String/Gauge Duality:

(re) discovering the QCD String in AdS Space

Zakopane, Poland June 2003

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Confronting String Theory with Lattice Gauge theory data

Three Lectures

I. Ancient Lore (circa 1970)

–Empirical Basis: (Before QCD)
–Covariant String Formalism
–Large N topology: (After QCD)

II. Gauge/Gravity Duality (circa 1995)

–Failure of the QCD String
–AdS/CFT correspondence for Super Strings
–Confinement, Instantons, Baryons and Finite T
–Hard versus Soft Processes: Wide Angle and Regge

III. String vs lattice spectra (circa 2000)

–Glueball Spectra
–Stretched String Spectra

Pedagogical Comments

- Solutions to String Theory are often simpler than the theory itself!
  - E.g. the simplest "Feynman diagram" or "Veneziano amplitude" is very physical.
  - Modern string courses spend O(500) pages before deriving it. This is not the easiest (or historical approach)!
- This is the reason I am going to follow a (pseudo) historical approach.
  - Of course in the end you should do it both ways: inductively and deductively
- As 'tHooft has said it is important to demystify string theory!

Some References:

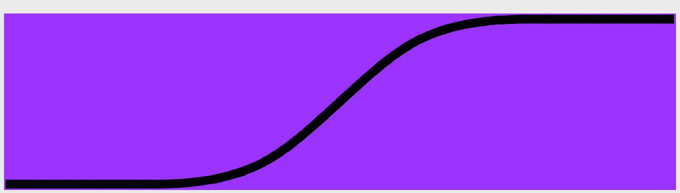
- Maldacena, hep-th/9711200 *The large N Limit of Superconformal Field Theories and Supergravity.*
- Brower,Tan and Mathur hep-th/0003115, ``*Glueball Spectrum for QCD from AdS Supergravity Duality*;
- Polchinski and Strassler, hep-th/0109174 *Hard Scattering and Gauge/String Duality*; hep-th/0209211 *Deep Inelastic Scattering and Gauge/String Duality*; Polchinski and Susskind, hep-th/00112204 *String Theory and the Size of Hadrons*
- Brower and Tan hep-th/0207144, ``*Hard Scattering in the M-theory dual for the QCD string*.
- Brower, Lowe and Tan het-th/0211201 *Hagedorn transition for strings on pp-wave and tori with chemical potentials*
- Brower and Tan het-th/02XXXX *Work in progress (hopefully) on the Stretched String in an AdS Black Hole.*

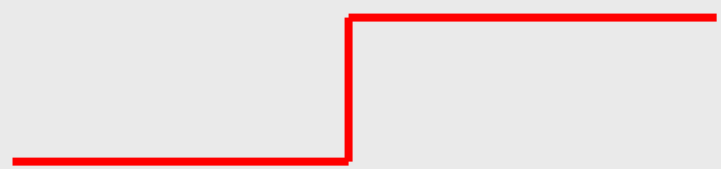
Basic Question behind these lectures:

Is QCD exactly equivalent (i.e. dual) to a Fundamental String Theory?

- Field Theories often have more than one "fundamental" formulation.
  - e.g. Sine Gordon QFT ←→ Massive Thirring QFT (Coleman-Mandelstam 1975)

$$\frac{1}{g^2}[\partial_\mu\phi\partial^\mu\phi+m^2(1-\sin(m\phi)] \quad \leftrightarrow \quad \bar{\psi}(i\gamma_\mu\partial_\mu-m)\psi-g(\bar{\psi}\gamma_\mu\psi)^2$$

