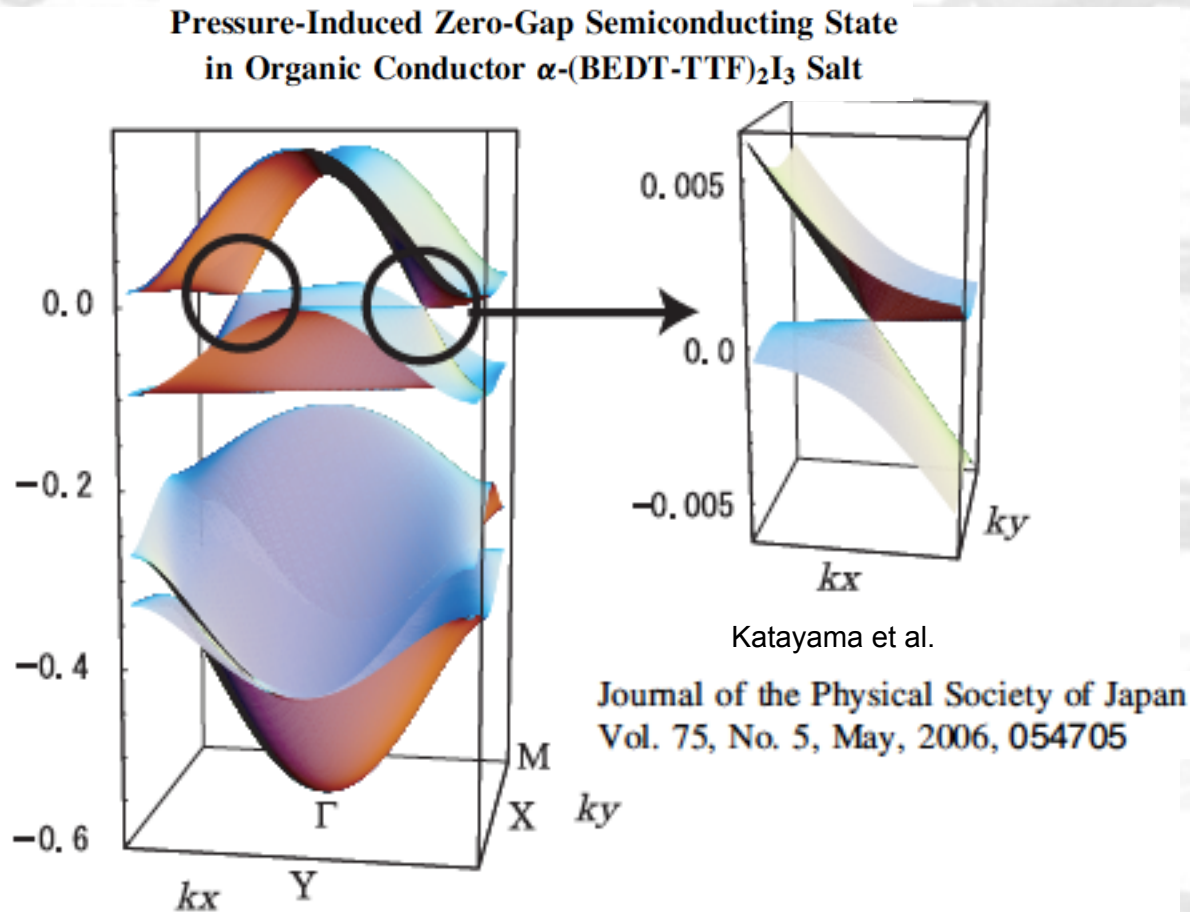
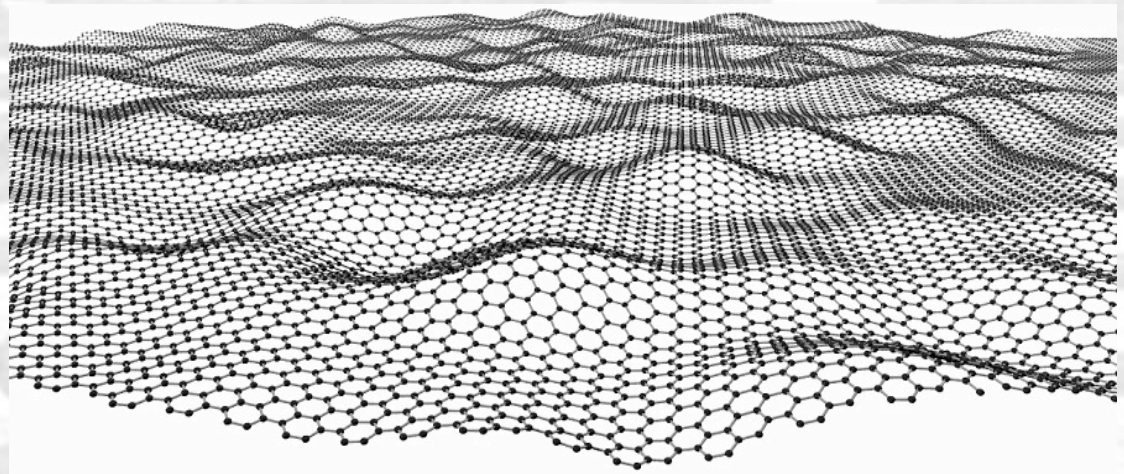
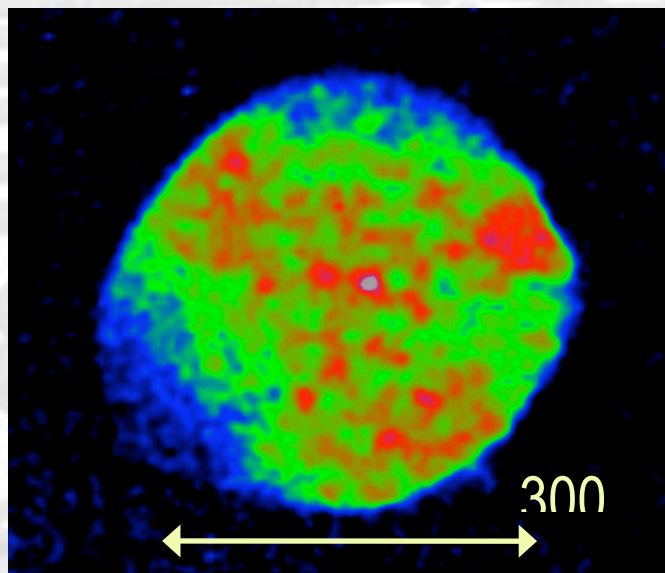


Fig. 2. Band dispersion of α -(ET)₂I₃ at $P_a = 4$ kbar in first Brillouin zone (left panel) and enlarged one around contact point at $k^0 = (0.602\pi, -0.353\pi)$ (right panel), where Fermi energy is taken as origin.

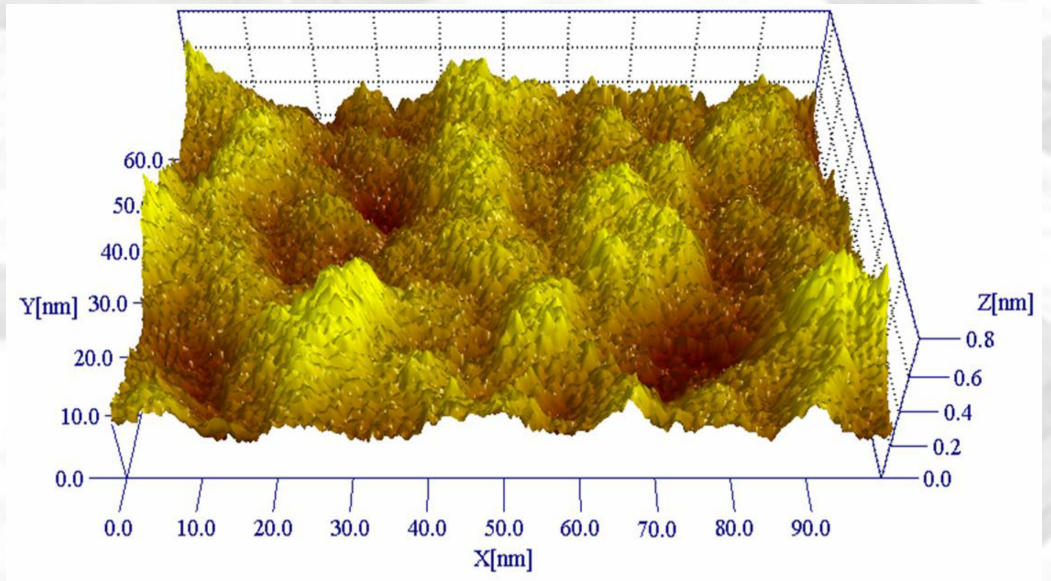
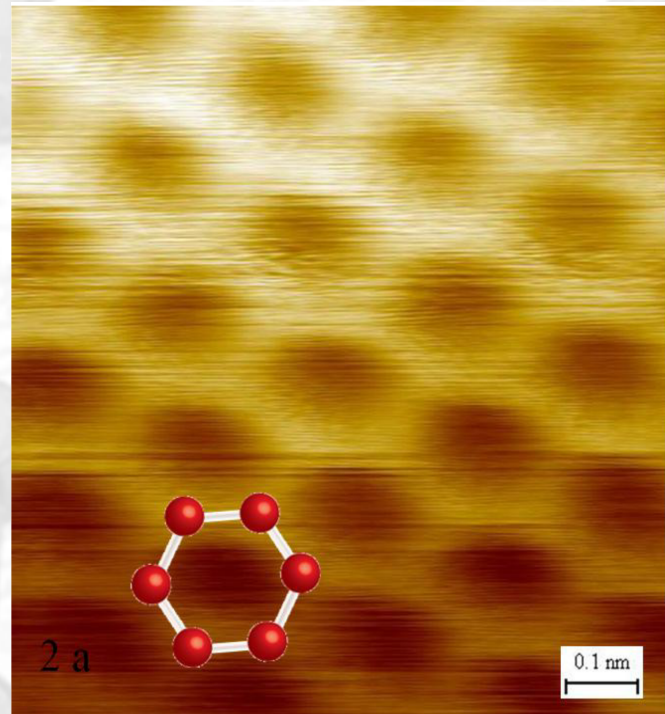
What is unique about Graphene ?



Lesson # 1: Graphene is soft !



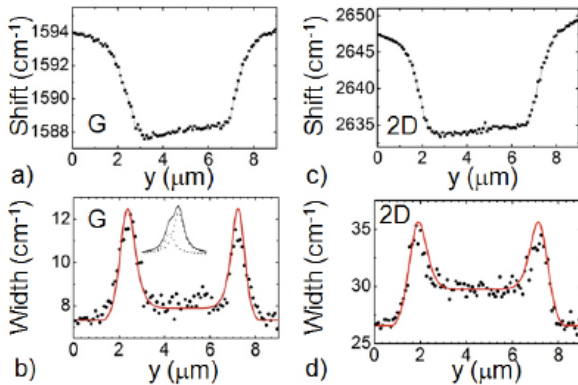
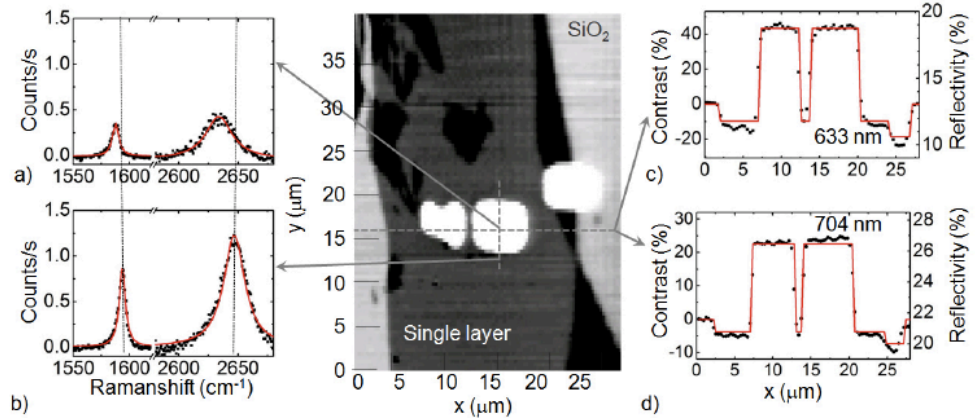
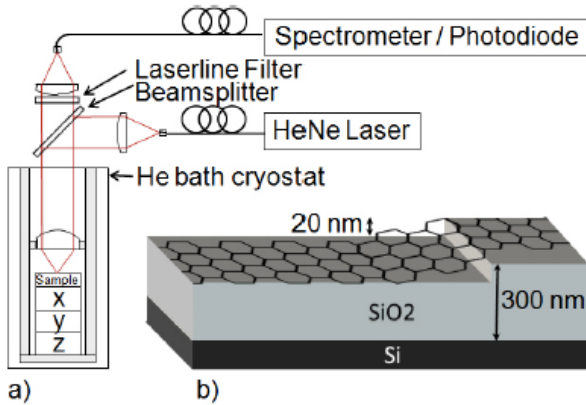
J. C. Meyer *et al.*, Nature **446**, 60 (07)



E. Stolyarova *et al.*, PNAS 104, 9209 (2007)

Ishigami *et al.*, Nano Letters (2007)

The physical properties are affected by its softness



Biaxial Strain in Graphene Adhered to Shallow Depressions

Constanze Metzger, Sebastian Rmi, Mengkun Liu, Silvia V. Kusminskiy, Antonio H. Castro Neto, Anna K. Swan and Bennett B. Goldberg
Nano Lett., **2010**, *10* (1), pp 6–10

Graphene Folds !

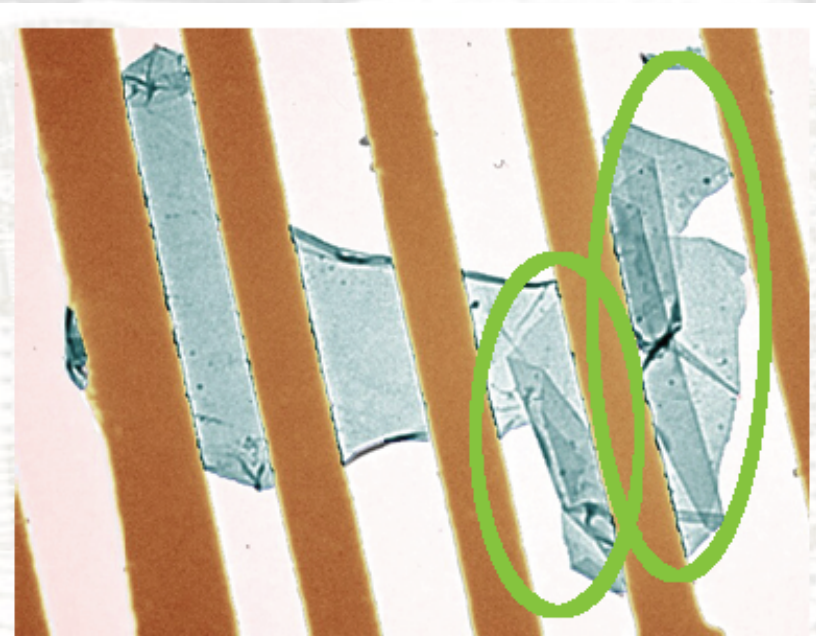


Fig. 3: Suspended graphene. Meyer *et al.*, Nature.

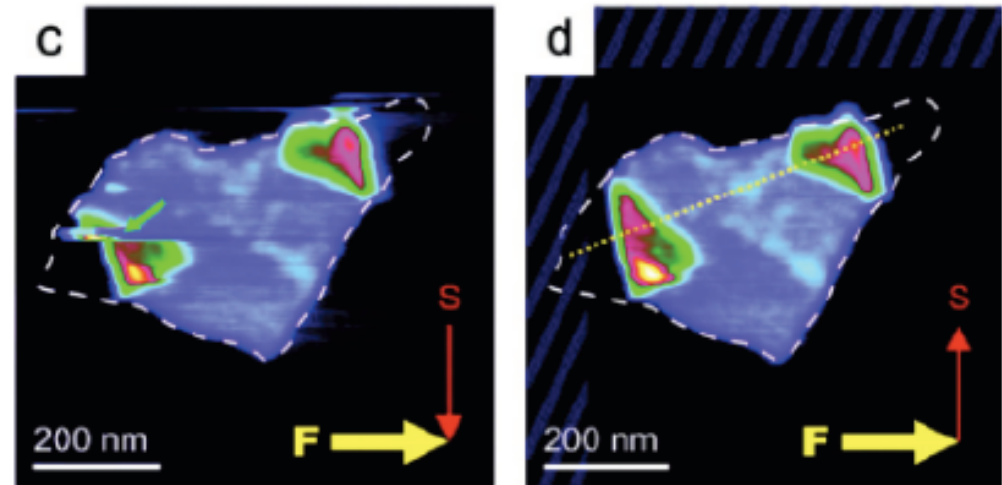
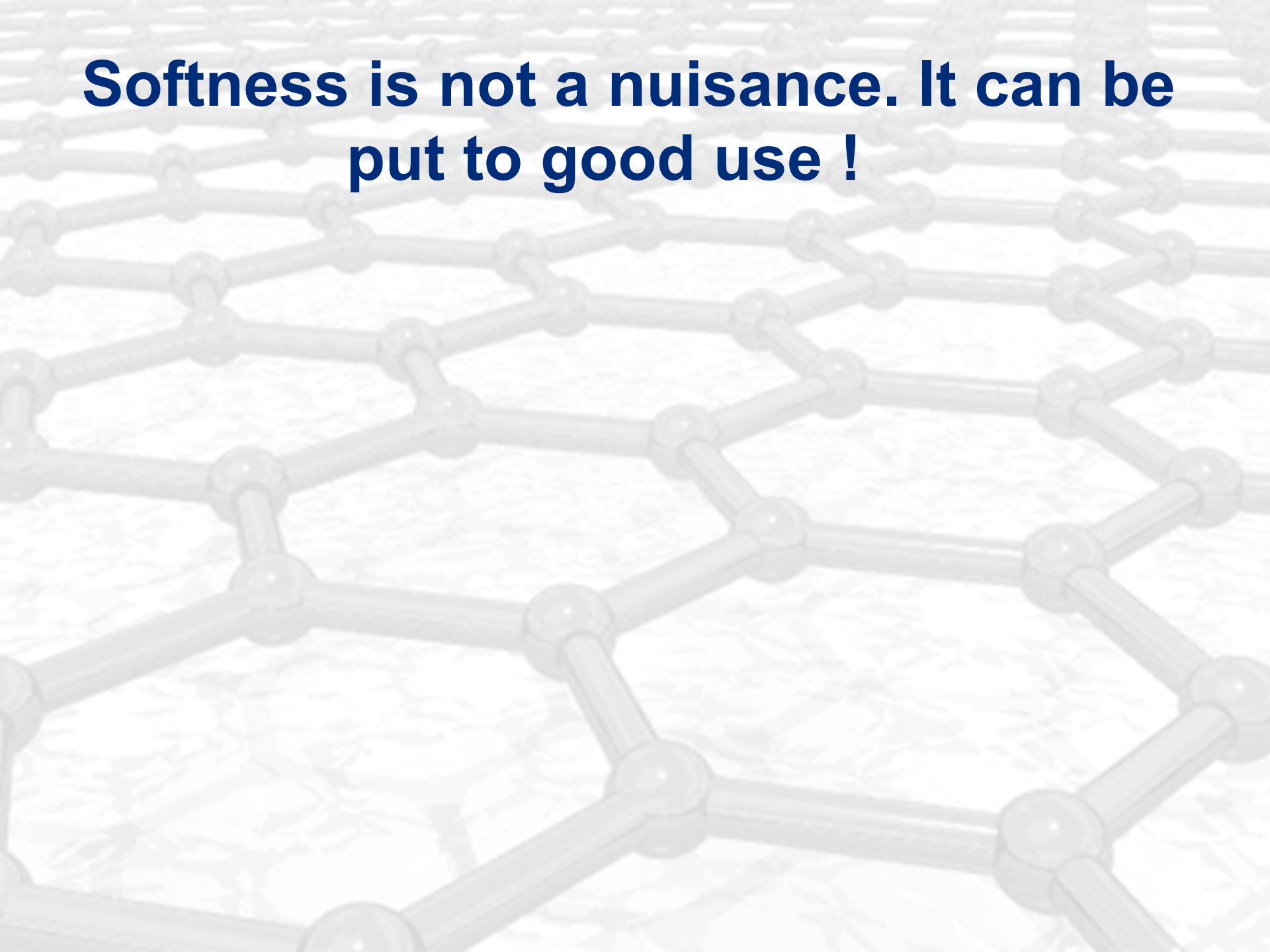


Fig. 4: AFM of graphene folds. Schniepp *et al.* Nano.

**Softness is not a nuisance. It can be
put to good use !**



Softness is not a nuisance. It can be put to good use !

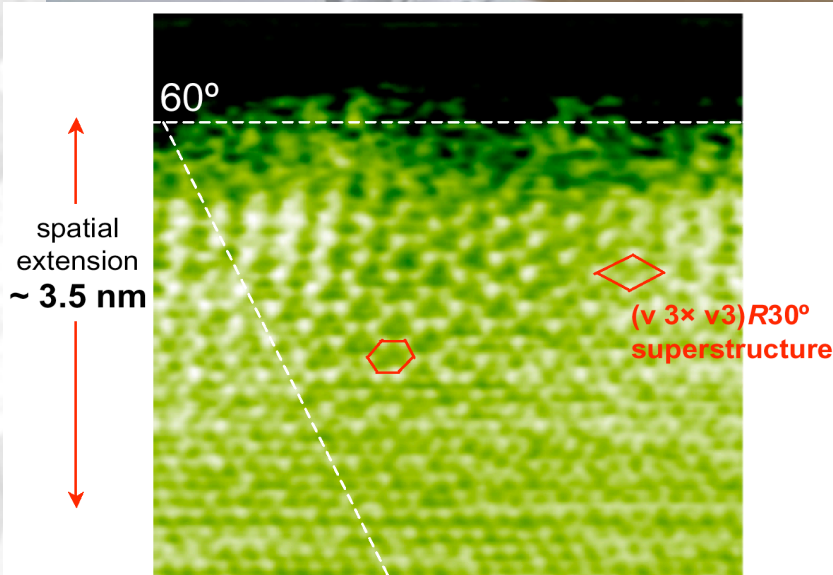
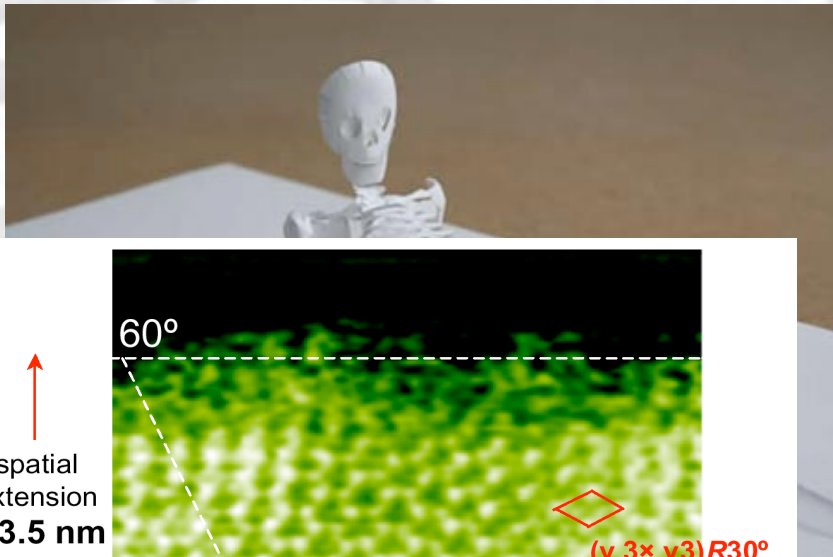


Paper-cutting



Origami

Softness is not a nuisance. It can be put to good use !



