

CENTRIPETAL ACCELERATION

Newton's first law of motion states that each body moves at a constant speed in a straight line unless a net external force acts upon the body. Thus if an object is moving about a circle, even at constant speed, then there must be a net external force acting on the object and the object must be accelerating. The acceleration on a revolving object points toward the center of the circular motion and is called centripetal acceleration. The term centripetal comes from Latin words meaning "center seeking."

There are many important examples of centripetal acceleration. For example, the Earth and Moon exert gravitational forces on each other, and the Moon experiences a centripetal acceleration towards the center of the Earth-Moon system. In today's lab a rubber stopper is connected to a string, and the rubber stopper is swung in a horizontal circle. The tension in the string causes the stopper to undergo centripetal acceleration. (If the string suddenly breaks, then the rubber stopper will move in a straight line in accordance with Newton's first law.)

If an object of mass m is rotating with constant speed v about a circle of radius r , then the centripetal acceleration on the object has the magnitude:

$$a = v^2/r \quad (1)$$

This equation is derived in Randall Knight, *Physics for Scientists and Engineers*, Section 7.2. In today's laboratory exercise the centripetal acceleration equation (1) will be tested experimentally.

Although equation (1) is useful, it is more convenient for today's lab to express the same result in terms of the time T needed for one complete revolution. The speed of the rotating body is calculated by dividing the circumference of the orbit by the period T .

$$v = C/T = 2\pi r/T$$

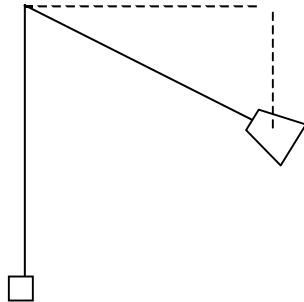
Substitute for v in equation (1) and simplify to get the following equation for centripetal acceleration:

$$a = 4\pi^2 r/T^2 \quad (2)$$

Apparatus and Free Body Diagrams

The apparatus for this lab consists of a rubber stopper of mass m_s , a piece of nylon string which is threaded through a long glass tube, and some steel washers with a total mass m_w . The steel washers are attached to a paper clip on one end of the string, and the rubber stopper is fastened to the other end of the string. A person should hold the glass rod and swing the rubber stopper at a uniform rate in a horizontal circle as shown in Figure 1. One should *not* touch the nylon string, but ideally the rubber stopper should be swung at a sufficient rate so that the steel washers do not move up or down. In other words, the weight of the steel washers at the bottom of the string should balance precisely the upward tension in the string so that the string in the glass tube does not move vertically.

Figure 1



Neglect all frictional forces and air resistance, and draw below a free body diagram of the steel washers.

If the steel washers are at rest, then from the preceding free body diagram we see that the upward force of tension in the string F_T equals the downward weight of the steel washers.

$$F_T = m_w g \quad (3)$$

Now draw below a free body diagram of the revolving rubber stopper.

From Newton's second law and the free body diagram of the revolving rubber stopper, the horizontal component of the string's tension on the revolving rubber stopper must be

$$m_s a = F_T \cos \theta \quad (4)$$

Substitute equation (2) for centripetal acceleration into (4).

$$m_s (4\pi^2 r / T^2) = F_T \cos \theta \quad (5)$$

If L is the length along the string from the top of the glass tube to the center of the rubber stopper (see Figure 1), then r is the perpendicular distance from the center of the rubber stopper to the glass tube and $r = L \cos \theta$. Substitute this expression for r into (5) and then divide out the cosine factor.

$$m_s 4\pi^2 L \cos \theta / T^2 = F_T \cos \theta$$

$$m_s 4\pi^2 L / T^2 = F_T \quad (6)$$

If the string does not stretch and if the string's weight is negligibly small, then F_T , the string's tension on the rubber stopper, has the same magnitude as the string's tension on the steel washers in eqn. (3). Substitute eqn. (3) into eqn. (6) and we obtain

$$m_s 4\pi^2 L / T^2 = m_w g \quad (7)$$

All of the quantities in eqn. (7) can be measured easily, and so it provides us with a test of the centripetal acceleration.

Procedure

1) It is helpful to work in groups of two or three. If there are three in a group, then one person must swing the rubber stopper, another should count the number of revolutions of the rubber stopper, and the third can measure the time with a stopwatch.

2) Place 8 steel washers (or a similar weight) on the clip at the bottom of the string, and practice rotating the rubber stopper in a uniform circular motion so that the stopper does not move radially either in or out.

Hints for Success

- a) It is often easiest if the radius of revolution r is about 20 centimeters.
 - b) *Do not move the hand too much* while swinging the stopper. Ideally the steel washers should be stationary.
 - c) Make an ink mark on the nylon string at the bottom of the glass tube. Practice and make certain that the ink mark remains at the same position as the rubber stopper is rotated.
 - d) Wear goggles so that the stopper does not hit anyone in the eye.
- 3) Use a scale to measure the mass of the rubber stopper m_s and the mass of the steel washers and paper clip m_w . Convert these data from grams to kilograms, if necessary, and record the data in the space above Table 1. For the weight $W_w = m_w g$ in Table 1, use $g = 9.80 \text{ meters/second}^2$.
- 4) After having practiced rotating the stopper in a horizontal plane, carry out three trials and record the data in Table 1. The length L in Table 1 should be measured along the string from the top of the glass tube to the center of the rubber stopper.
- 5) It is often most accurate to measure t the total time for a predetermined number of revolutions, perhaps 20 revolutions. The period T for one complete revolution is then obtained by dividing t by the number of revolutions n , i.e. $T = t/n$. For example, if there are 20 revolutions of the stopper in 10 seconds, then T equals $10 \text{ seconds}/20 = 0.5 \text{ seconds}$.
- 6) Calculate the value of F_T using eqn.(6), which is based on the centripetal acceleration equation.
- 7) Each group should do three trials for the same weight of steel washers W_w and find the average value of F_T .

Table 1

m_s = Mass of rubber stopper _____ kilograms

m_w = Mass of steel washers and paper clip _____ kilograms

Trial	Weight of washers $W_w = m_w g$ (N)	Length L (meters)	Total time t (sec)	# of rev.	Time of 1 rev. $T = t/n$ (sec)	F_T based on centripetal acceleration (eqn. 6) (N)
1						
2						
3						

Average value of $F_T =$ _____ N

Results

1) Is the equation for centripetal acceleration justified? Is the average value of F_T equal to the weight of steel washers W_w in agreement with eqn.(7)? What is the percentage error?

2) If time permits, then repeat the experiment for a different mass of hanging steel washers and record the data in Table 2.

3) What were the major sources of error in this experiment?