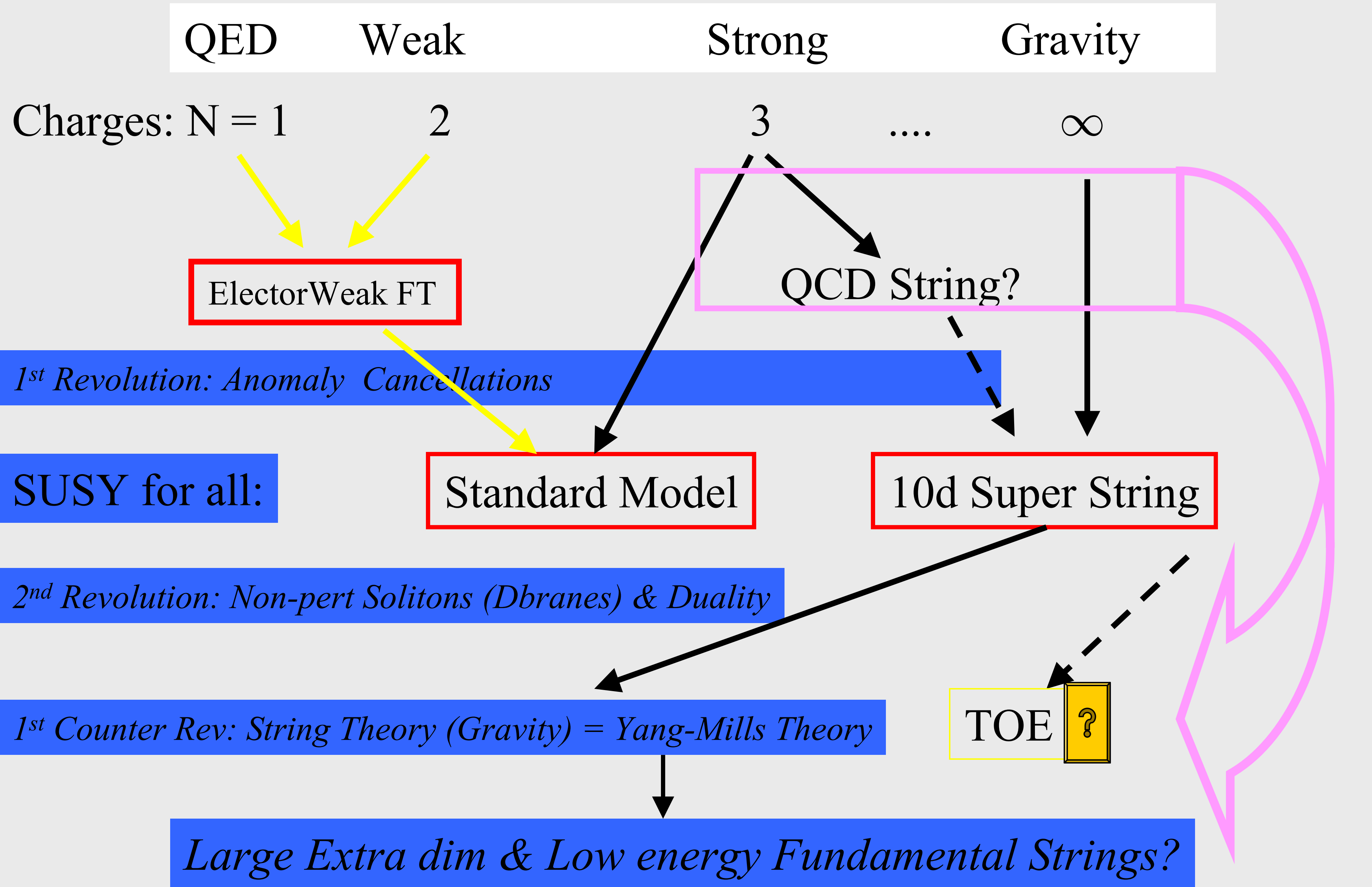




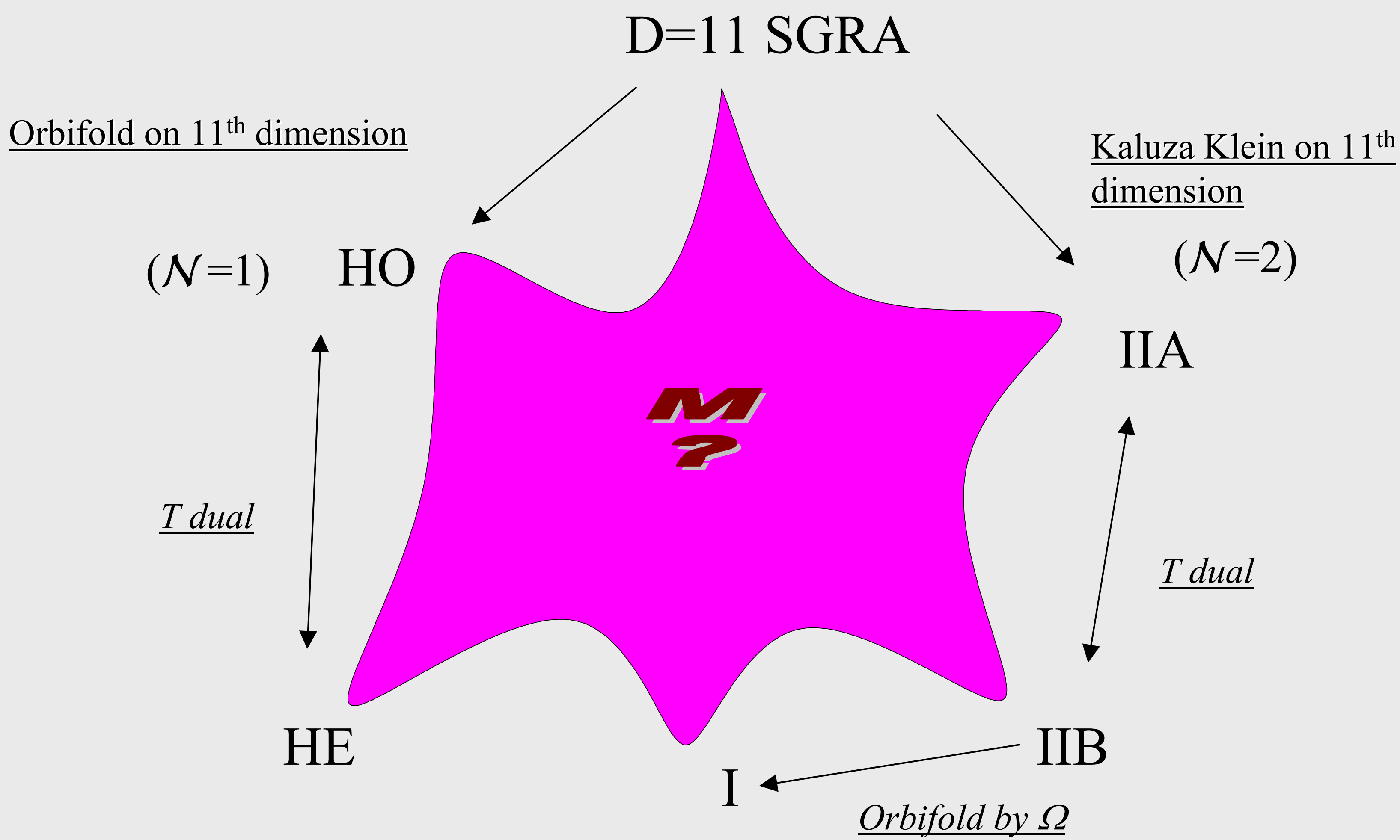
–The classical kink "solitons" can be replaced by sharp "elementary" fermion

– 3d Ising ←→ 3d Z2 Gauge Theory, Olive-Monoten for N = 4 SYM, etc.......

Relativistic Quantum Theories



Hexagon of Theory Space!



(“We dance around in a ring and suppose the secrete sits in the middle and knows” T. S. Eliot ?)

Lecture I – *Ancient Lore*

- Before QCD
  - Why and How was String theory invented?
  - Regge and Dolen-Horn-Schmid Duality
  - Veneziano's Pion amplitude
  - Guessing the covariant String formulation.

- After QCD
  - 1/N expansion is the QCD string theory:
  - weak coupling – asymptotic freedom
  - strong coupling – confined phase
  - χ Lagrangian

The discovery of strings --- "Not by Accident"

- circa 60's Local field theory was failing!
  - Elect/Mag → Success QED (divergent) perturbation theory
  - Weak → Failure No QFT field theory (d=6 J-J term)
  - Strong → Failure No QFT (Pions obey nice geometrical χL)
  - Gravity → Failure No QFT (Graviton obeys nice geometrical EH L)

1 out 4 is not impressive!

- Hadronic (Strong Nuclear) force was most troubling.
- Many "particles" of higher and higher spin with naive $A_J \simeq s^J$
- Can't all have their own "elementary" field!
- All hadrons are "bound states" on Regge sequences (aka Bootstrap)