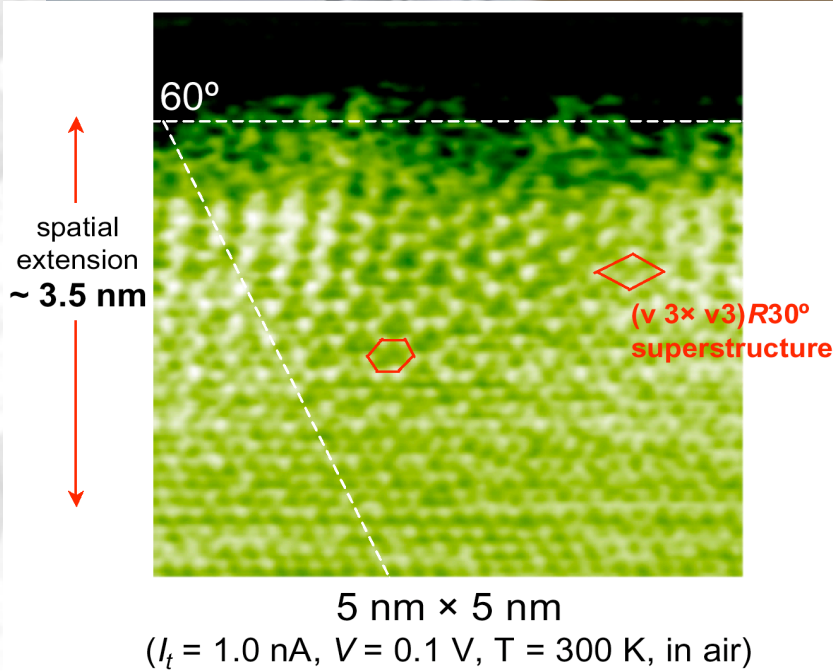
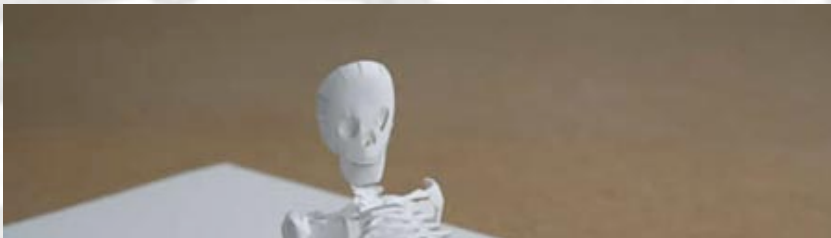
5 nm × 5 nm

($I_t = 1.0$ nA, $V = 0.1$ V, $T = 300$ K, in air)

Origami

Rough edges are a “killer” for graphene nanotechnology !

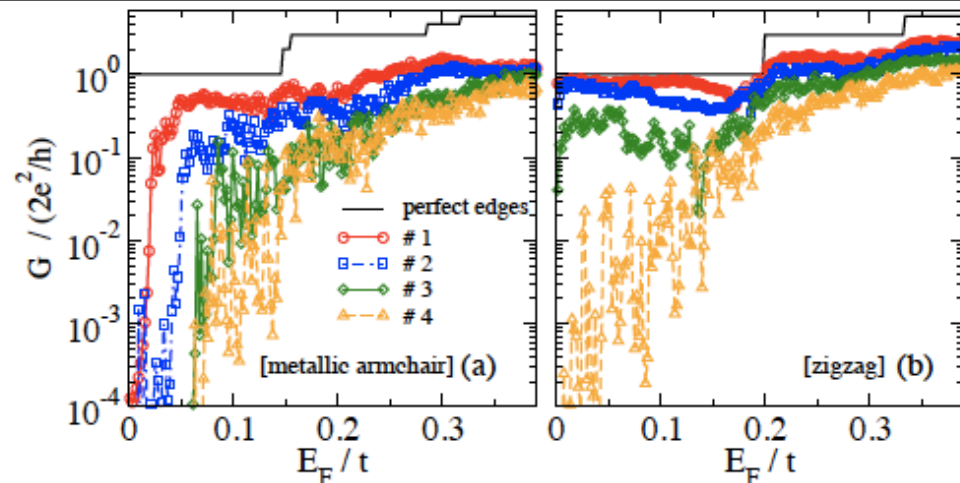
Softness is not a nuisance. It can be put to good use !



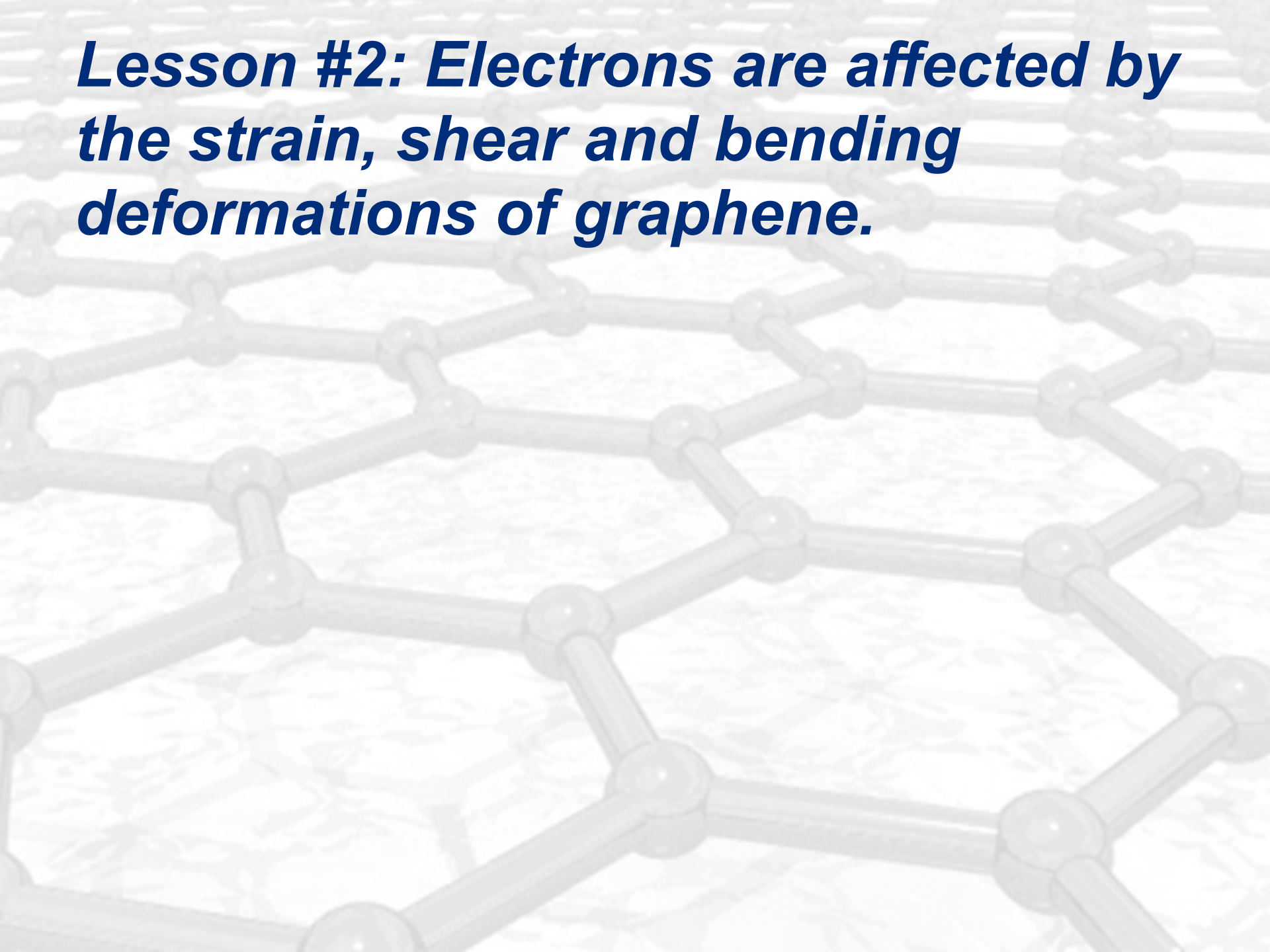
C. Lewenkopf, E. Mucciolo, AHCN, Phys. Rev. B 77, 081410R (2008)

E. Mucciolo, AHCN, C. Lewenkopf, Phys. Rev. B 79, 075407 (2009)

Rough edges are a “killer” for graphene nanotechnology !



Lesson #2: Electrons are affected by the strain, shear and bending deformations of graphene.



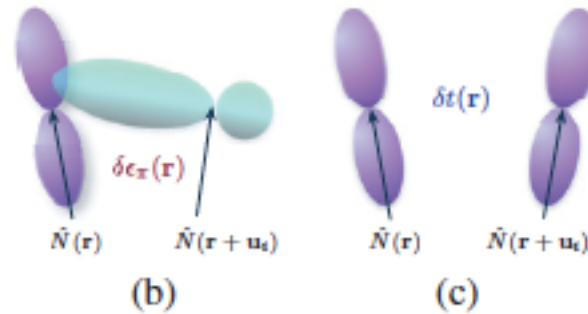
Lesson #2: Electrons are affected by the strain, shear and bending deformations of graphene.

Graphene as an electronic membrane

EPL, 84 (2008) 57007

doi: 10.1209/0295-5075/84/57007

EUN-AH KIM¹ and A. H. CASTRO NETO^{2(a)}



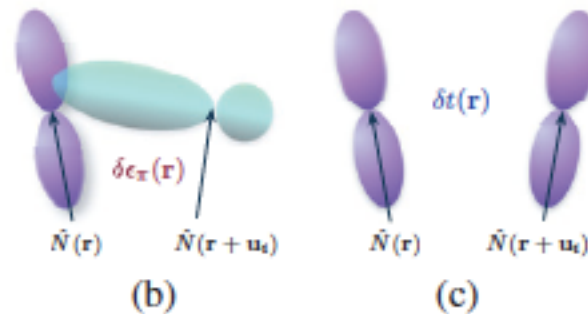
Lesson #2: Electrons are affected by the strain, shear and bending deformations of graphene.

Graphene as an electronic membrane

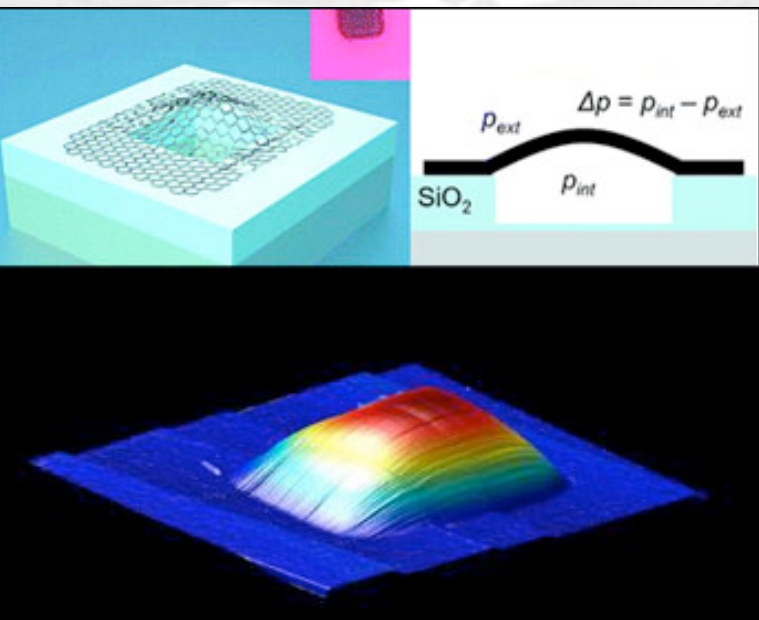
EPL, 84 (2008) 57007

doi: 10.1209/0295-5075/84/57007

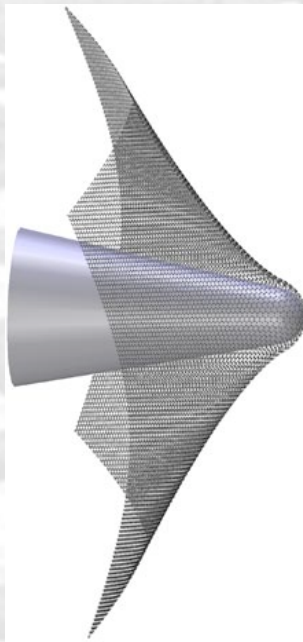
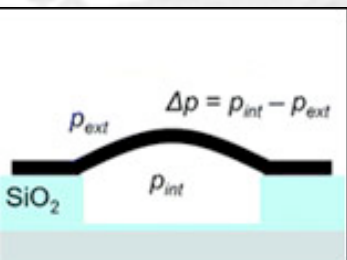
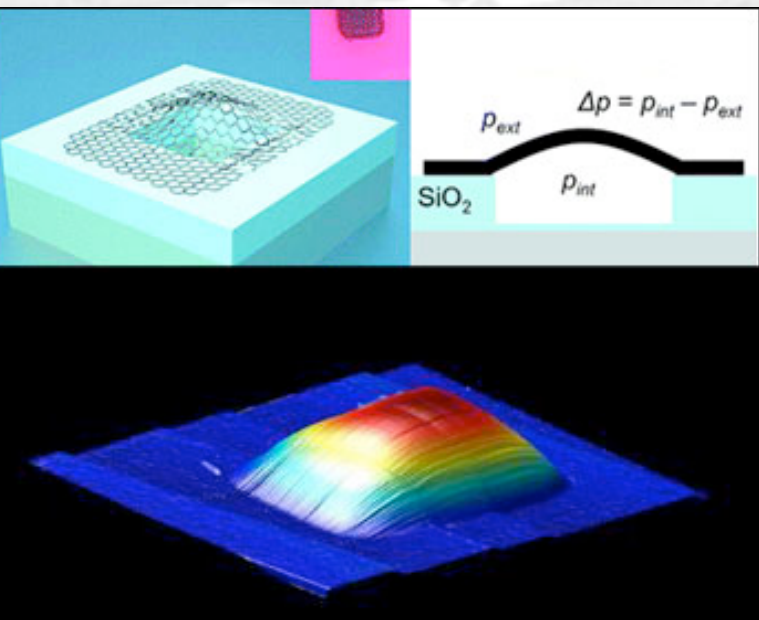
EUN-AH KIM¹ and A. H. CASTRO NETO^{2(a)}



$$H = \int dr \Psi^\dagger(\mathbf{r}) \left\{ \vec{\sigma} \cdot \left[i v_F \nabla + \vec{\mathcal{A}}(\mathbf{r}) \right] - \mu + \Phi(\mathbf{r}) + U(\mathbf{r}) \right\} \Psi(\mathbf{r})$$

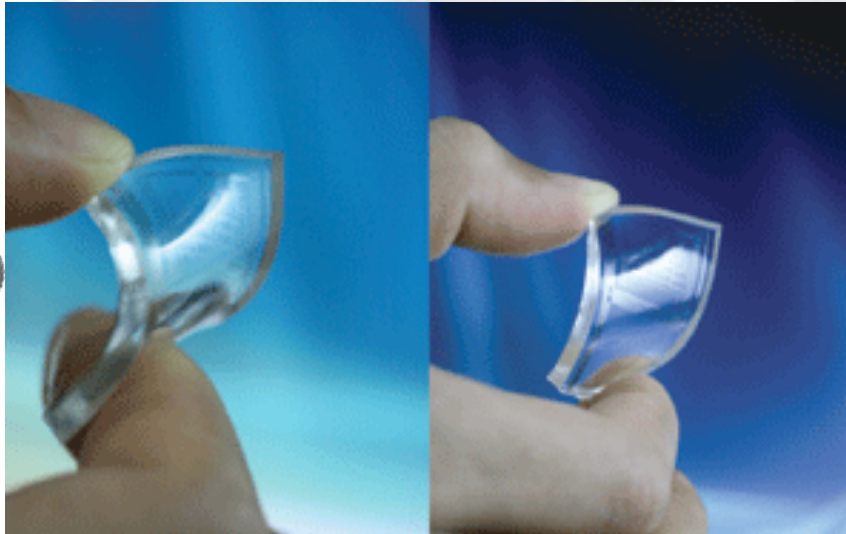
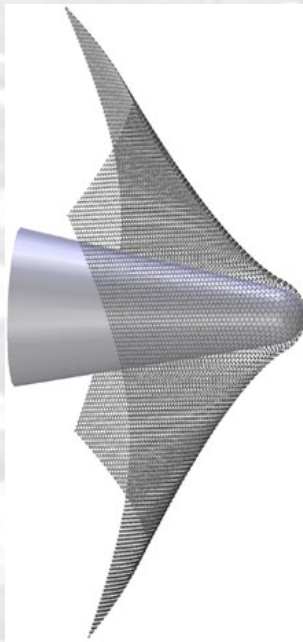
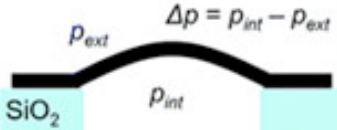
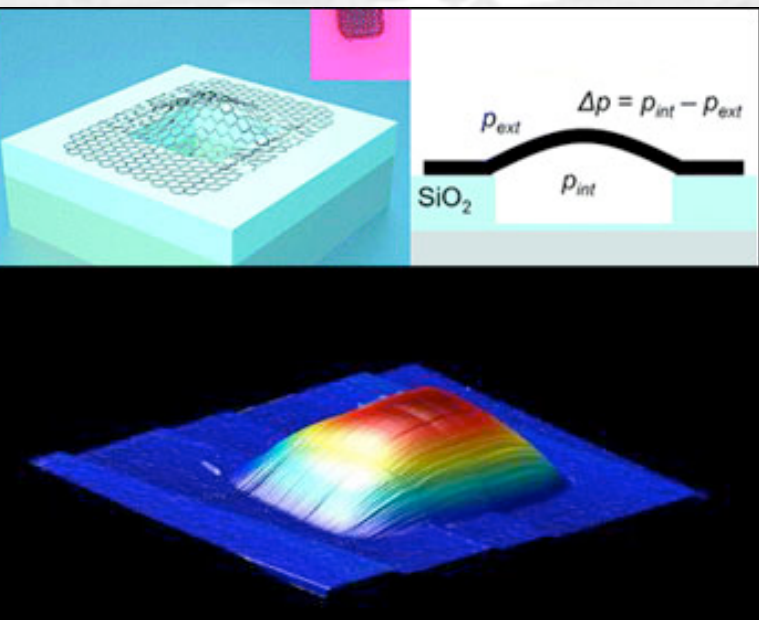


J. S. Bunch et al., Nano Lett. 8, 2458 (2008).



J. S. Bunch et al., Nano Lett. 8, 2458 (2008).

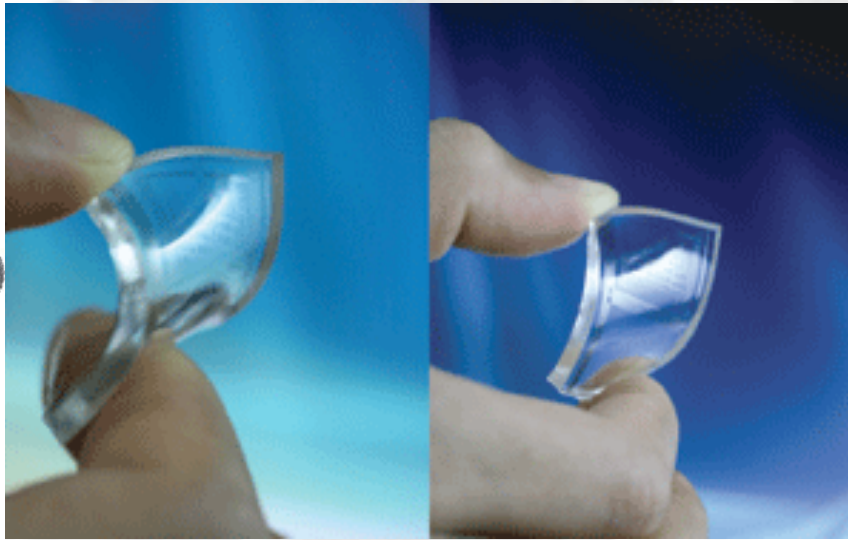
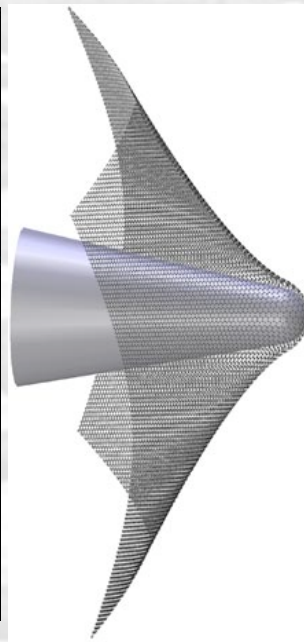
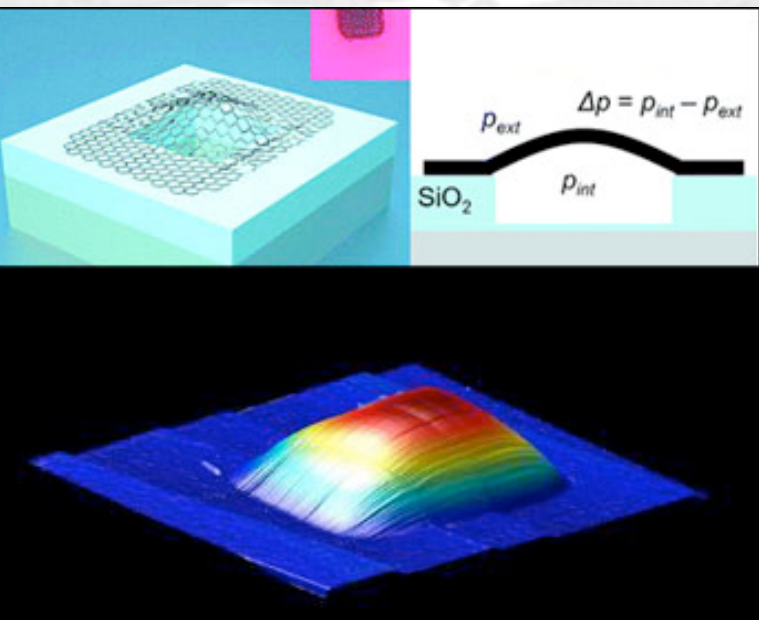
C. Lee et al. Science 321, 325 (2008)



J. S. Bunch et al., Nano Lett. 8, 2458 (2008).

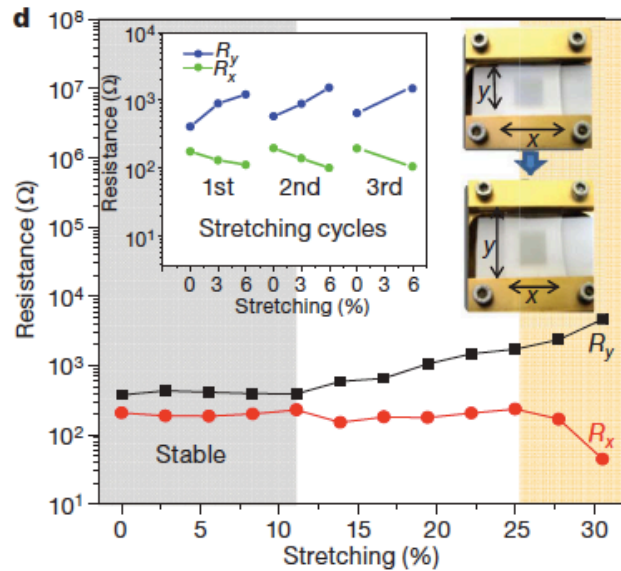
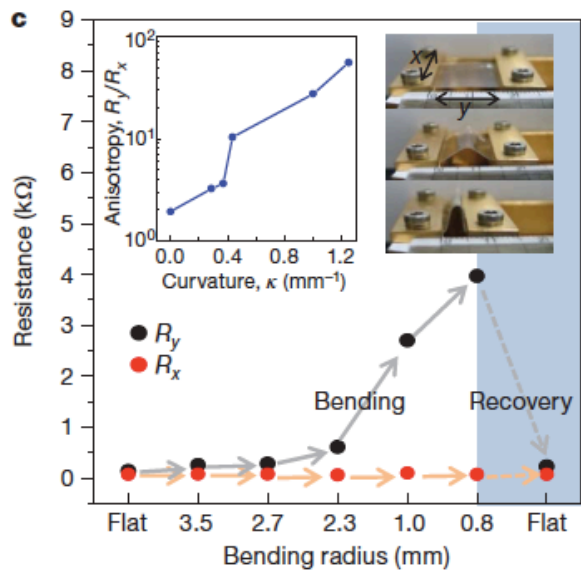
C. Lee et al. Science 321, 325 (2008)

