Subject: Forces (Free Body Diagrams; F = ma)

Used for: To find the acceleration of one of more objects when they feel one or more forces.

Prior knowledge: Math: trigonometry, vectors

The FBD recipe!
Often a Free Body Diagram is useful or necessary to solve a problem that involves forces. Follow these steps, and you'll solve any problem with little difficulty.

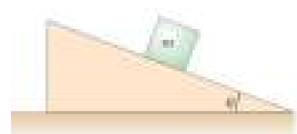
1. Draw one Free Body Diagram for each object (see below for what is a good FBD).

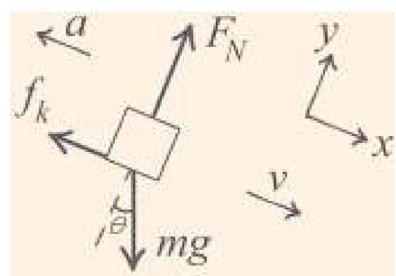
2. Break the forces up into components.

3. For each object and each direction, write down ΣF = (sum of forces) = ma.

4. Solve this set of equations. If there are N unknowns then you need N equations. Often you need to include additional equations like $f_k = \mu_k F_N$ to solve the problem completely.

How to draw the perfect FBD
The better your drawing, the easier it is to solve a problem. Try to stick to the rules below. As an example, consider a block that slides down an incline, but slows down because of friction.





- Keep it simple; often you can represent an object with a simple box or circle. Don't represent it as a point (because the forces often start at a certain point of the object, which could produce a torque)

- Don't make the diagram too small. You'll often add a lot of stuff to it.

- Draw the forces, make them touch the object. See table below for a list of possible forces. Don't break them up in components yet; it often makes your diagram messy.

- It often helps to draw a small coordinate system (with x and y) near the diagram. Each object can have its own coordinate system.

- It often helps to include any angles that are given.

- Draw an arrow with v or a in your diagram, if you know their direction. Don't let it touch the object! Only forces should touch the object.

- If you have several objects, use the same symbol for quantities with the same magnitude! For example, if two objects are connected with a rope, you should use the same F_T for the tension that each object feels. Similarly, if the objects have different masses, you need to use m_1g for one object and m_2g for the other (and not mg for both of them, because the m's are different).

List of forces and how to find them:

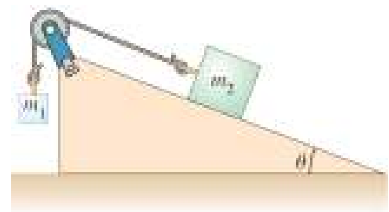
Equation	**Name**	**Direction**	**Notes**
$F_g = mg$	Gravity; weight	Downwards (towards center of the Earth).	$g = 9.8$ m/s^2 on the Earth
F_N	Normal force	Perpendicular (i.e. normal) to the surface.	Equal to whatever force is needed to prevent the object from falling through the floor. Use the FBD recipe to find F_N .
F_T	Tension	In direction of the rope or string (away from the object).	Is either given already, or needs to be found via the FBD recipe.
$F_s = kx$	Spring force (Hooke's law)	Opposite to the direction of x	x is indicates how much the spring is compressed or stretched, and $x = 0$ if it's not stretched or compressed at all. k is called the <u>spring constant</u>.
$f_k = \mu_k F_N$	Kinetic friction	Opposite to the direction of the velocity	μ_k is the <u>coefficient of kinetic friction</u> and F_N the normal force.
f_s	Static friction	Parallel to the surface, but the exact direction depends on what is needed to prevent object from moving.	Equal to whatever force is needed to prevent object from moving. However, the max value of f_s is: $f_{s,\max} = \mu_s F_N$ with μ_s the <u>coefficient of static friction</u> and F_N the normal force.
$F_G = G\dfrac{m_1 m_2}{r^2}$	Gravitational force	From center of one mass towards the center of the other	Use this equation instead of $F_g = mg$ when the r is approximately the radius of the Earth or larger (i.e. for planets and such).
$F_B = \rho_f V_s g$	Buoyancy force	Upwards	ρ_f is the density of the fluid, and V_s the part of the volume of the object that is submerged. When an object of mass m is floating, $F_B = \rho_f V_s g = mg$.

Circular motion:
When an object moves in a circle of radius r and with speed v, then the net force ΣF this object is experiencing should be:

$$\sum F = \frac{mv^2}{r}$$

This is called the centripetal force.

Extended example:
Given the situation as shown in the figure. Determine the acceleration with which the blocks move. Assume there is no friction.

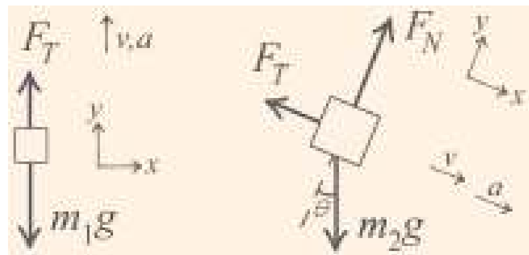


The problem asks for the acceleration, and therefore "Forces" is the best way to approach this problem (because if we find the net force, we've found the acceleration – via $\Sigma F = ma$.).

1. <u>Draw one Free Body Diagram for each object.</u>

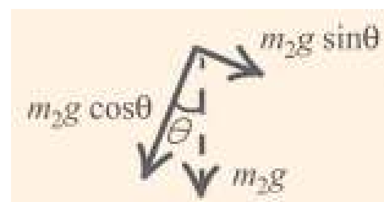
We can ignore the pulley (it's not mentioned that we should take it into account, and we know nothing about it, so we can ignore it).

Note that the tension F_T is the same for both objects, so we use the same symbol (and different symbols for the weights m_1g and m_2g). Also v and a are the same for both objects, and their directions are as indicated.



2. <u>Break the forces up into components.</u>

Because of the way we've chosen our coordinate system in step 1, we only need to break up the gravitational force of m_2 into components.



3. <u>For each object and each direction, write down ΣF = (sum of forces) = ma.</u>

We have two objects, but m_1 only has forces in one direction. We should therefore write down <u>three</u> equations:

Mass m_1: $\sum F_y = F_T - m_1 g = m_1 a$

Mass m_2:
$$\sum F_x = m_2 g \sin\theta - F_T = m_2 a$$
$$\sum F_y = F_N - mg\cos\theta = 0$$

4. <u>Solve this set of equations. If there are N unknowns then you need N equations.</u>

We have 3 equations and 3 unknowns (F_T, F_N, and a), so this is solvable. Often this is the hardest part, because it requires a lot of math and practice.

- First "solve" for F_T using the equation for m_1 as: $F_T = m_1 a + m_1 g$
- Next, plug this into the upper equation for m_2: $m_2 g \sin\theta - \left(m_1 a + m_1 g\right) = m_2 a$
- We now have an equation with only one unknown, which is easier to solve:

$$m_2 g \sin\theta - m_1 g = m_1 a + m_2 a = (m_1 + m_2) a \qquad \Rightarrow \qquad a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$$