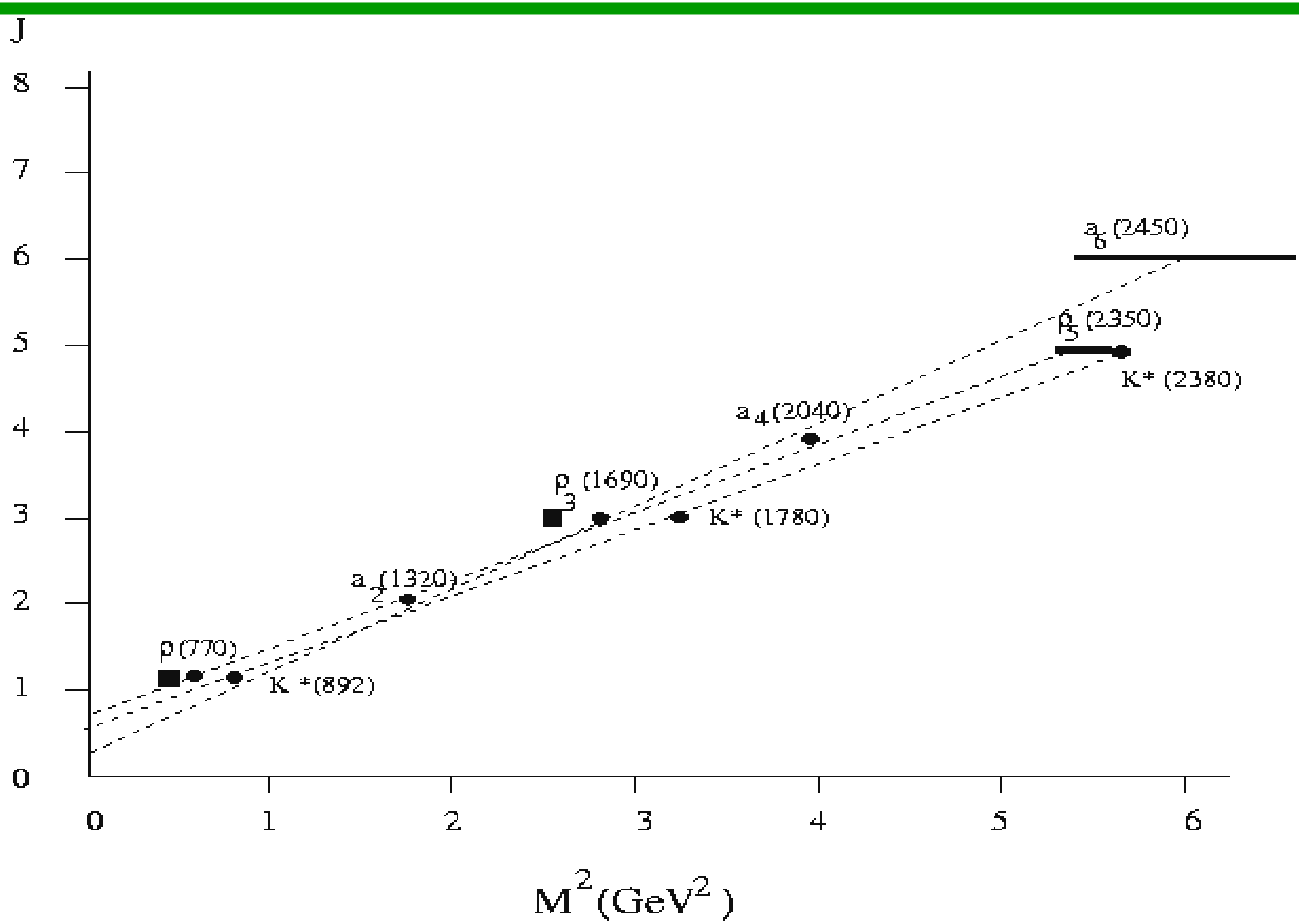


FIG. 1. Meson (ρ, K^* and a) Regge trajectories constructed from recent tabulated data (dark circles and error bars, PDG 2000). Boxes are model TDA predictions for the ρ trajectory.