K.S.Kim et al., Nature 2009

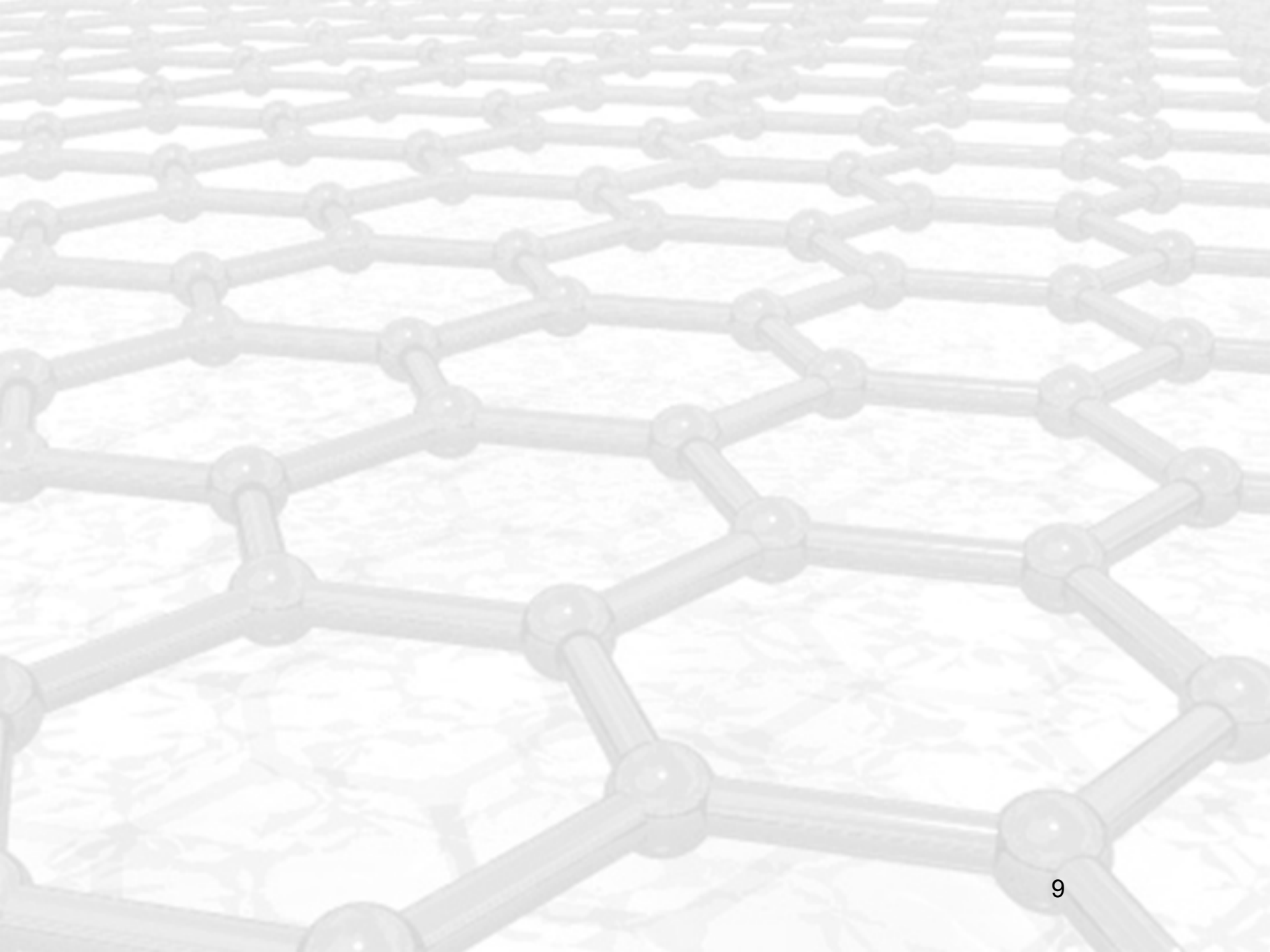


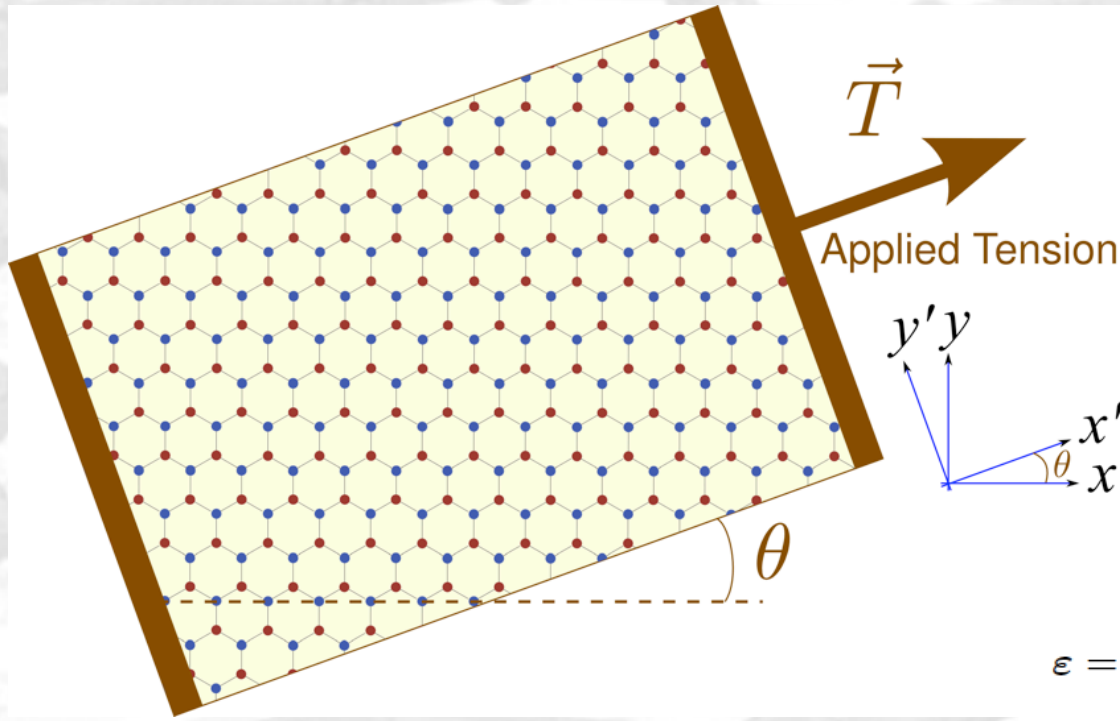
J. S. Bunch et al., Nano Lett. 8, 2458 (2008).

C. Lee et al. Science 321, 325 (2008)



K.S.Kim et al., Nature 2009





$$\mathbf{T} = T \mathbf{e}_{x'}$$

$$\varepsilon'_{ij} = S_{ijkl} \tau'_{kl} = T S_{ijkl} \delta_{kx} \delta_{lx} = T S_{ijxx}$$

$$\varepsilon'_{xx} = T S_{xxxx}, \quad \varepsilon'_{yy} = T S_{xyxy}$$

$$\varepsilon = T S_{xxxx} \quad \sigma = -S_{xyxy}/S_{xxxx}$$

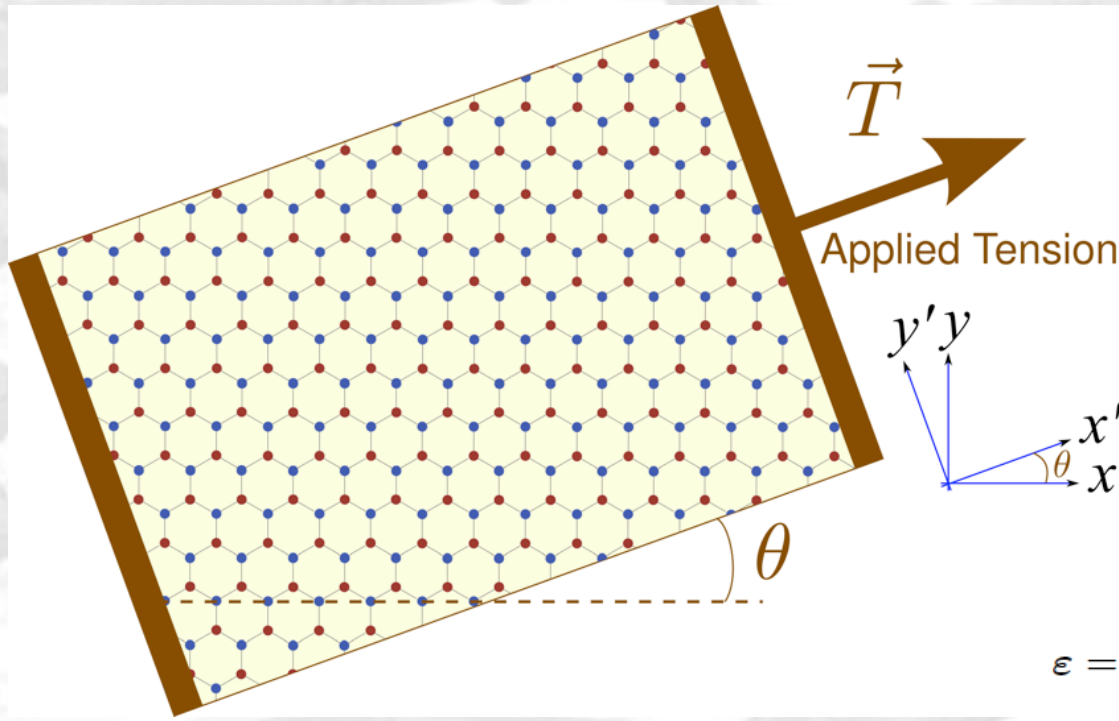
$$\sigma = 0.165$$

$$\varepsilon' = \varepsilon \begin{pmatrix} 1 & 0 \\ 0 & -\sigma \end{pmatrix}$$

$$\mathbf{T} = T \cos(\theta) \mathbf{e}_x + \sin(\theta) \mathbf{e}_y$$

$$\tau_{ij} = C_{ijkl} \varepsilon_{kl}, \quad \varepsilon_{ij} = S_{ijkl} \tau_{kl}$$

$$\varepsilon = \varepsilon \begin{pmatrix} \cos^2 \theta - \sigma \sin^2 \theta & (1 + \sigma) \cos \theta \sin \theta \\ (1 + \sigma) \cos \theta \sin \theta & \sin^2 \theta - \sigma \cos^2 \theta \end{pmatrix}$$



$$\mathbf{T} = T \mathbf{e}_{x'}$$

$$\varepsilon'_{ij} = S_{ijkl} \tau'_{kl} = T S_{ijkl} \delta_{kx} \delta_{lx} = T S_{ijxx}$$

$$\varepsilon'_{xx} = T S_{xxxx}, \quad \varepsilon'_{yy} = T S_{xyxy}$$

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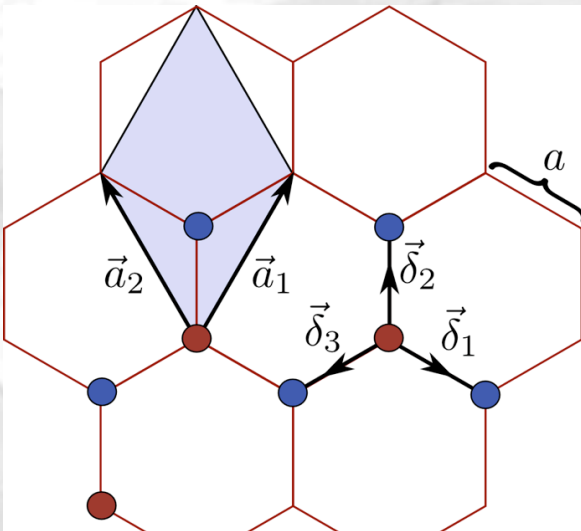
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$$\varepsilon = \varepsilon \begin{pmatrix} \cos^2 \theta - \sigma \sin^2 \theta & (1 + \sigma) \cos \theta \sin \theta \\ (1 + \sigma) \cos \theta \sin \theta & \sin^2 \theta - \sigma \cos^2 \theta \end{pmatrix}$$



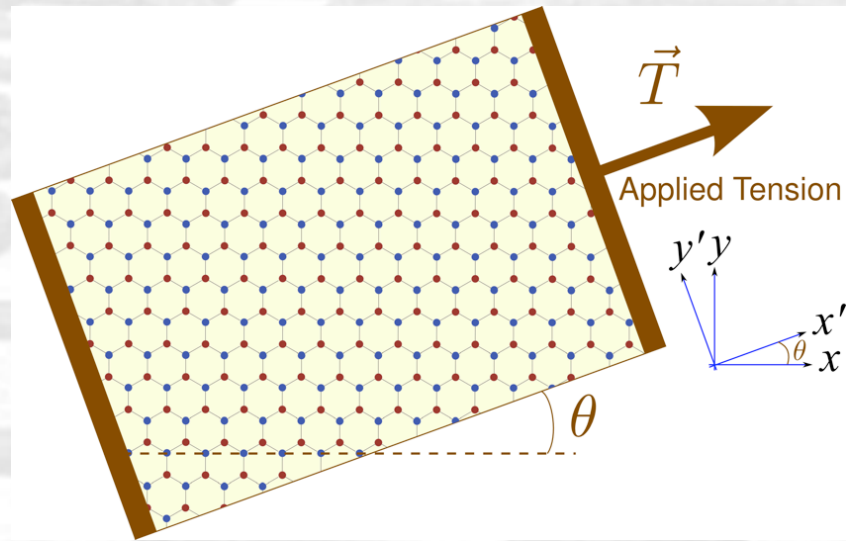
$$\delta_{\alpha} = (1 + \varepsilon) \cdot \delta_{\alpha}^0$$

$$V_{pp\pi}(l) = t_0 e^{-3.37(l-a_0)} \quad a_0 = 1.42 \text{ \AA}$$

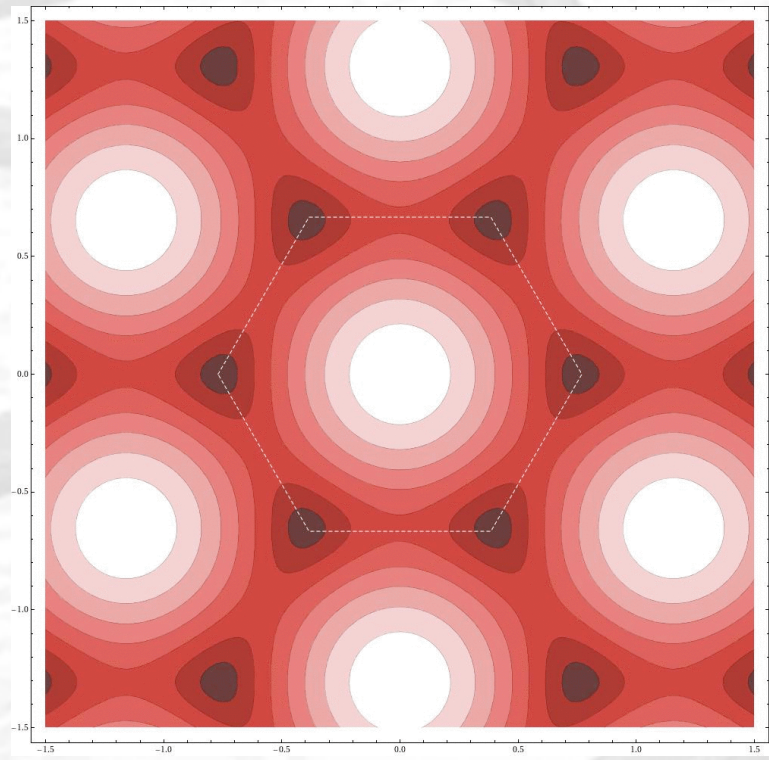
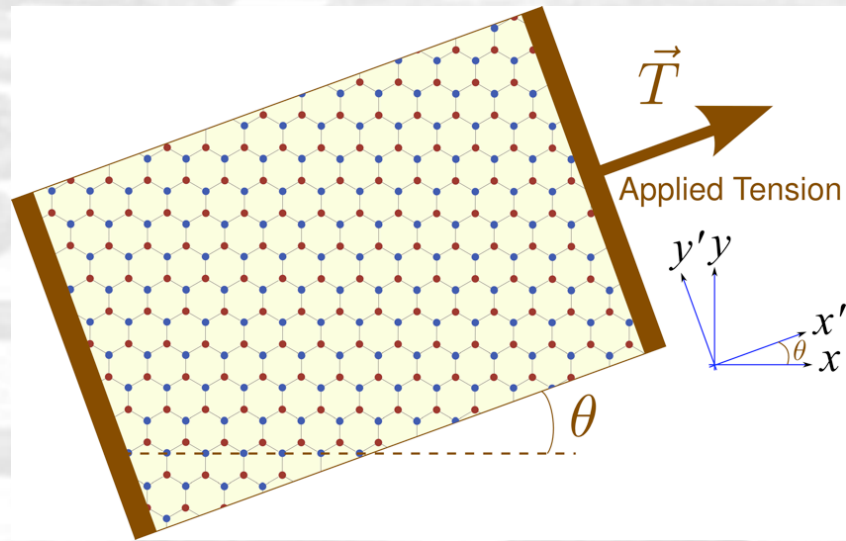
$$dV_{pp\pi}/dl = -6.4 \text{ eV/\AA}$$

$$t_0 = t(\delta_{\alpha}^0) \approx 2.7 \text{ eV}$$

$$E(k_x, k_y) = \pm |t_2 + t_3 e^{-ik \cdot \mathbf{a}_1} + t_1 e^{-ik \cdot \mathbf{a}_2}|$$

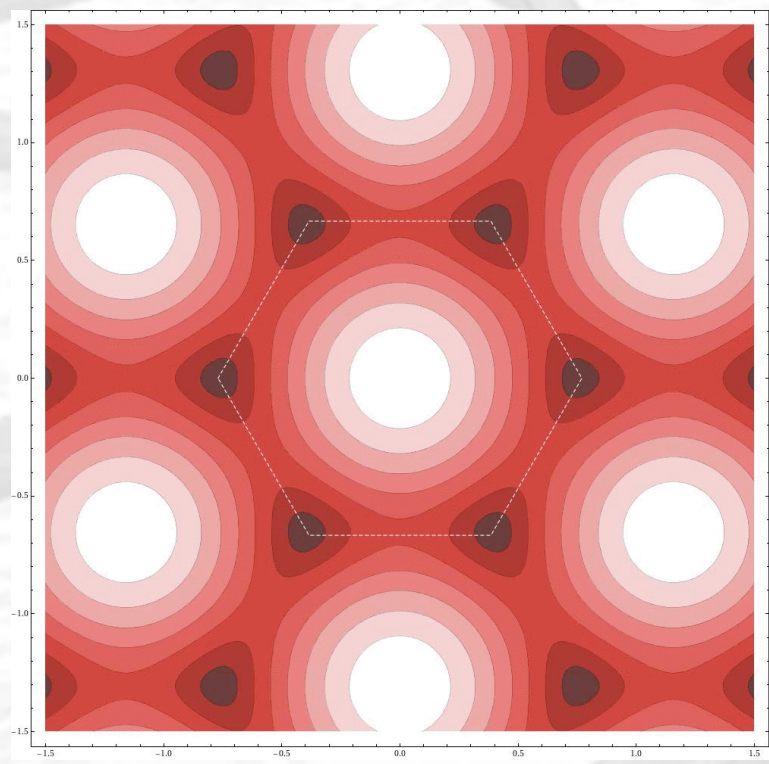
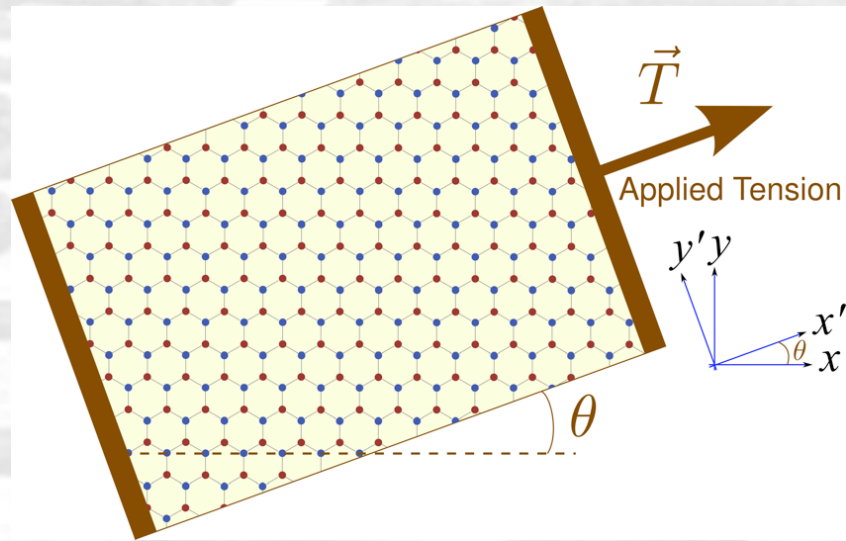


$$E(k_x, k_y) = \pm |t_2 + t_3 e^{-ik \cdot \mathbf{a}_1} + t_1 e^{-ik \cdot \mathbf{a}_2}|$$

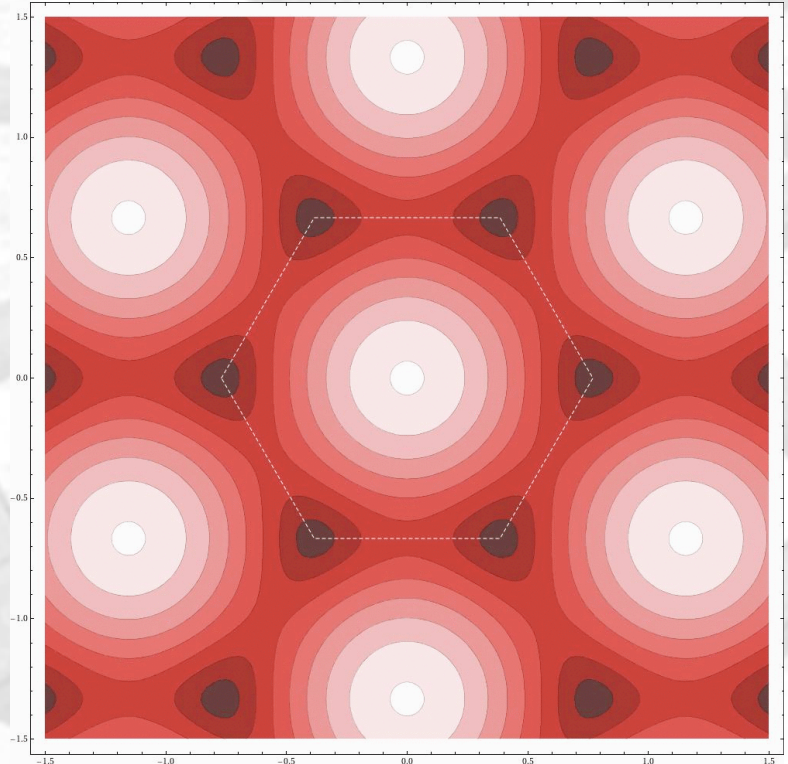


Armchair direction

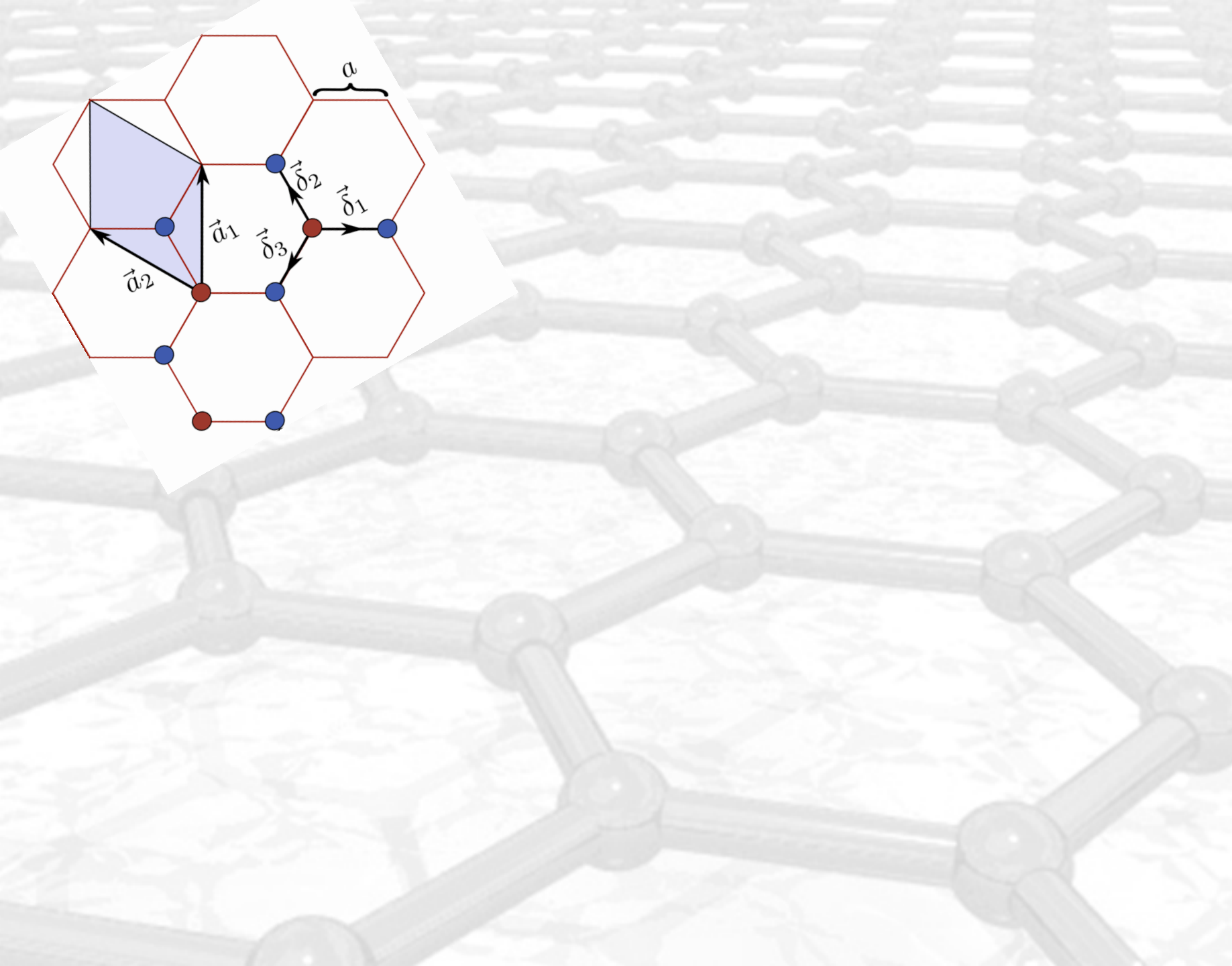
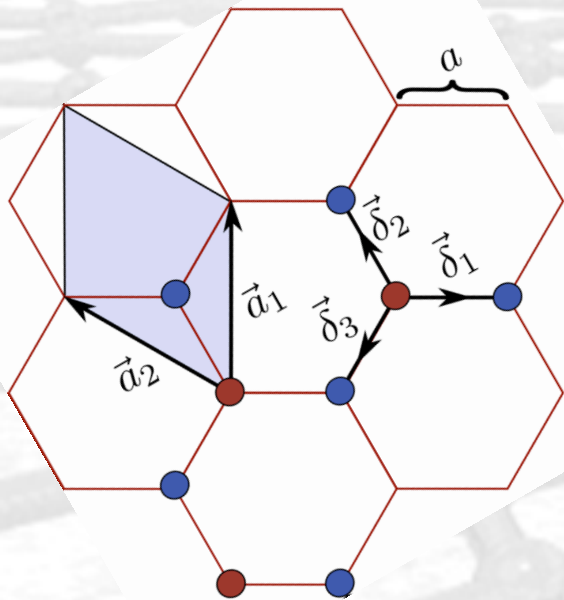
$$E(k_x, k_y) = \pm |t_2 + t_3 e^{-ik \cdot \mathbf{a}_1} + t_1 e^{-ik \cdot \mathbf{a}_2}|$$

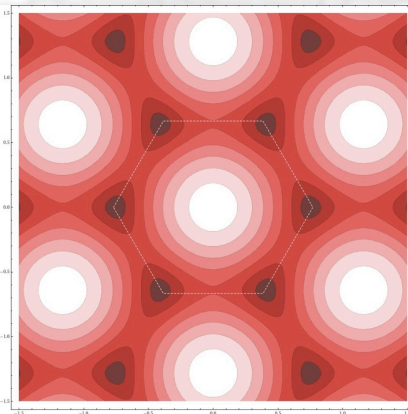
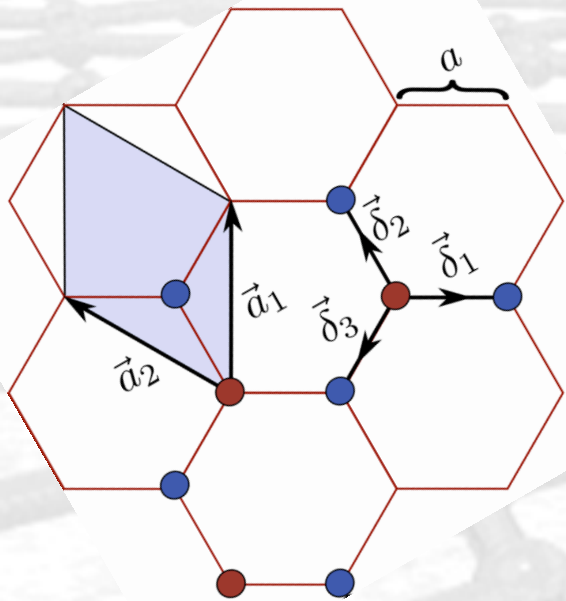


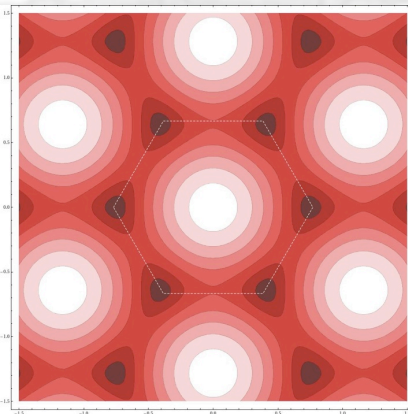
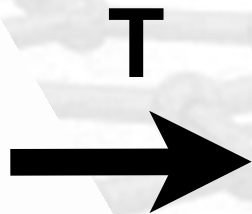
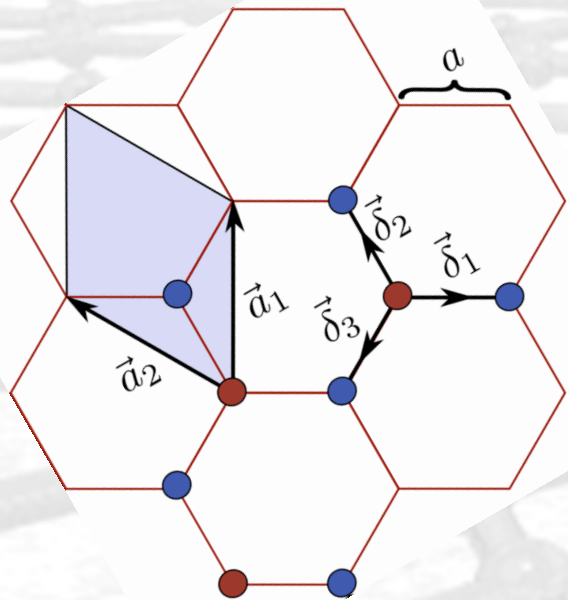
Armchair direction

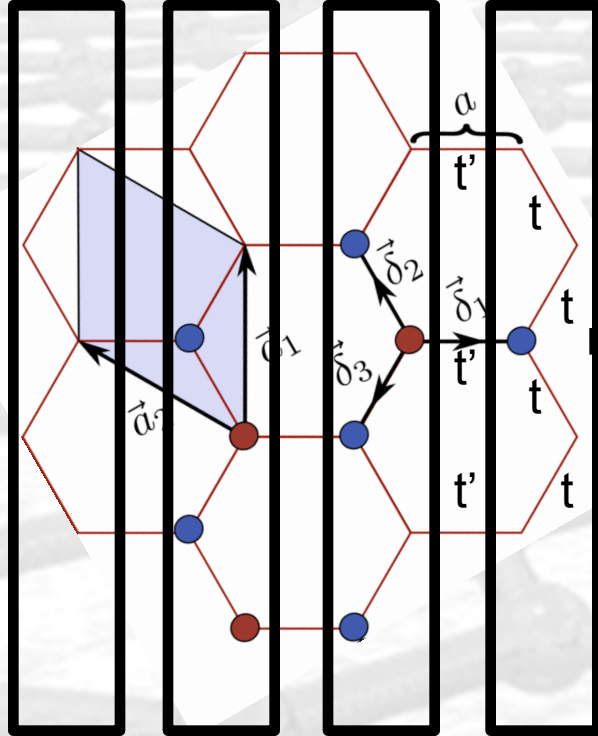


Zigzag direction

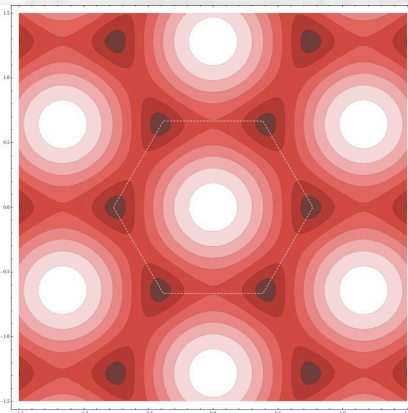


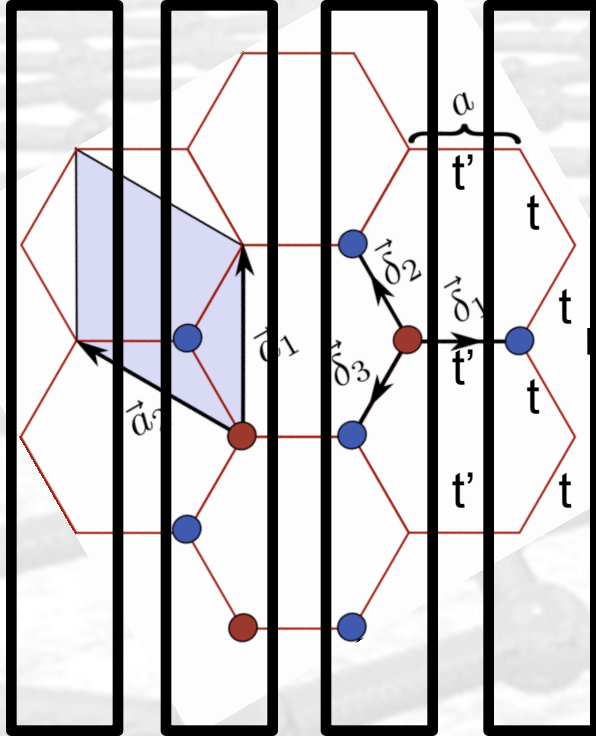




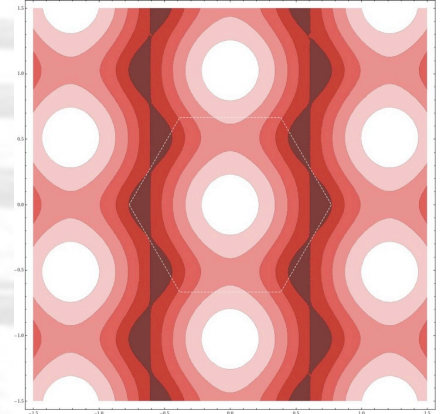
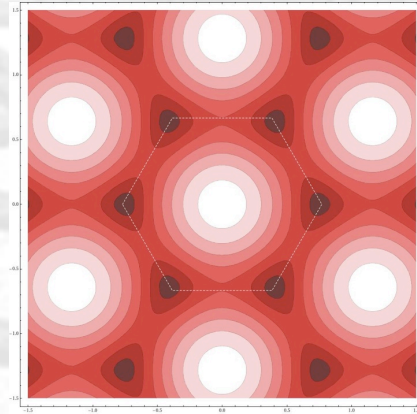


T





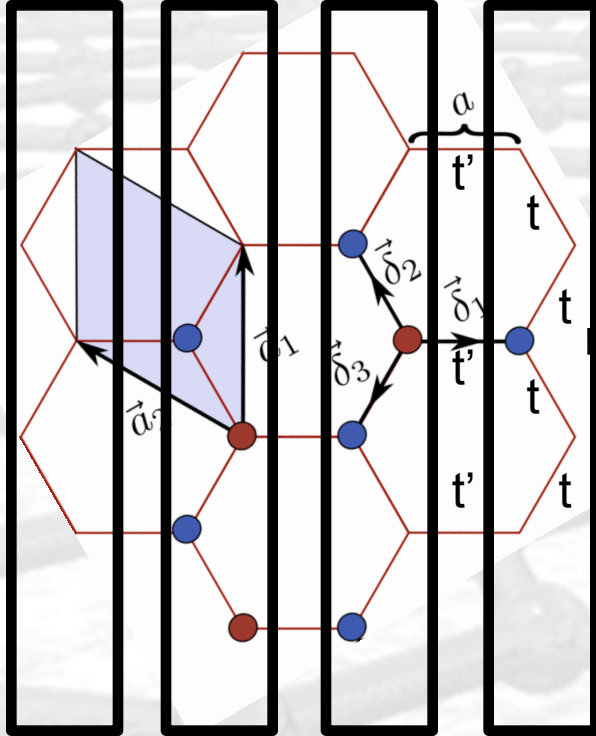
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“one-dimensionalization”

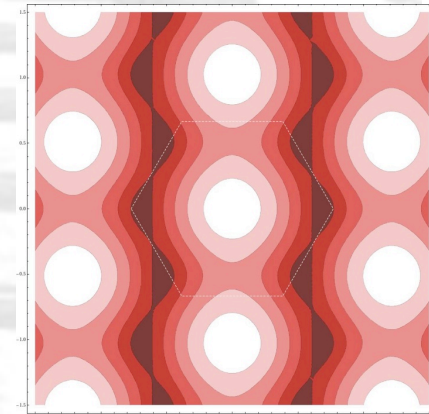
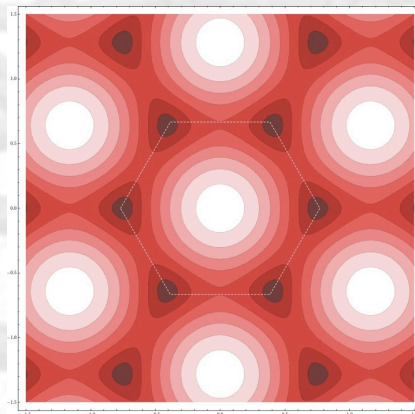
$t' \ll t$

Weakly Coupled Luttinger Liquids



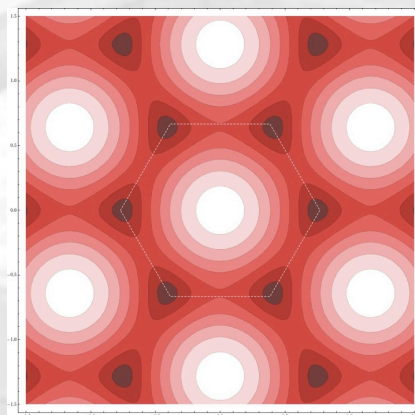
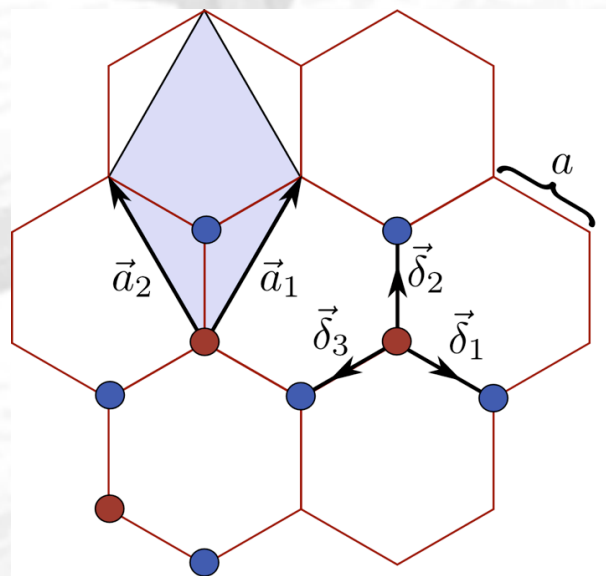
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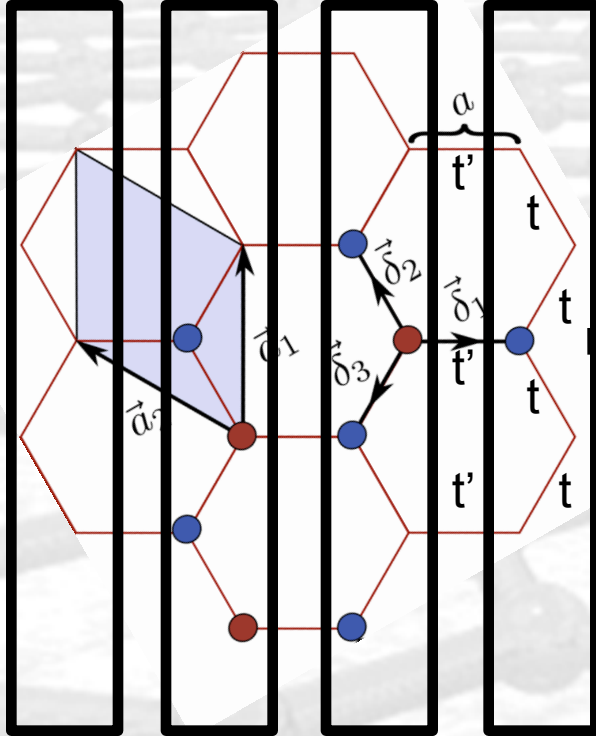
“one-dimensionalization”



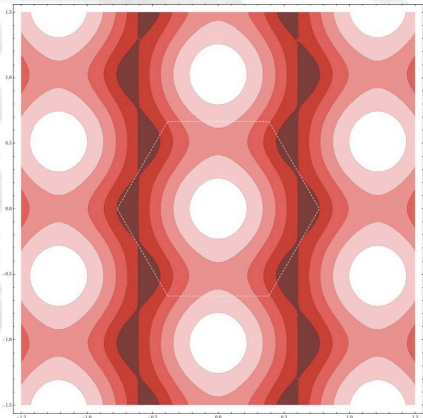
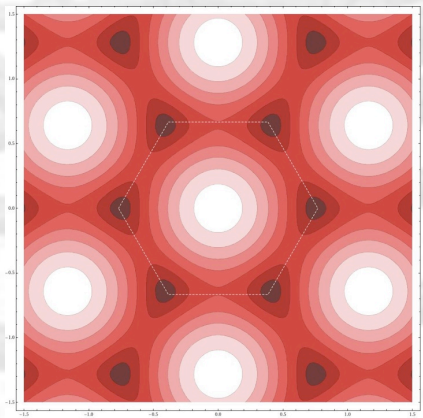
$t' \ll t$

Weakly Coupled Luttinger Liquids





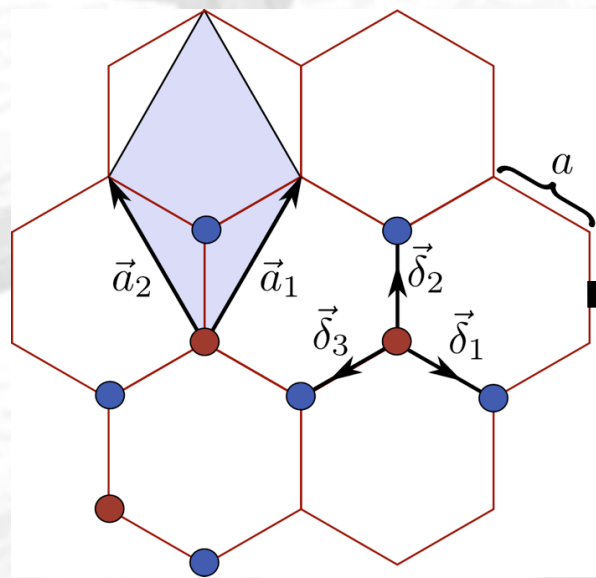
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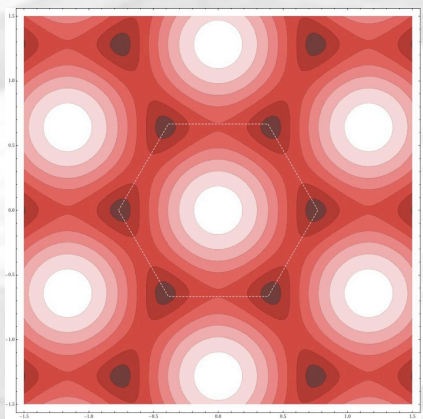
“one-dimensionalization”

$t' \ll t$

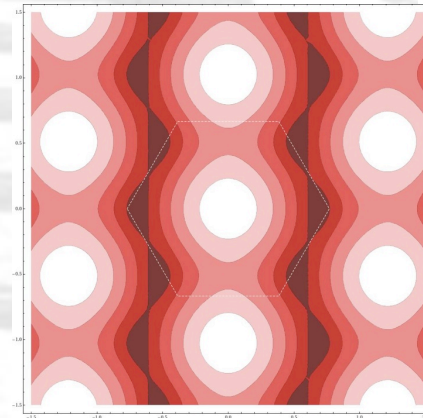
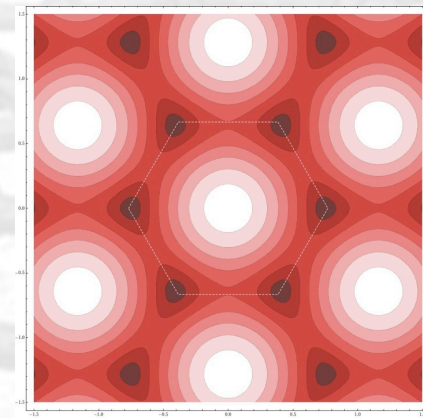
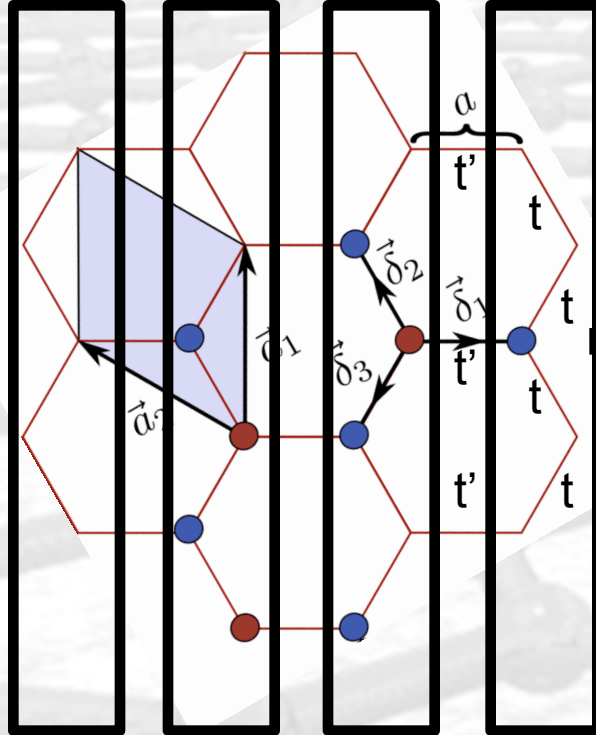
Weakly Coupled Luttinger Liquids