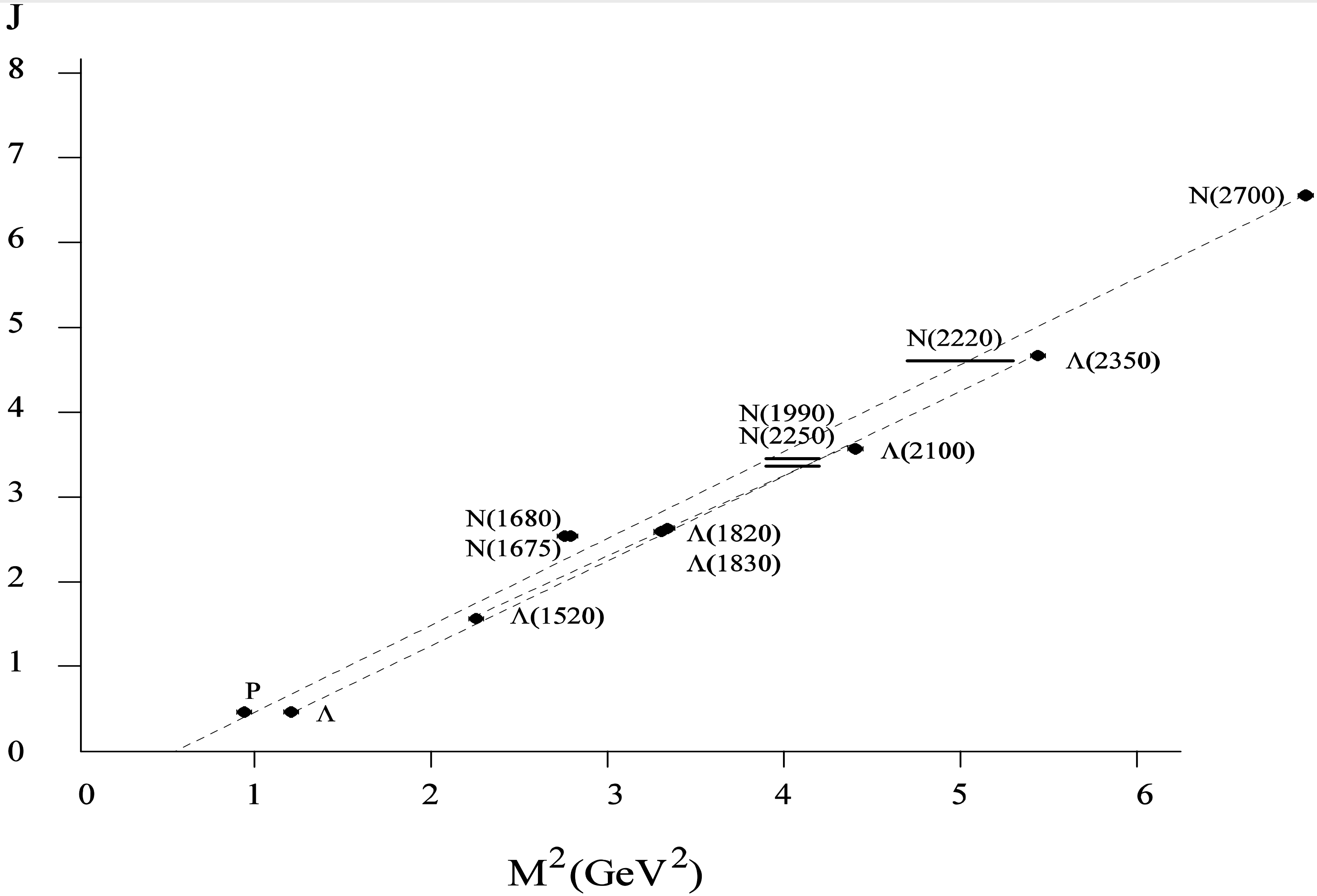


FIG. 2. Baryon (N and Λ) Regge trajectories constructed from recent tabulated data (dark circles and error bars, PDG 2000).

Regge: $J = \alpha(t) \simeq \alpha' t + \alpha_0$.

Common parameterization:

$$A_{\pi^+\pi^- \to \pi^+\pi^-}(s,t) \simeq \Gamma[1 - \alpha_\rho(t)](-\alpha' s)^{\alpha_\rho(t)}$$

Dolan-Horn-Schmid duality (Phys.Rev. 166, 1768 (1968):

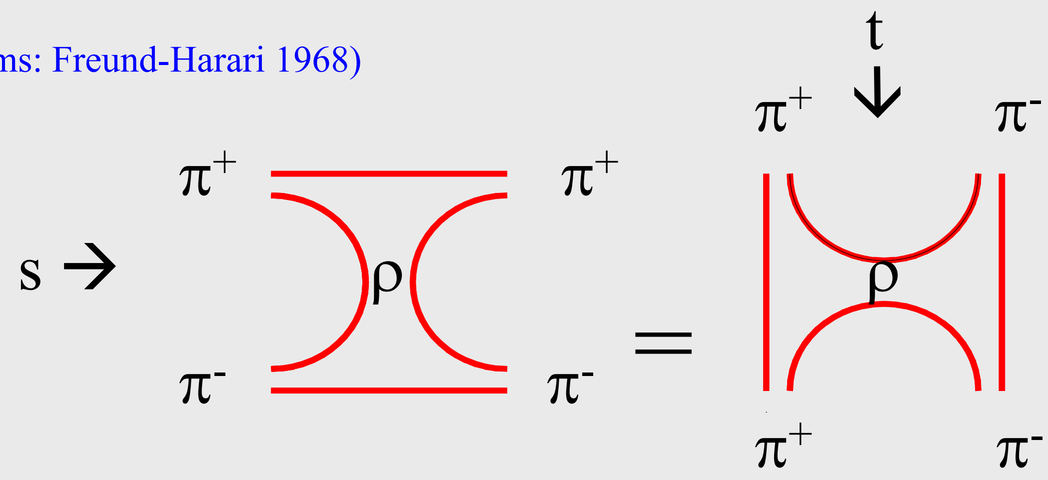
t-channel Regge $(-s)^{\alpha(t)}$ interpolates s-channel resonances