T

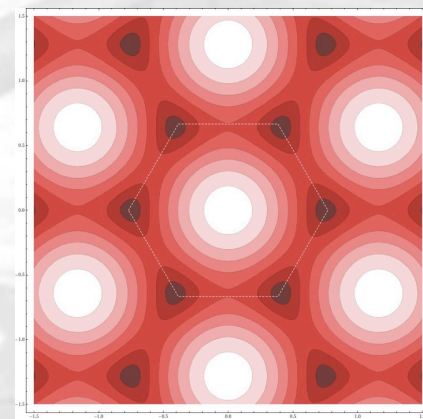
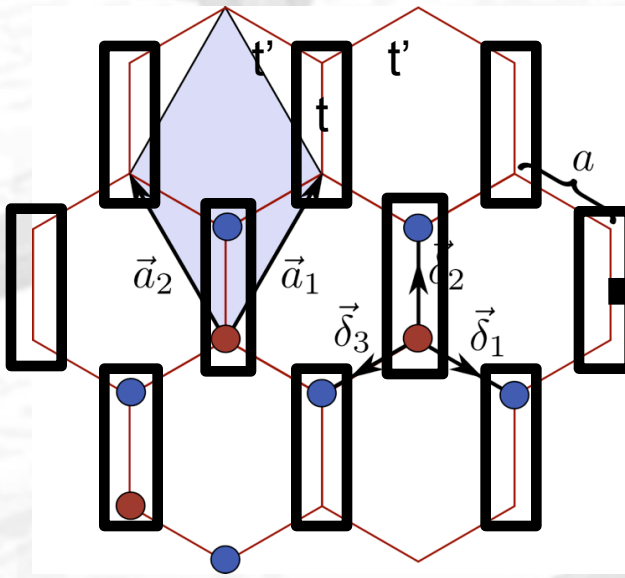


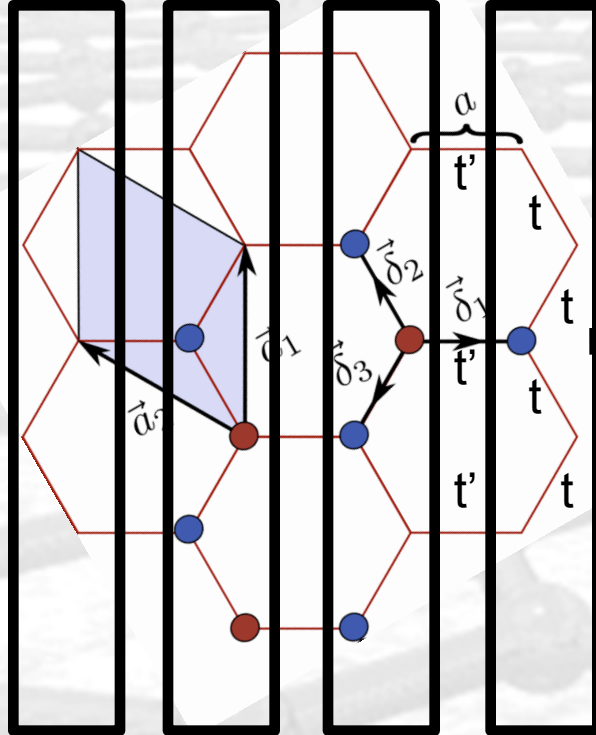
“one-dimensionalization”



$$t' \ll t$$

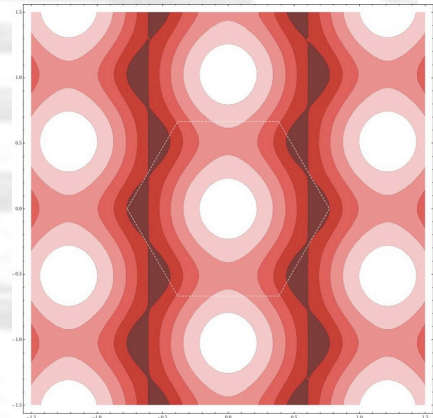
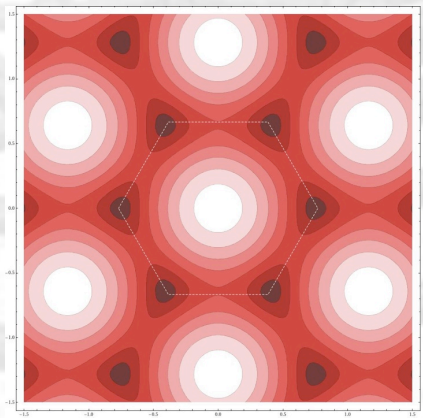
Weakly Coupled Luttinger Liquids





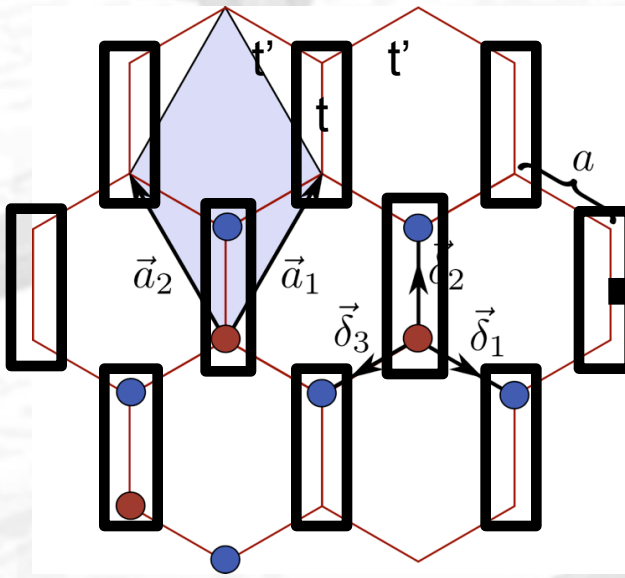
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“one-dimensionalization”

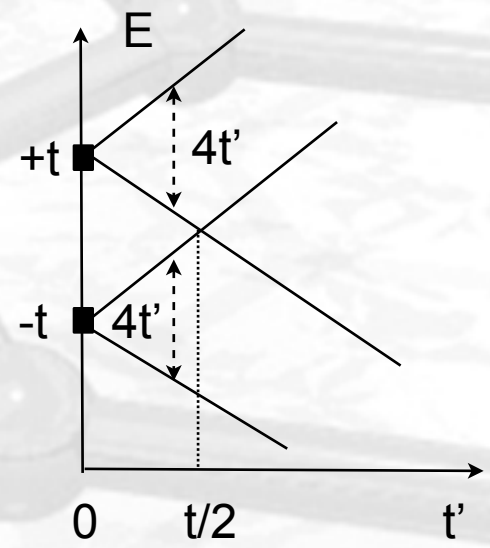


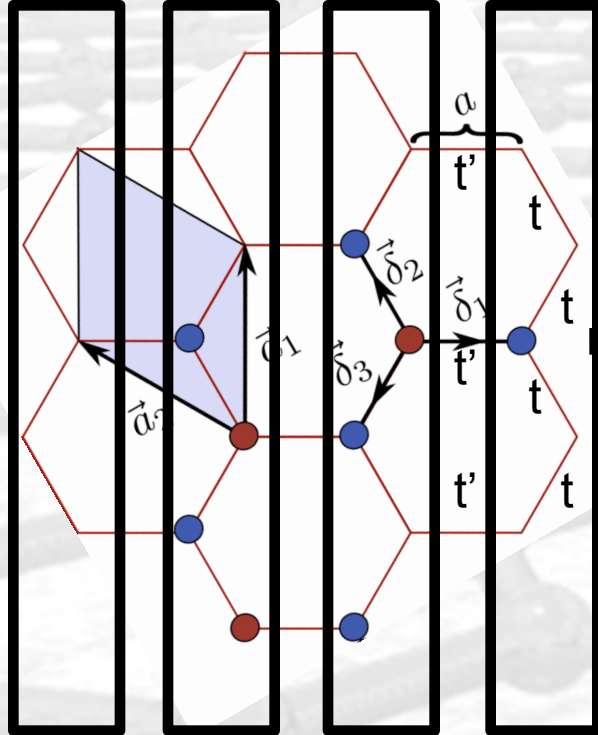
$t' \ll t$

Weakly Coupled Luttinger Liquids

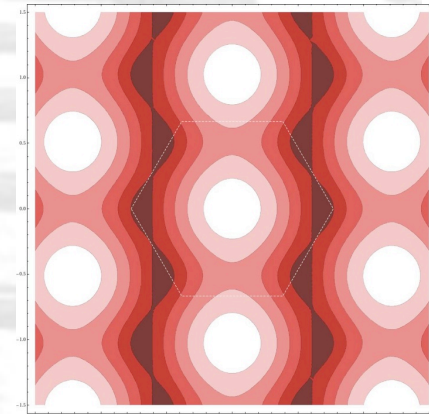
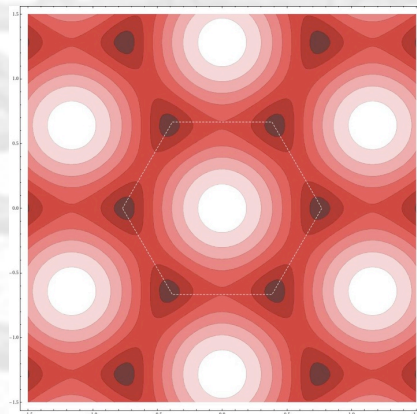


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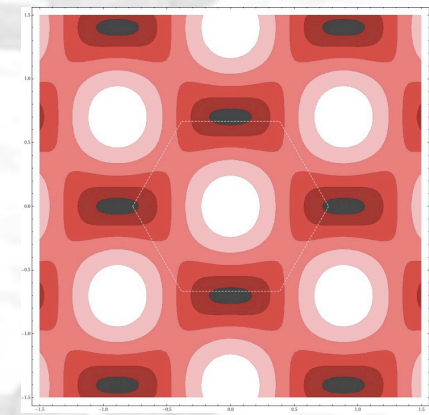
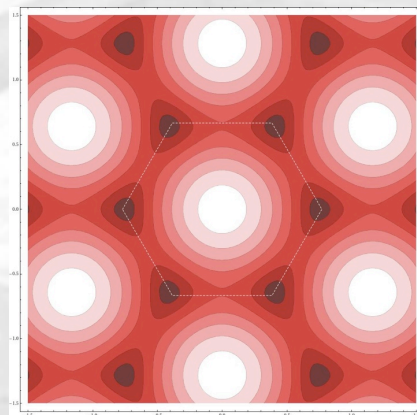
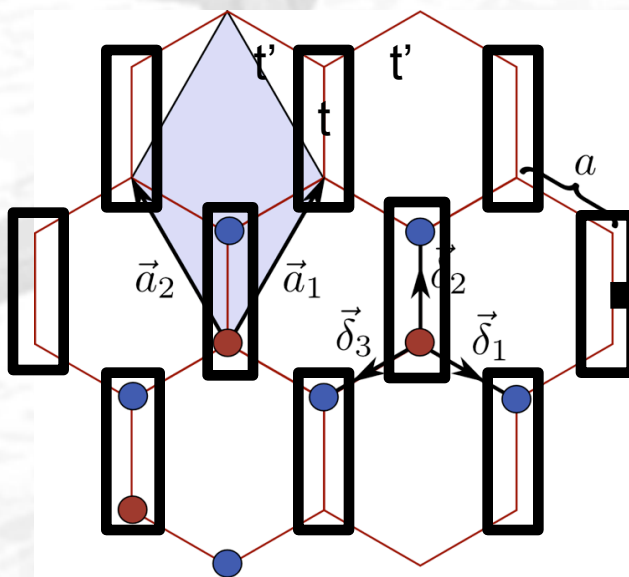


“one-dimensionalization”



$t' \ll t$

Weakly Coupled Luttinger Liquids

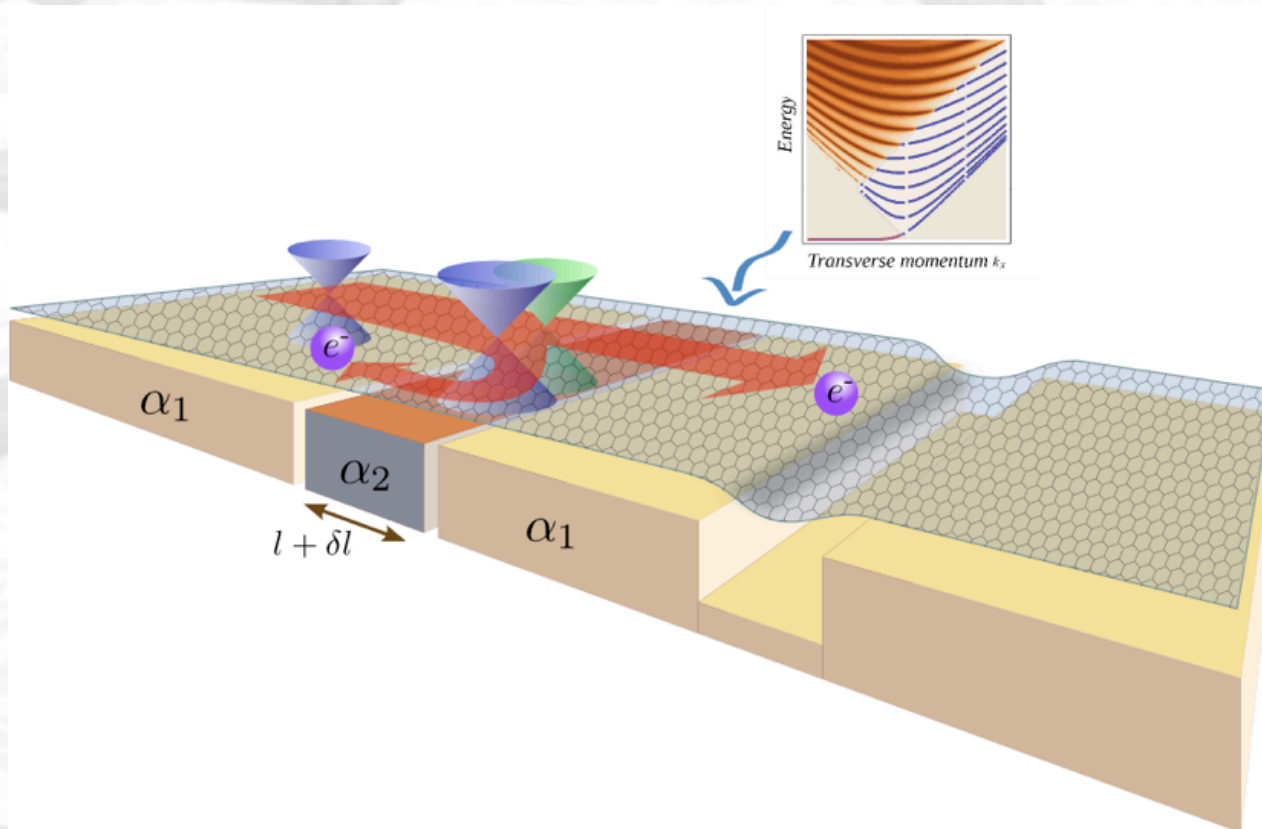
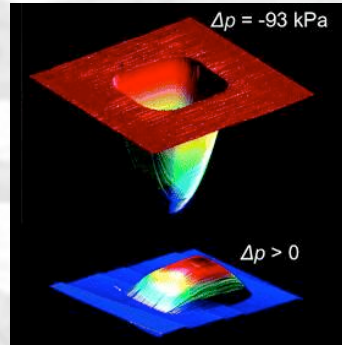
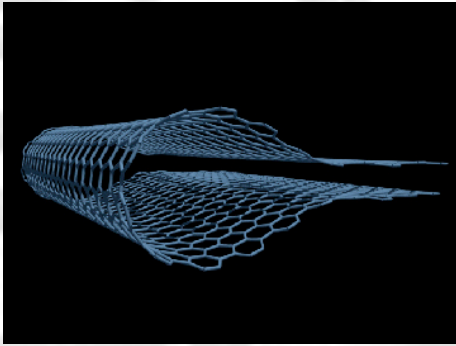


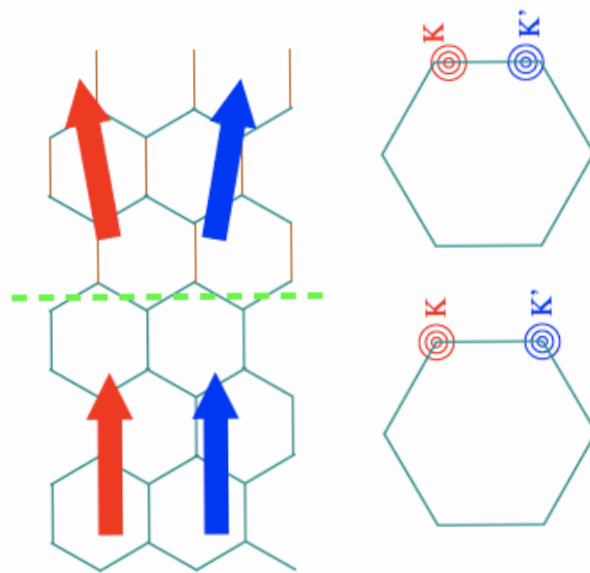
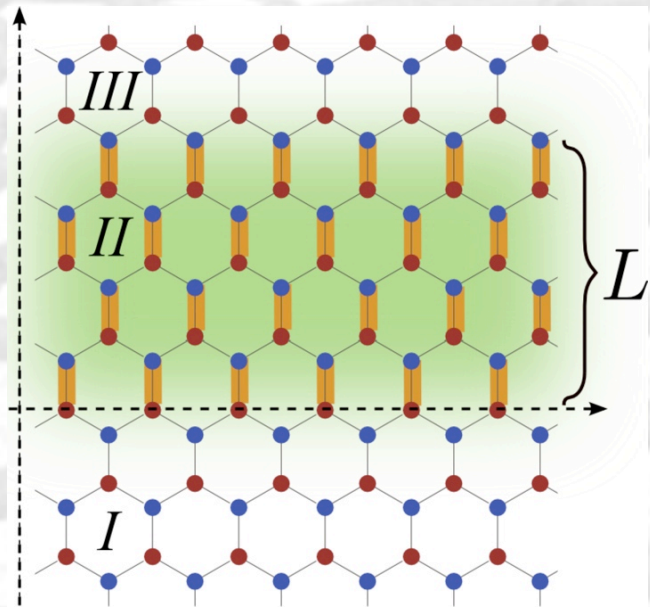
“zero-dimensionalization”

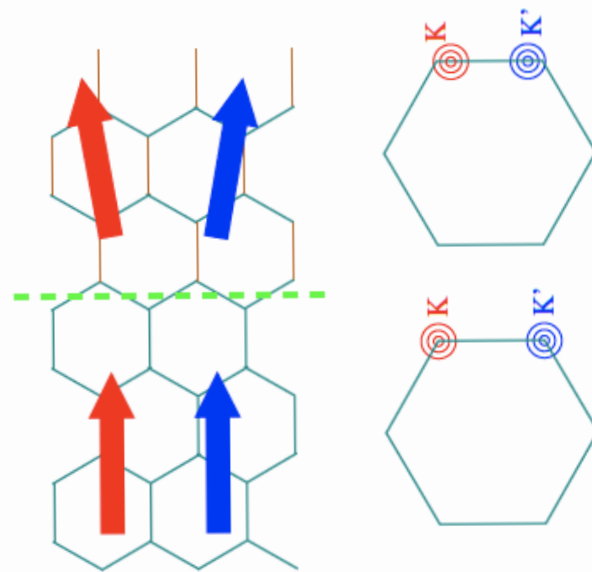
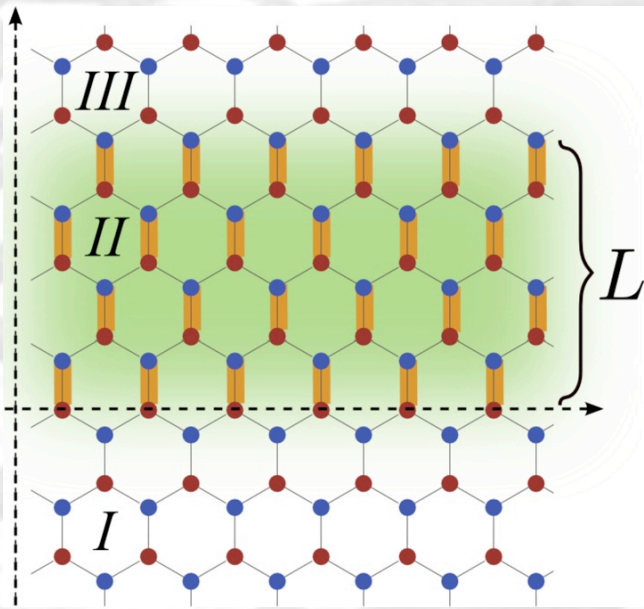
Dimerization

“ORIGAMI” ELECTRONICS

Vitor M. Pereira and A. H. Castro Neto, Phys. Rev. Lett. 103, 046801 (2009)



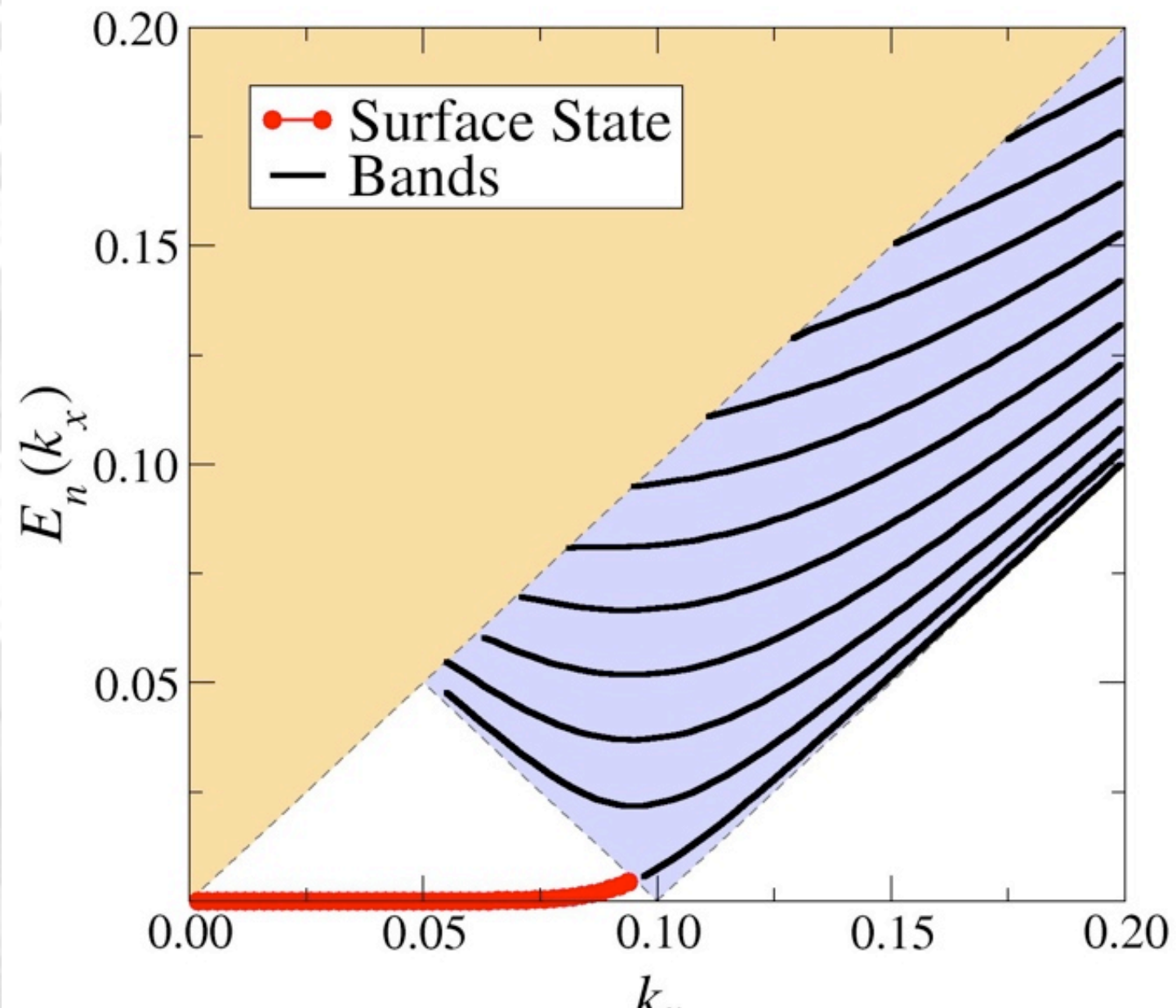


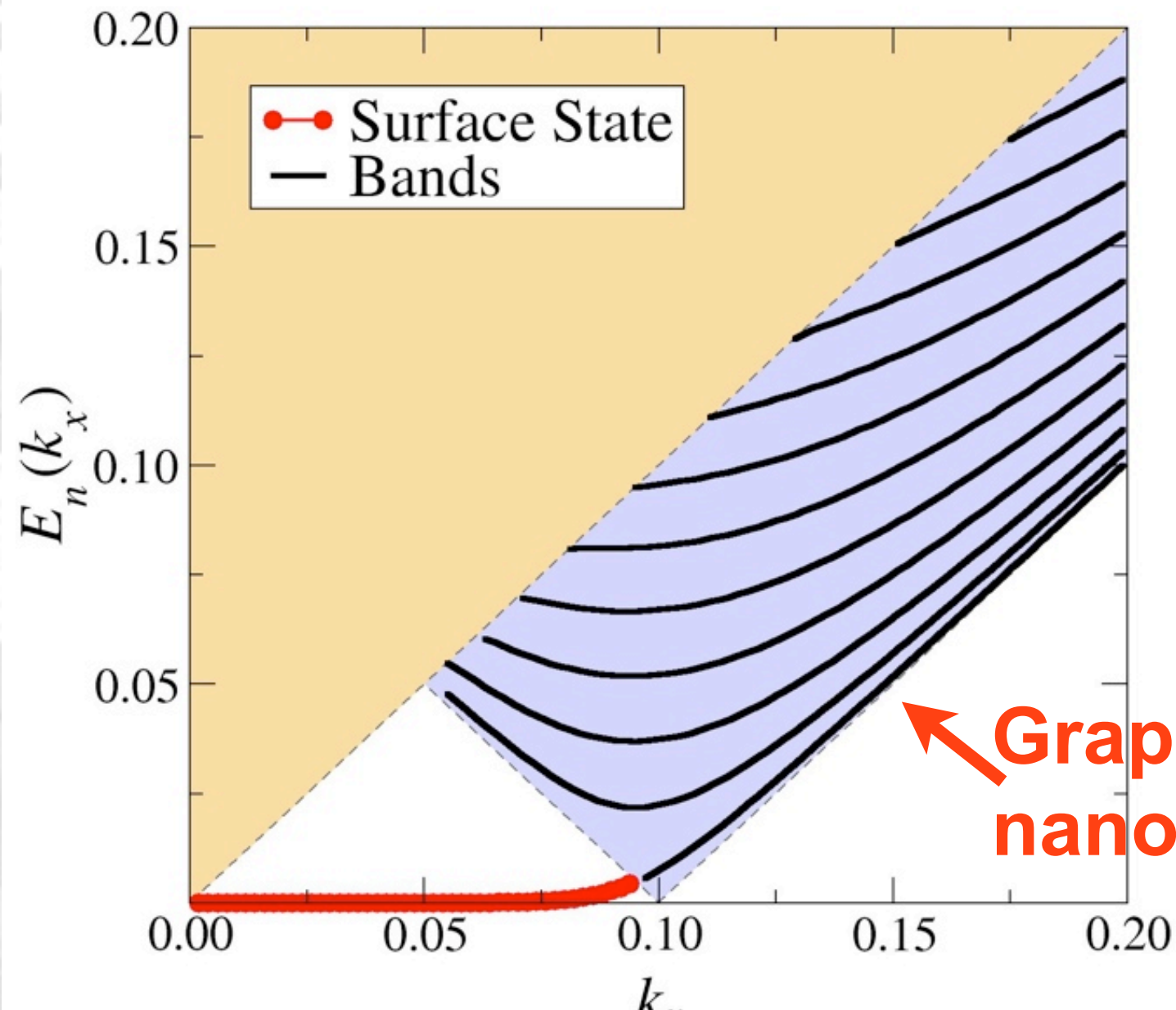


$$H = v_F \int dr \Psi^\dagger \begin{bmatrix} \boldsymbol{\sigma} \cdot (\mathbf{p} - \frac{1}{v_F} \mathcal{A}) & 0 \\ 0 & -\boldsymbol{\sigma} \cdot (\mathbf{p} + \frac{1}{v_F} \mathcal{A}) \end{bmatrix} \Psi ,$$

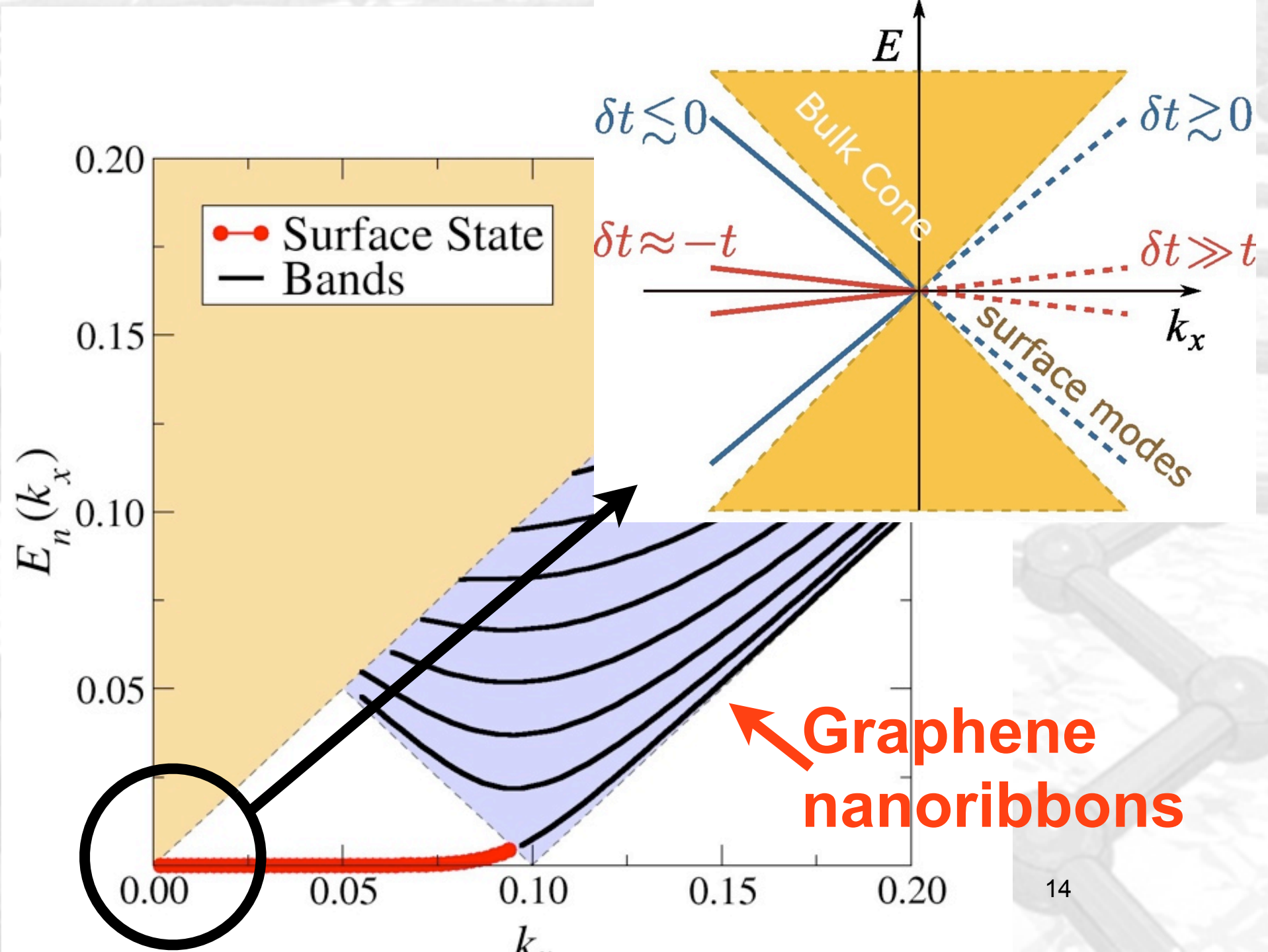
$$\mathcal{A}(\mathbf{r}) = \mathcal{A}_x(\mathbf{r}) - i\mathcal{A}_y(\mathbf{r})$$

$$\mathcal{A}(\mathbf{r}) = \delta t \theta(y) \theta(L - y) \mathbf{u}_x .$$



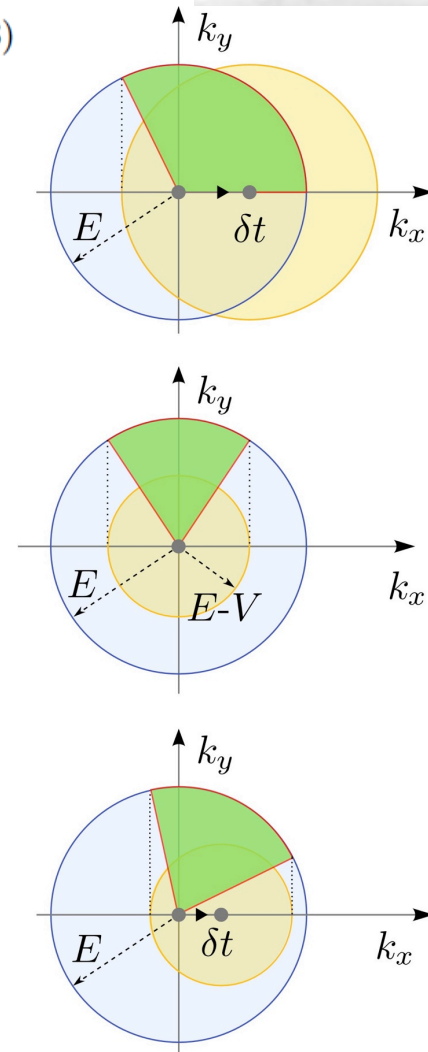
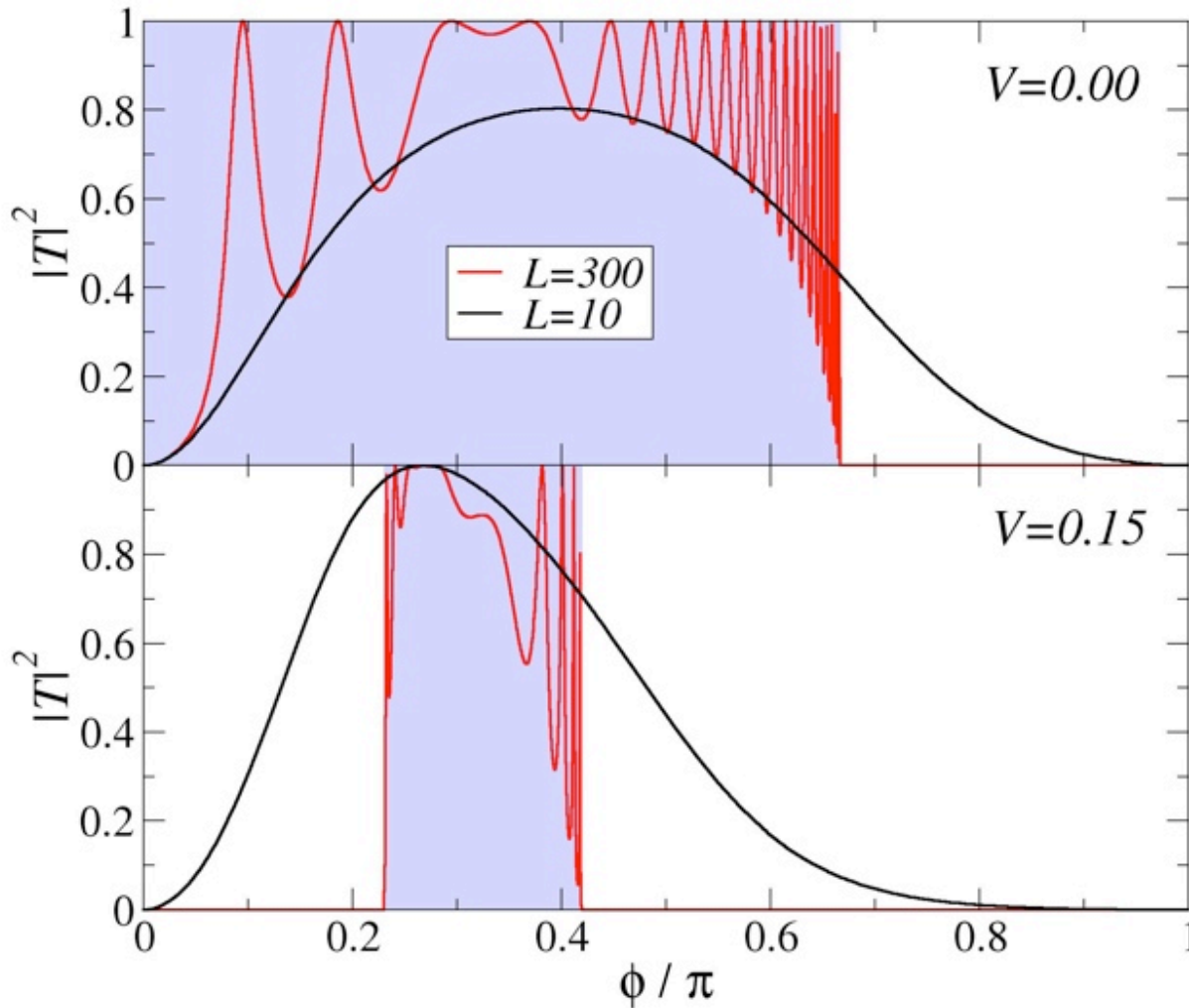


Graphene nanoribbons



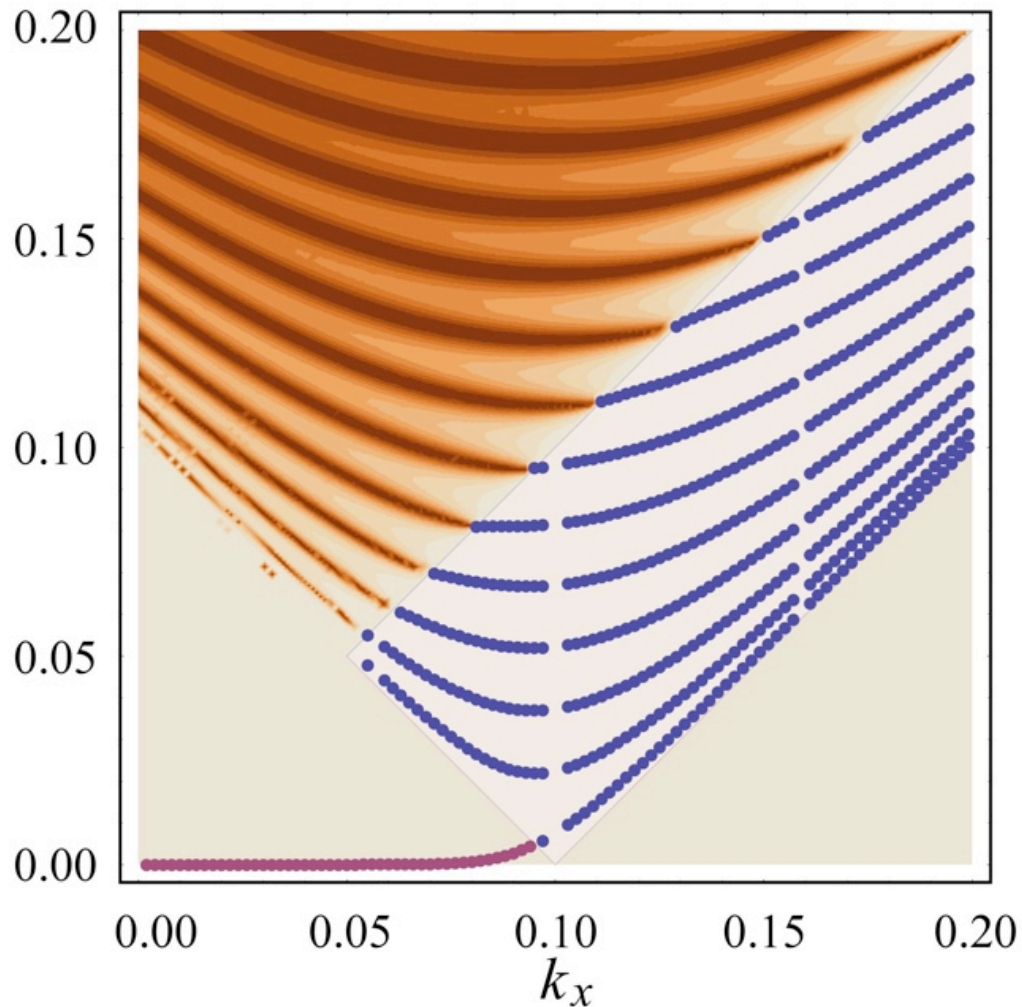
$$T = \frac{e^{-ik_y L} \sin \phi \sin \varphi}{\cos(q_y L) \sin \phi \sin \varphi + i \sin(q_y L) (\cos \phi \cos \varphi - 1)}$$

M. M. Fogler, F. Guinea, and M. I. Katsnelson, Phys. Rev. Lett. 101, 226804 (2008)



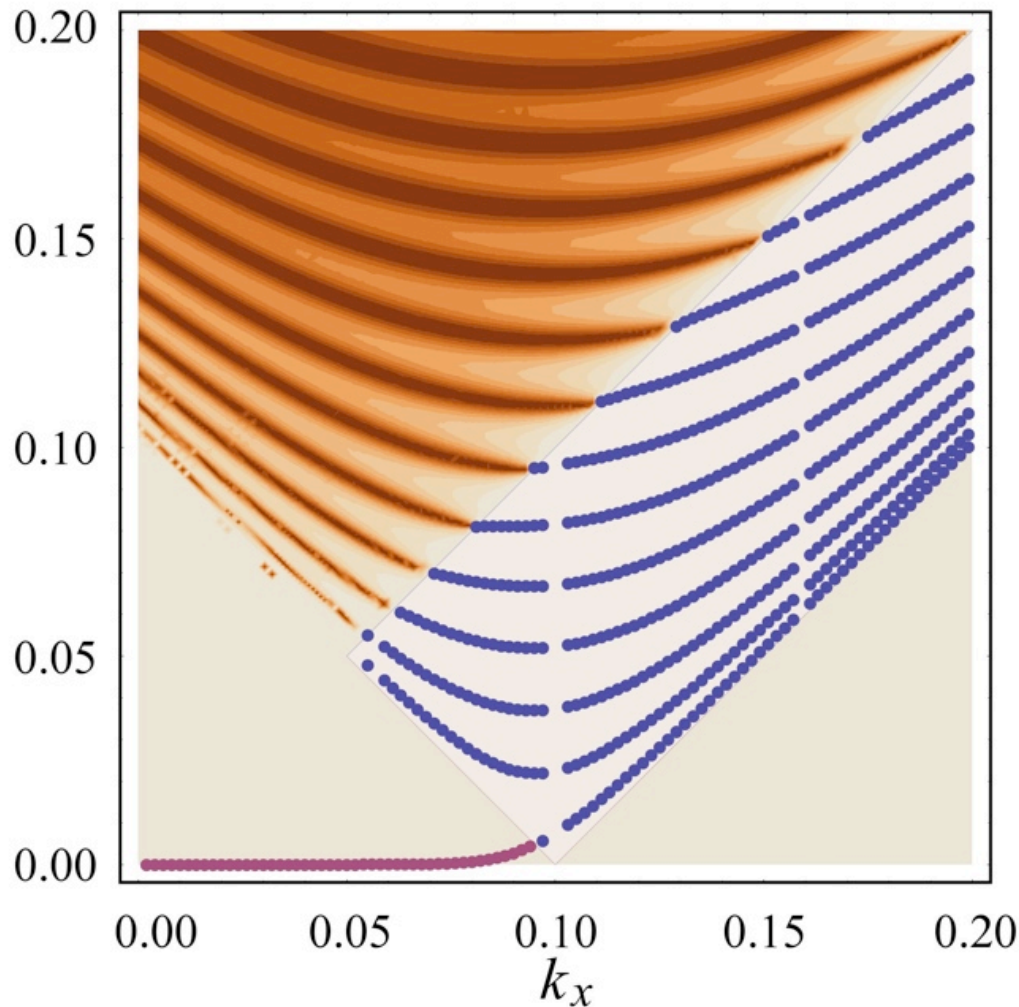
$$T = \frac{e^{-ik_y L} \sin \phi \sin \varphi}{\cos(q_y L) \sin \phi \sin \varphi + i \sin(q_y L) (\cos \phi \cos \varphi - 1)}$$

M. M. Fogler, F. Guinea, and M. I. Katsnelson, Phys. Rev. Lett. 101, 226804 (2008).

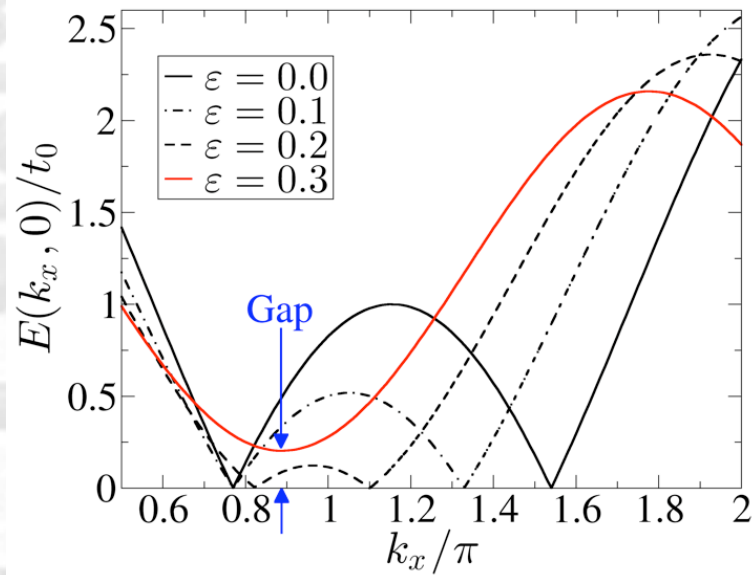
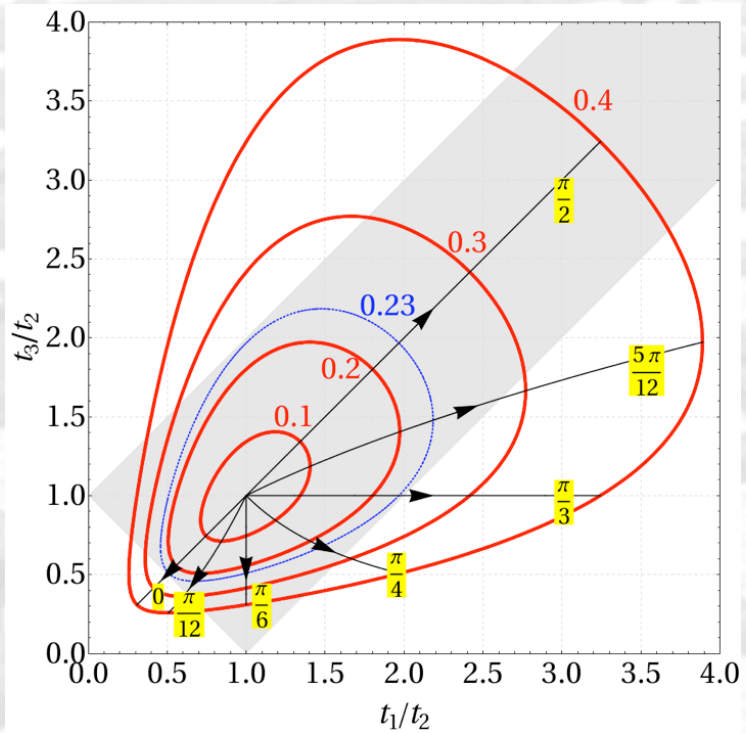


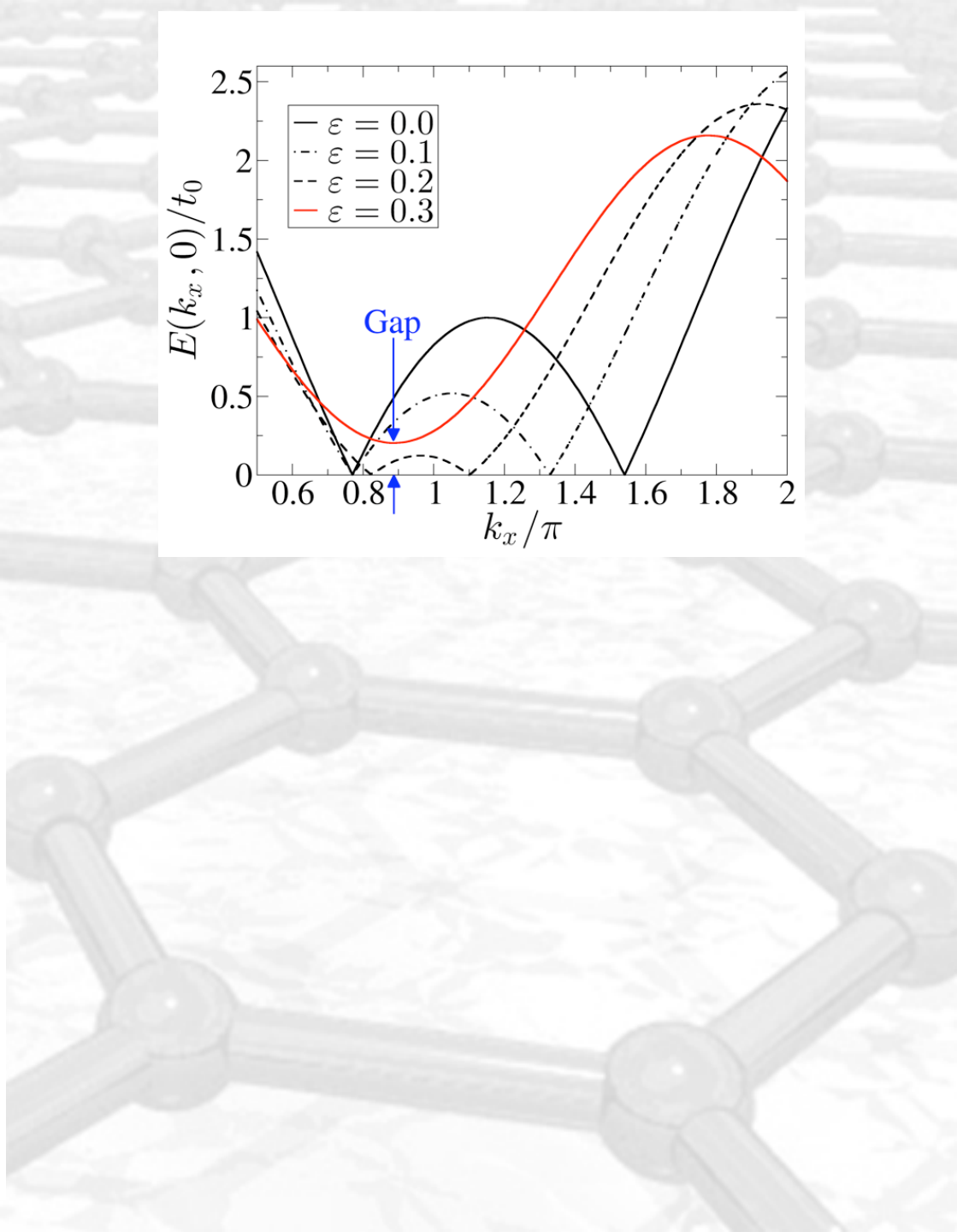
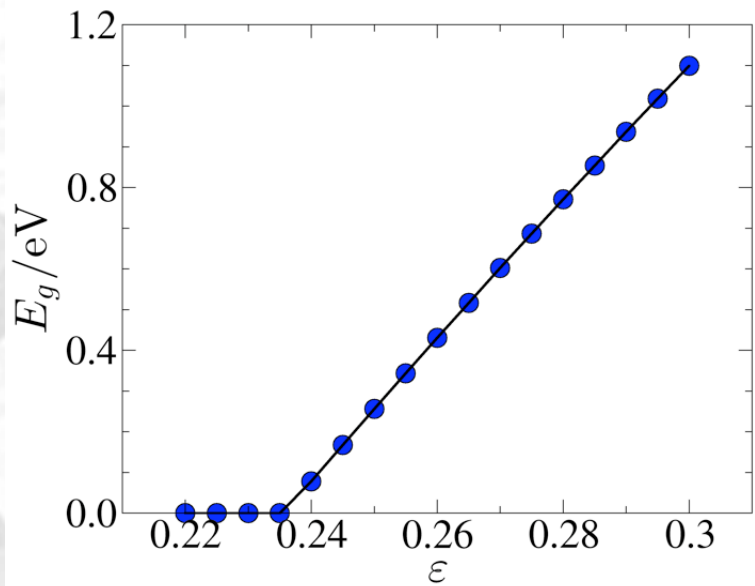
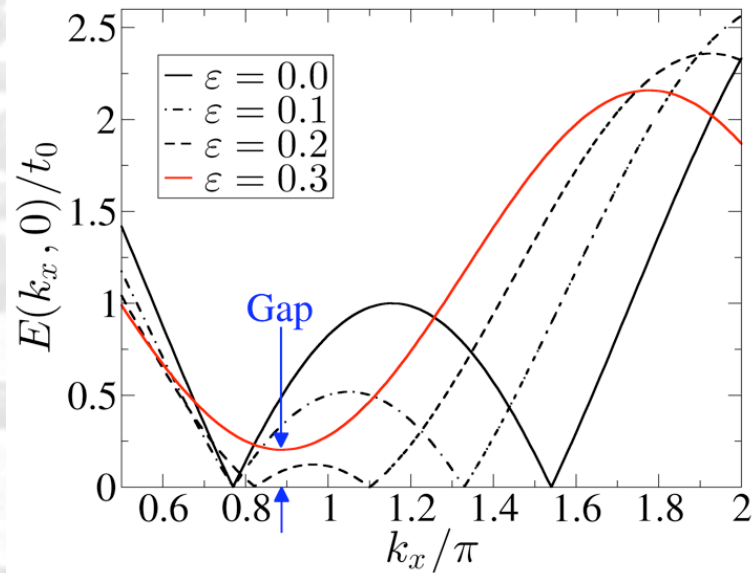
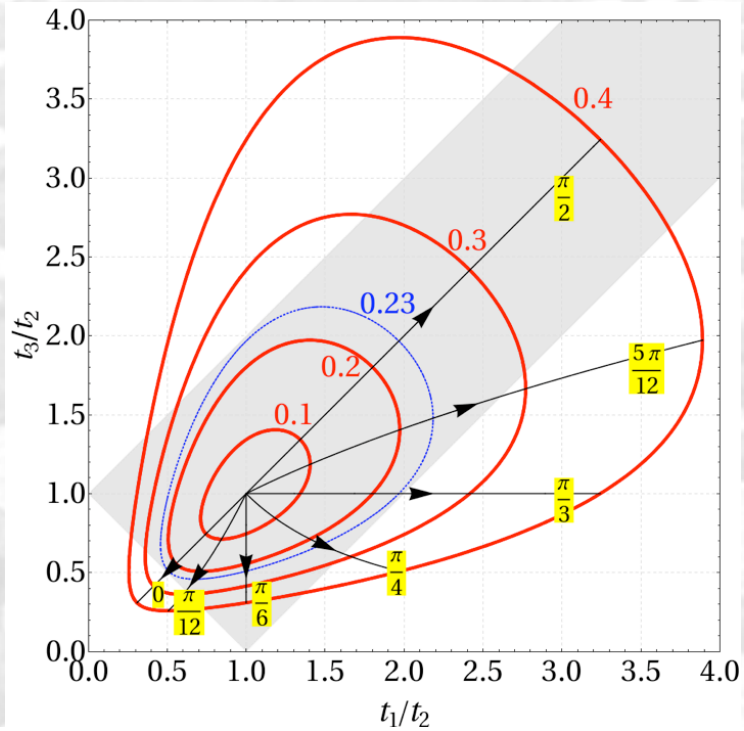
$$T = \frac{e^{-ik_y L} \sin \phi \sin \varphi}{\cos(q_y L) \sin \phi \sin \varphi + i \sin(q_y L) (\cos \phi \cos \varphi - 1)}$$

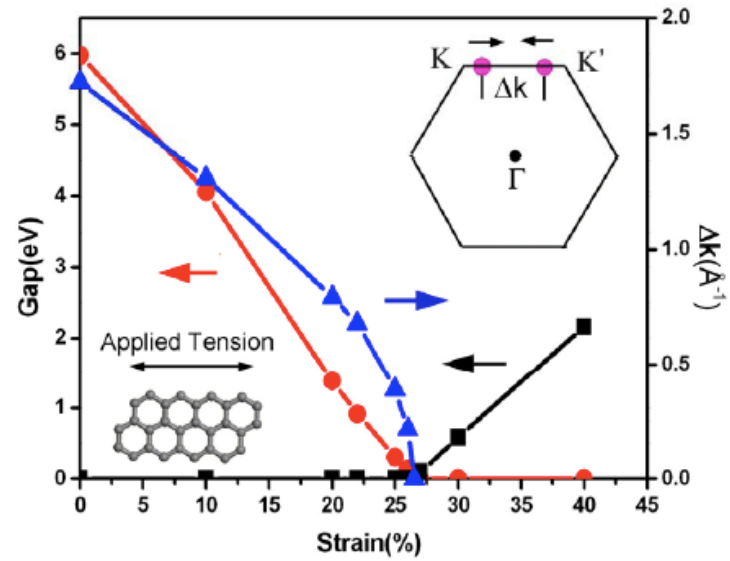
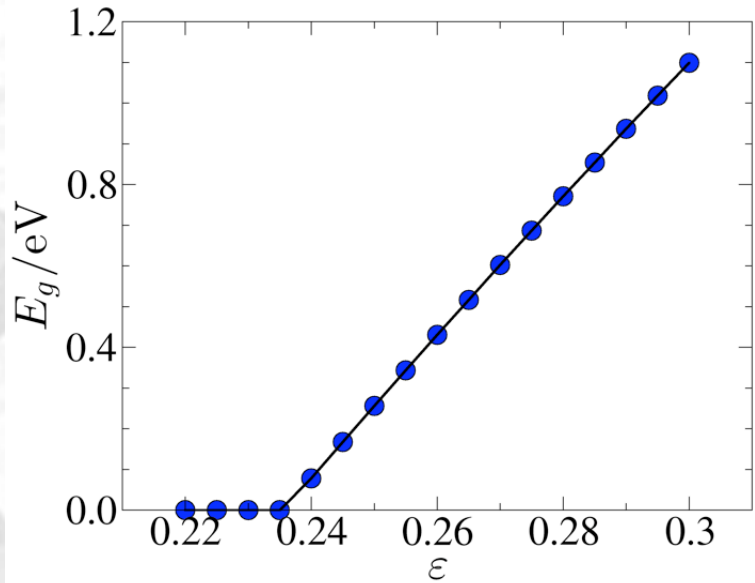
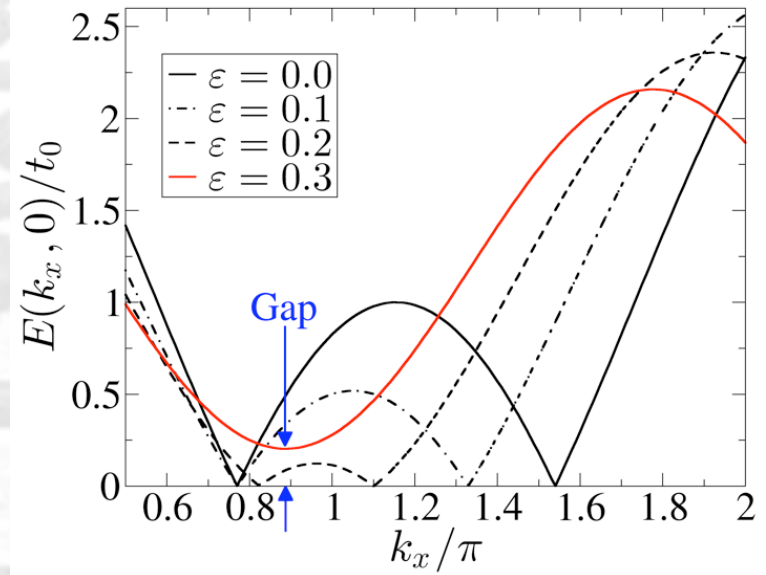
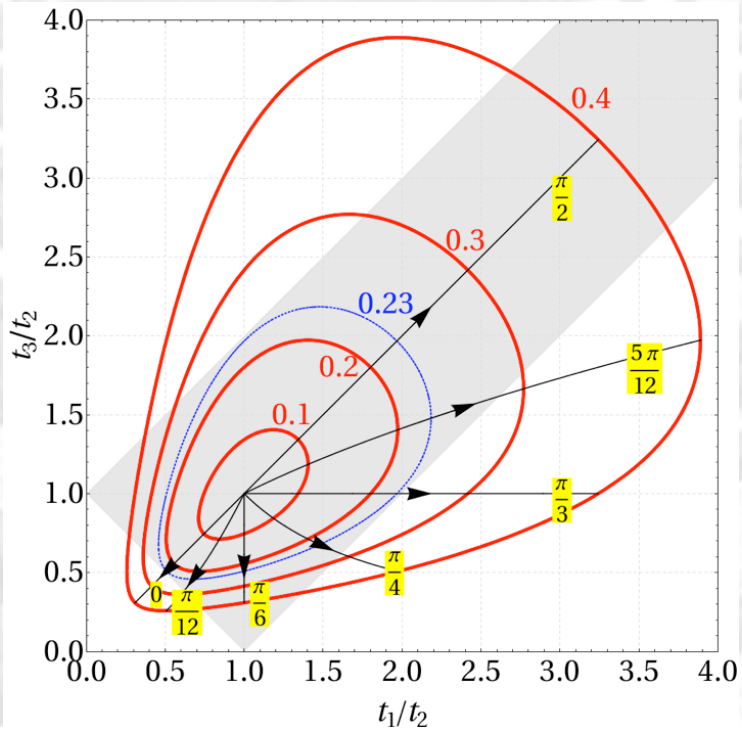
M. M. Fogler, F. Guinea, and M. I. Katsnelson, Phys. Rev. Lett. 101, 226804 (2008).



$L \approx 20\text{nm}$
 $E_g \approx 50\text{ meV}$
 $\delta t/t \sim 10\%$
 strain of $\sim 5\%$

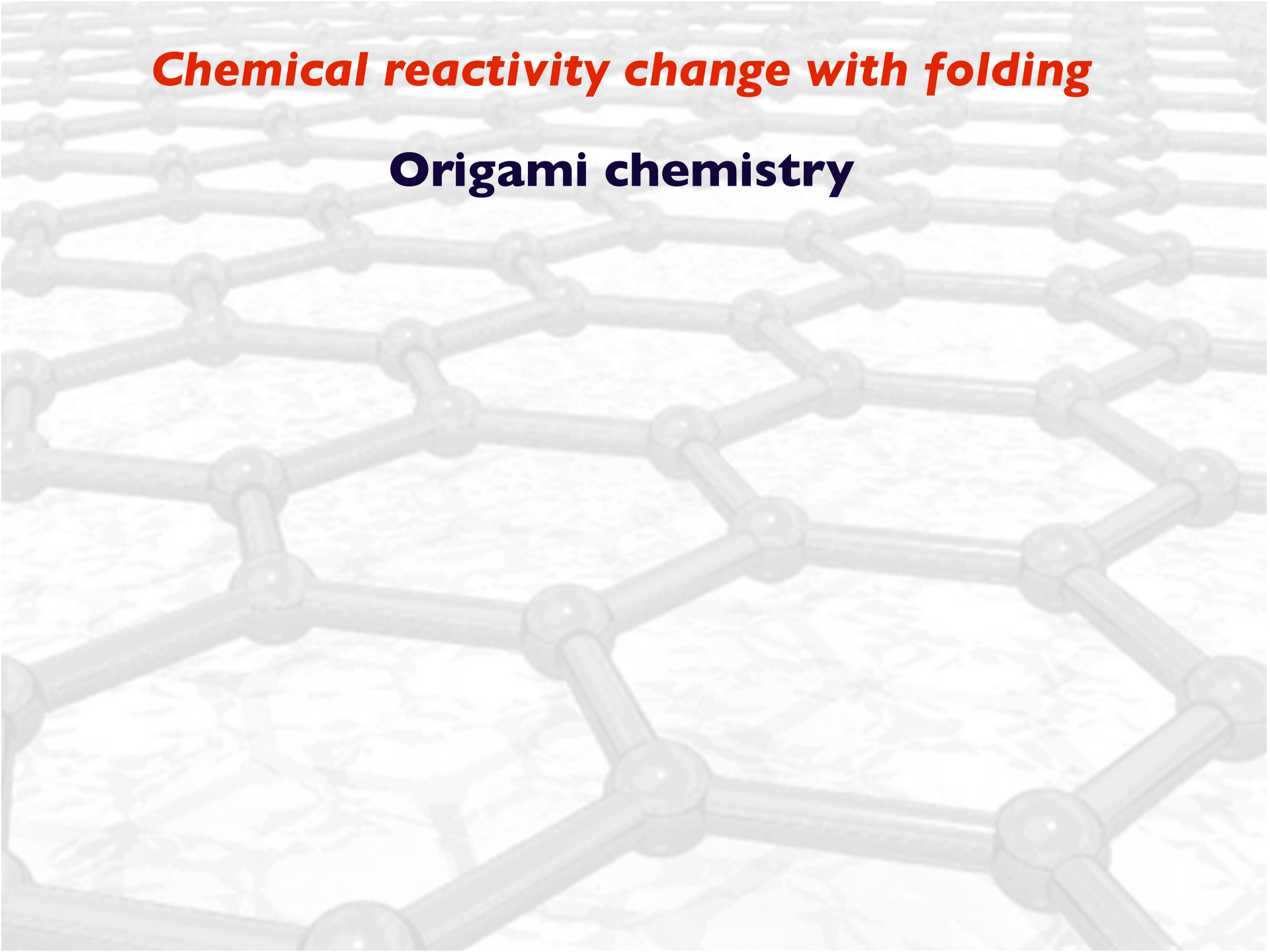






Chemical reactivity change with folding

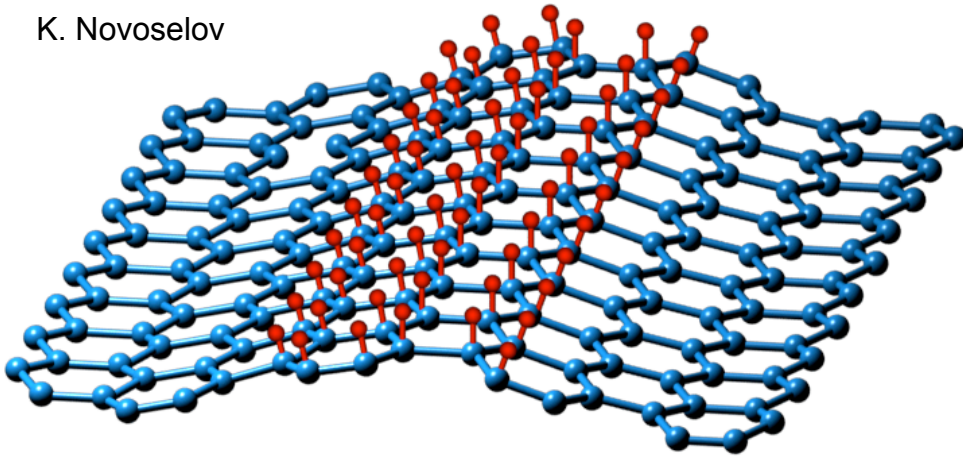
Origami chemistry



Chemical reactivity change with folding

Origami chemistry

K. Novoselov

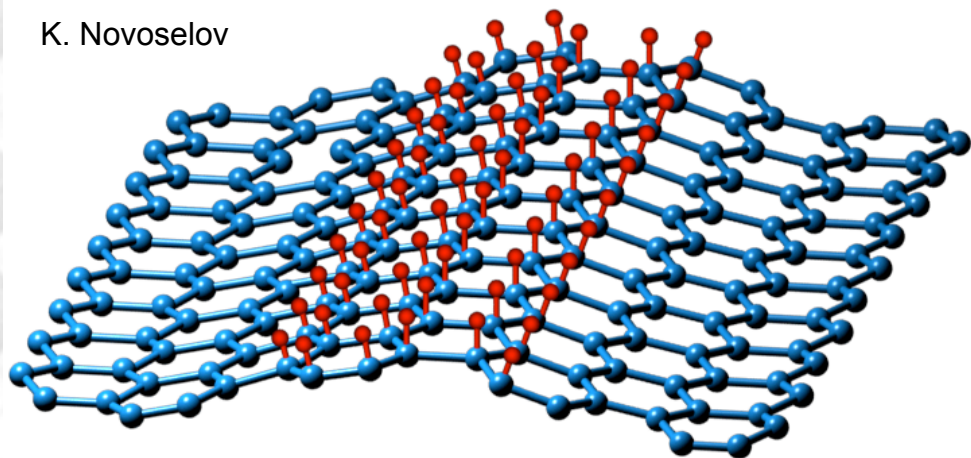


Graphane

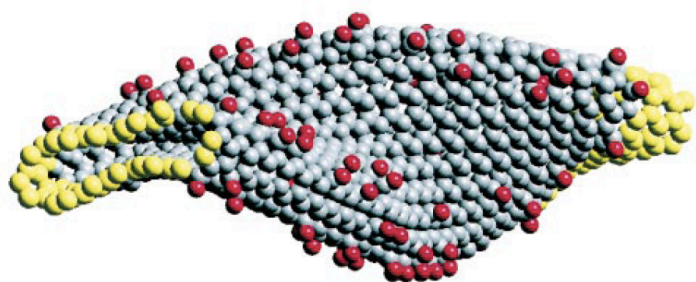
Chemical reactivity change with folding

Origami chemistry

K. Novoselov



Graphane



Srivastava et al., J. Phys. Chem. B 103, 4330 (1999)

Graphene Mechanics: correct analogy not so much like this...



1 micrometer square of graphene = sheet of a plastic wrap as big as a living room

Graphene Mechanics: ...but more like so!



New uses for
Plastic Wrap
Graphene

Use # 3,427:
Bike Happening Costume

Use # 3,428:
Bicycle Decoration

Buckling, Shear, Crumpling and Wrinkles

Due to its 2-dimensionality graphene

- 1 Cannot withstand **compression**
- 2 Cannot withstand **shear**

No Threshold for Buckling Instability



Inext. Memb.



Textiles



Paper

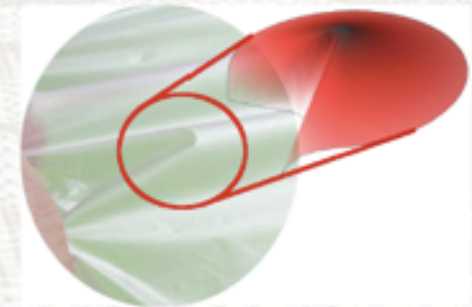


Metal

Universal aspects of Geometry and Physics of Wrinkling

- 1 Almost all area is locally flat;
- 2 Network of ridges linked/terminated by vertices;
- 3 **Elastic energy strongly focused in vertices.**

Witten, Rev. Mod. Phys. (2007), Vaziri & Mahadevan, PNAS (2008).



Vertices have universal shape \Rightarrow **Conical Singularities**



Are There Wrinkling Singularities in Graphene? Plenty!

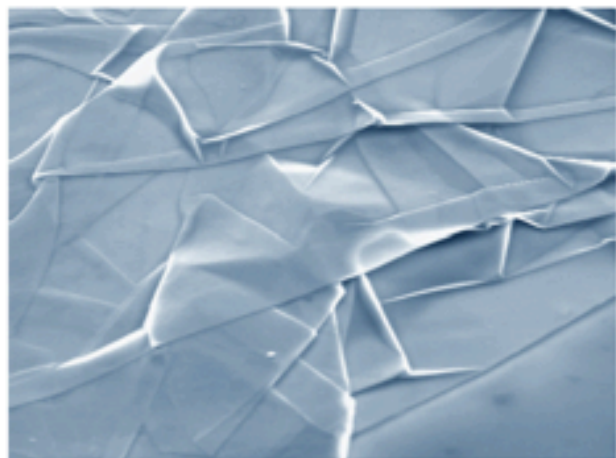


Fig. 5: Graphene on SiO₂ (A. Geim).

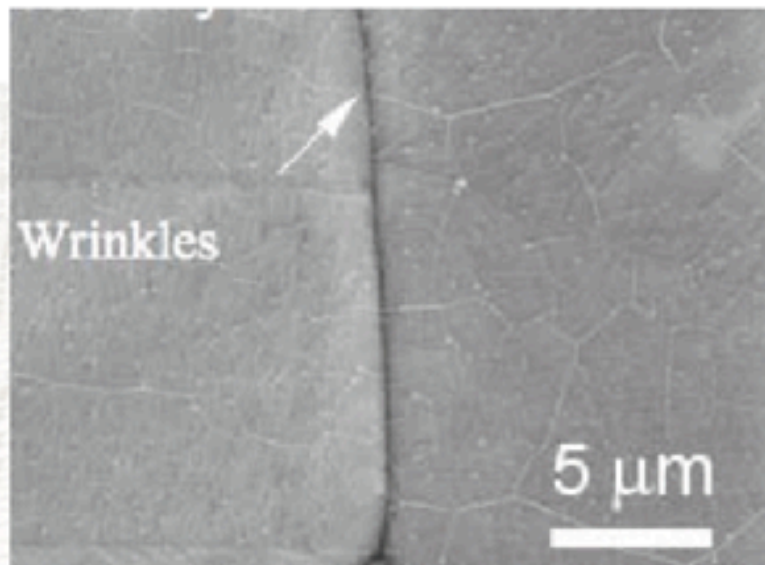


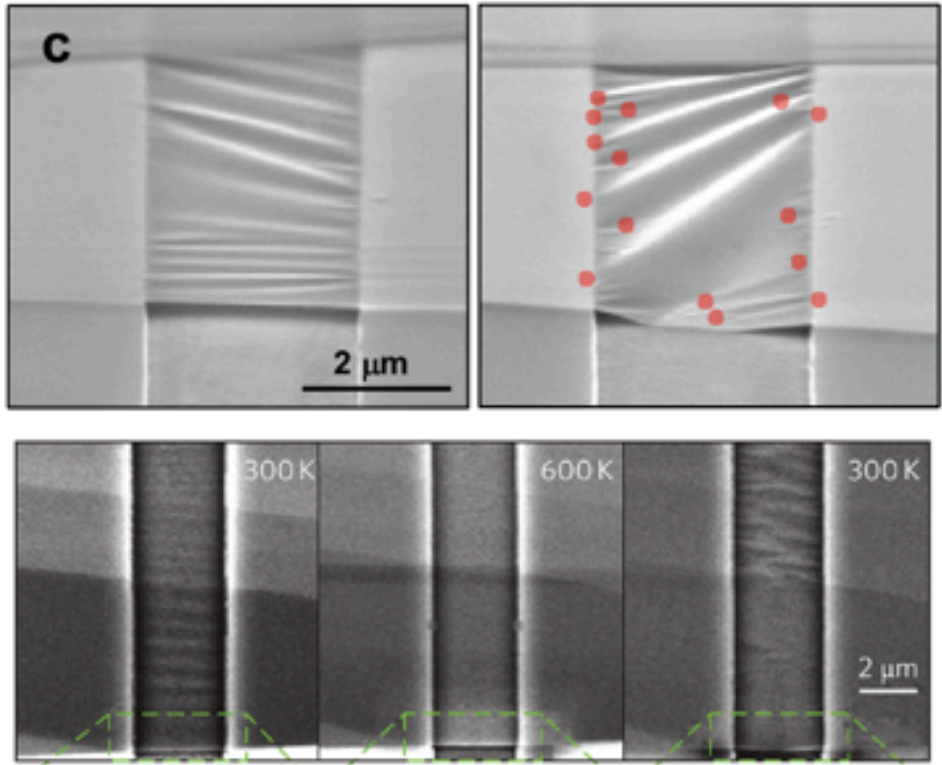
Fig. 6: CVD of graphene on Cu (R. Ruoff).

LETTERS
PUBLISHED ONLINE: 26 JULY 2009 | DOI: 10.1038/NNANO.2009.191

nature
nanotechnology

Controlled ripple texturing of suspended graphene and ultrathin graphite membranes

Wenzhong Bao¹, Feng Miao¹, Zhen Chen², Hang Zhang¹, Wanyoung Jang², Chris Dames² and Chun Ning Lau^{1*}



The figure consists of two main panels. The top panel, labeled 'C', shows two side-by-side micrographs of suspended graphene. The left micrograph shows a flat membrane with a 2 μm scale bar. The right micrograph shows the same membrane with a controlled ripple texture, indicated by red dots. The bottom panel shows a series of three micrographs of a suspended graphene membrane at different temperatures: 300 K, 600 K, and 300 K. The 600 K image shows a pronounced ripple texture. A green dashed line at the bottom indicates the profile of the ripples. A 2 μm scale bar is present in the bottom right of the 300 K images.

Fig. 7: Typical wrinkles in suspended graphene.

Simulation of Graphene Wrinkling and Singularities

- Unprecedented possibilities for mechanical simulation of ultra-thin sheets and plates.
- Accurate and efficient bond-order potentials available.

Atomistic simulation of graphene

- Biaxial Shear
- $T = 0$
- REBO + MD relaxation

Buckles just as a thin plate would.

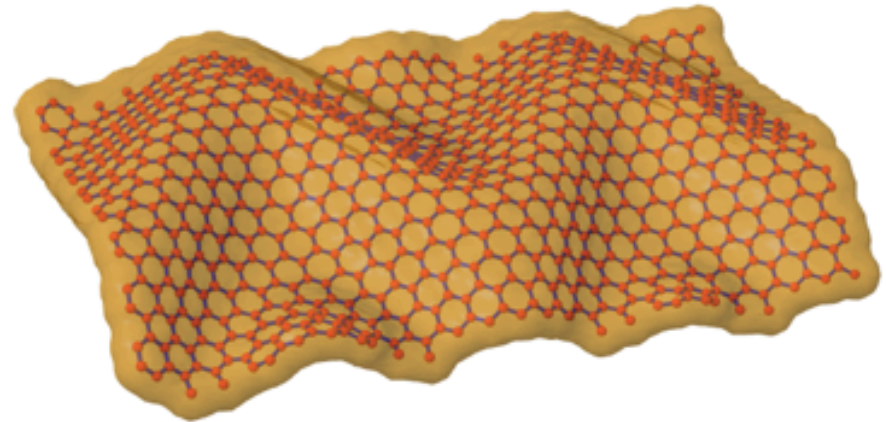


Fig. 8: Relaxed structure under biaxial shear.

Isolated Conical Singularities

- Same curvature profile;
- No strain except at the vertex.

Target for electron scattering



Fig. 9: Relaxed structure under biaxial shear.

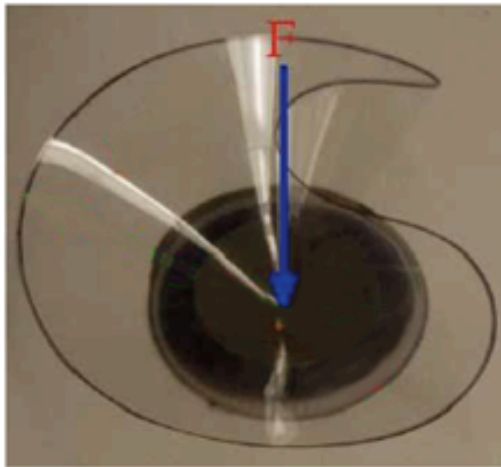


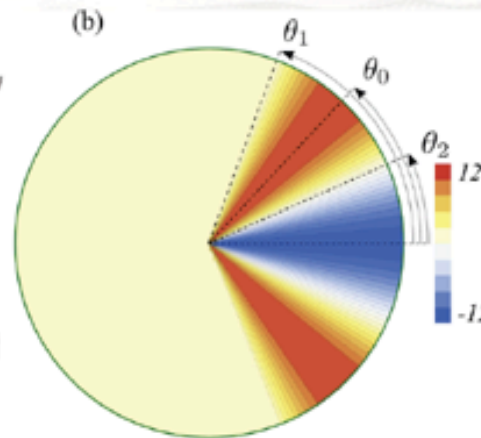
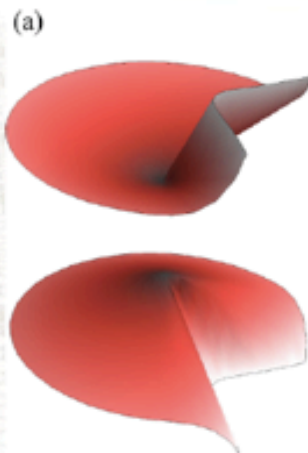
Fig. 10: Mellado *et al.*, 0912.3778

- 1 Indentation of a free, flat membrane;
- 2 Apply NL elasticity to obtain deformation field:

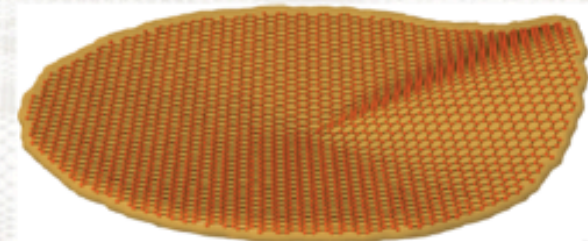
$$\mathbf{u}(\rho, \theta) = u_\rho(\rho, \theta)\mathbf{u}_\rho + u_\theta(\rho, \theta)\mathbf{u}_\theta + \zeta(\rho, \theta)\mathbf{z}$$
- 3 Vertical displacement is a generalized cone:

$$\zeta(\rho, \theta) = \rho \psi(\theta)$$
- 4 Extract **exact solution** for $\psi(\theta)$, $\mathbf{u}(\rho, \theta)$.

$\mathbf{u}(\rho, \theta)$ **universal** \rightarrow **independent** of material parameters!



Cauchy-Born Hypothesis

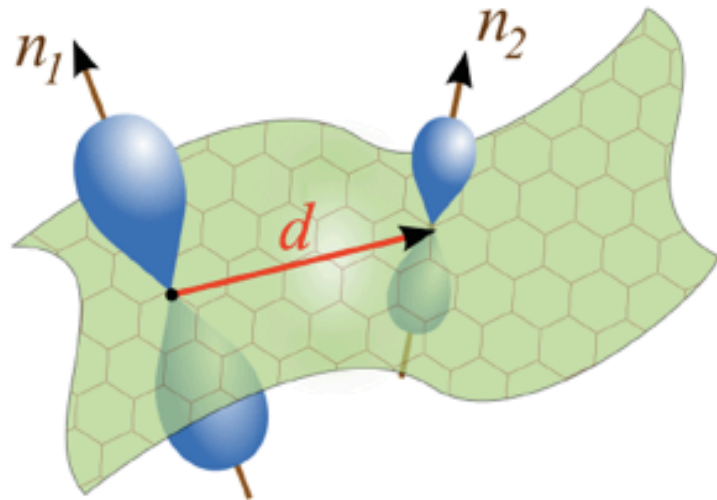


Carbon lattice follows $\mathbf{u}(\rho, \theta)$

Fig. 11: Analytical Profile of $\mathbf{u}(\rho, \theta)$



Dirac Electrons on a Curved Surface



Arbitrary p_z orbitals in 3D space

$$t_{12} = V_{pp\pi} \mathbf{n}_1 \cdot \mathbf{n}_2 + (V_{pp\sigma} - V_{pp\pi})(\mathbf{n}_1 \cdot \hat{\mathbf{d}})(\mathbf{n}_2 \cdot \hat{\mathbf{d}})$$

See also Isacsson PRB (2008)

Remember: we know the exact geometry of the conical surface from the elasticity solution.

Differential geometry on the Conical Surface to obtain \mathbf{n}_i , \mathbf{d}_j :

$$t_{ij} = t + \delta t_{ij} \rightarrow \delta t_{ij} \approx -V_{pp\pi}^0 \frac{1}{2} |(\delta_0 \cdot \nabla) \nabla \zeta|^2 + \frac{V_{pp\pi}^0/3 - V_{pp\sigma}^0/4}{d_0^2} [(\delta_0 \cdot \nabla)^2 \zeta]^2$$

But, at low energies, graphene electrons behave as Dirac fermions:

$$H \approx v_F \boldsymbol{\sigma} \cdot [\mathbf{p} - \frac{1}{v_F} \mathcal{A}(\mathbf{r})] + [3t'_0 + \Phi(\mathbf{r})] \sigma^0 \quad (\mathcal{A}(\mathbf{r}), \Phi(\mathbf{r}) \propto \delta t_{ij})$$

$$\Phi(\mathbf{r}) = \alpha \text{Tr}^2 [K^i_j] - \beta \det [K^i_j] \quad (K^i_j = \text{Curvature Tensor, Gauss II})$$

$$\alpha = 9a_0^2 V_{pp\pi}^0 / 8 + 27a_0^2 V_{pp\sigma}^0 / 32 \approx 1.5 \text{eV}\text{\AA}^2, \beta = 3a_0^2 V_{pp\pi}^0 + 9a_0^2 V_{pp\sigma}^0 / 8 \approx 3 \text{eV}\text{\AA}^2$$

Semiclassical Transport

Scattering rate

$$S(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} |V_{\mathbf{k}, \mathbf{k}'}|^2 \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}), \quad V_{\mathbf{k}, \mathbf{k}'} = \Phi_{\mathbf{k}-\mathbf{k}'} \frac{1 + \exp(i\phi_{\mathbf{k}} - i\phi_{\mathbf{k}'})}{2}$$

Effective Potential

$$\Phi(r) = \frac{\alpha}{r^2} \times [\psi(\theta) + \psi''(\theta)] \quad (\text{supercritical!}) \quad \Phi_q \sim \log(qr_0) + \dots$$

Conductivity

$$\sigma = \sigma_0 G_{2,4}^{3,1} \left(4k_F^2 r_0^2 \mid 0, 0, 0, -2 \right)^{-1}, \quad \sigma_0 = \frac{v_F^2 \hbar e^2}{2\pi^3 n_i v_0^2}$$

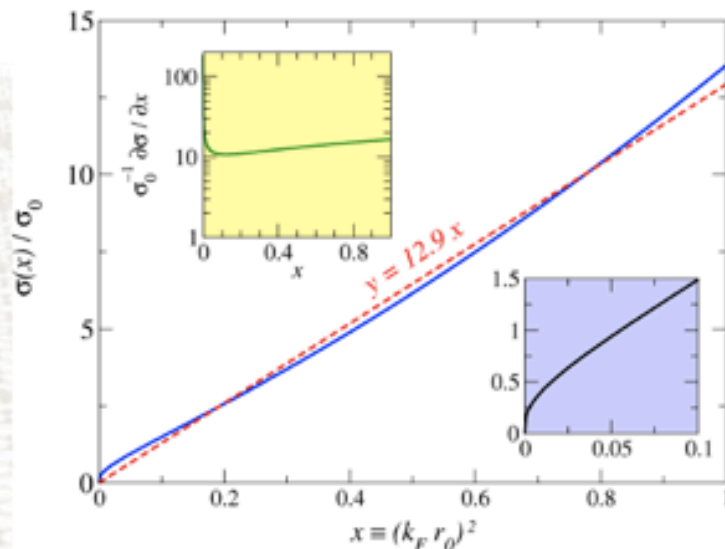


Fig. 12: DC conductivity vs density.

DC Conductivity

- Quasi-linear as function of n_e ;
- Log-singularity at the origin;
- Same physics as mid-gap states.



Local Electronic Structure

Sanity check

Go back to the full tight-binding, and analyze local electronic structure.

$$H = \sum_{\langle i,j \rangle} t_{ij} c_i^\dagger c_j + \sum_{\langle\langle i,j \rangle\rangle} t'_{ij} c_i^\dagger c_j + \text{H. c.}$$

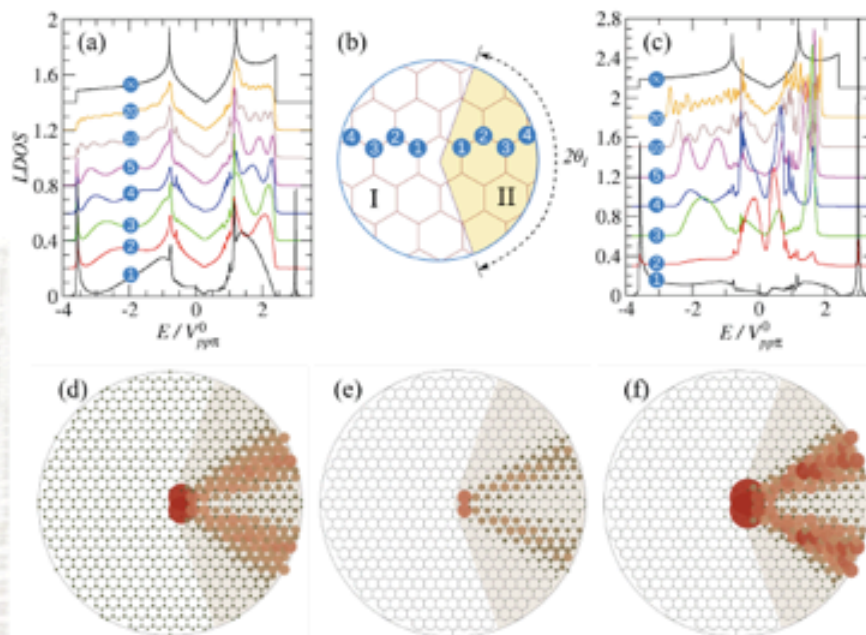


Fig. 13: LDOS nearby a CS.

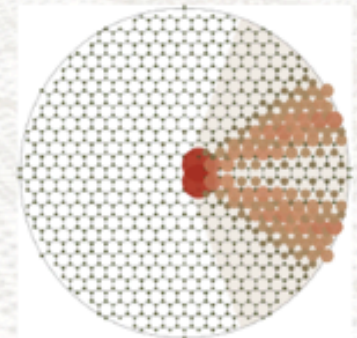
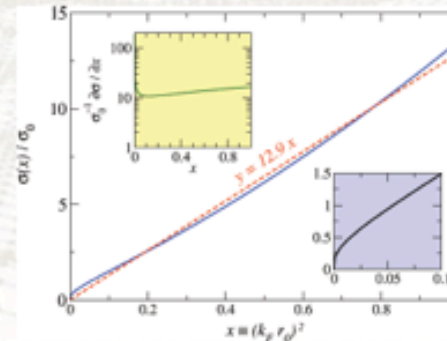
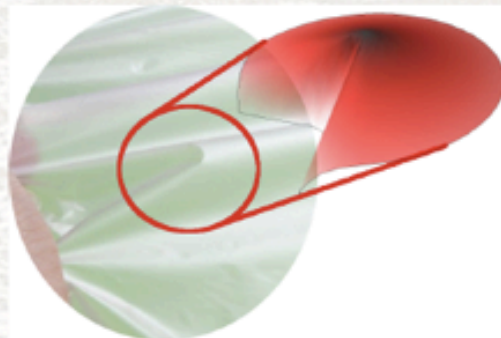
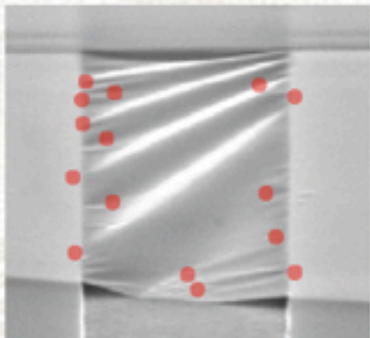
Local Electronic Structure

- Apex can be strongly confining;
- Charge density concentrated around apex;
- Extra leakage around rays of null curvature;
- Pseudo magnetic oscillations in LDOS vs E .

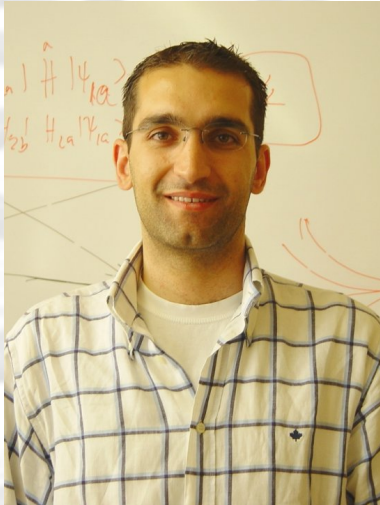


Conclusions

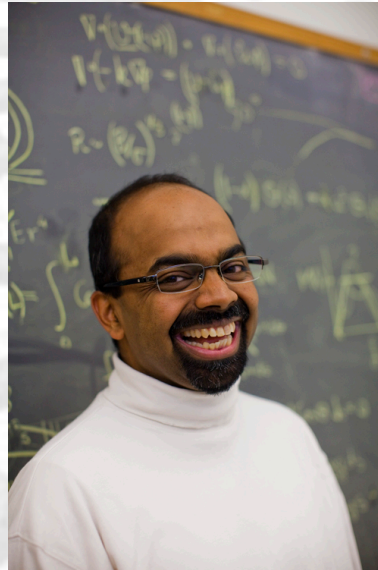
- 1 Graphene **wrinkles**. Often and easily.
- 2 Universal aspects of wrinkling in graphene \Rightarrow **Conical singularities**.
- 3 Graphene's **electrons couple to structural deformations** and curvature.
- 4 Conical singularities act as **strong scatterers** yielding
 - Linear-in-density σ ;
 - Confining centers;
 - Variable pseudo magnetic fields.
- 5 Even in the absence of any impurities, mobility limited by wrinkling.
- 6 **Pseudomagnetic fields** interfere with Landau quantization.
- 7 Further possibilities in the realm of strain engineering.



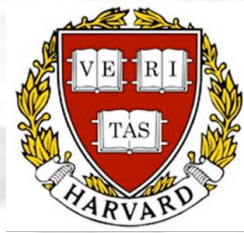
Collaborators on graphene's wrinkles



Vitor Pereira



L. Mahadevan



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