$$\beta(t)(-\alpha' s)^{\alpha(t)} \simeq \sum_r \frac{g_r^2}{s - (M_r - i\Gamma_r)^2}$$

- Search for small parameter: Width/Mass = $\Gamma_\rho / m_\rho \simeq 0.1$

Duality for π π scattering

(Duality diagrams: Freund-Harari 1968)



•Crossing symmetry, A(s,t) = A(t,s), and Dolen-Horn-Schmid duality suggested:

An symmetric generalization in the zero width approximation!

(Veneziano Model!)

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]}$$

Two Point Function

$$A(s,t) = g^2 \frac{\Gamma[-\alpha_\rho(t)]\Gamma[-\alpha_\rho(s)]}{\Gamma[-\alpha_\rho(s) - \alpha_\rho(t)]} = g^2 \int_0^1 dx\, x^{-1-\alpha(s)}(1-x)^{-1-\alpha(t)}$$

Duality:

$$= -g^2 \sum_{r=0}^{\infty} \frac{(\alpha(t)+1)(\alpha(t)+2)\cdots(\alpha(t)+r)}{r!} \int_0^1 dx\, x^{-1-\alpha(s)+r}$$

$$= -\sum_{r=0}^{\infty} \frac{P_r(-\alpha' t)}{\alpha' s - r} \sim \Gamma(-\alpha(t))(-\alpha' s)^{\alpha(t)}$$

The poles are approximated by a smooth power if you let s>> -t and stay a little away from the real axis!

N point Function

with the 2 point function rewritten as

$$\int_{x_1}^{x_3} \frac{dx_2}{(x_4 - x_3)(x_4 - x_1)(x_3 - x_1)} \prod_{1\le i<j\le 4} (x_j - x_i)^{2\alpha' p_j p_i}$$

with $x_1 = 0$, $x_3 = 1$, $x_4 = \infty^{\dagger}$

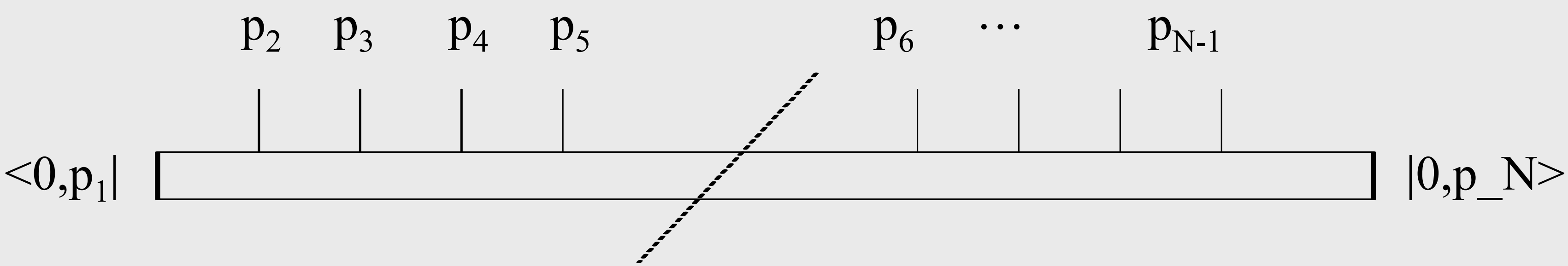
The obvious generalization to the open string tachyonic N point function:

$$A_N(p_1, \cdots, p_N) = g^{N-2} \int \frac{dx_2 dx_3 \cdots dx_{N-2}}{(x_N - x_{N-1})(x_N - x_1)(x_{N-1} - x_1)} \prod_{1\le i<j\le N} (x_j - x_i)^{2\alpha' p_j p_i}$$

where the integrations region is restricted to $x_1 \le x_2 \le x_3 \le \cdots \le x_N$

(† There is no infinity, if the exponent of x_4 vanishes ($-2 + 2\alpha' p_4 (p_1 + p_3 + p_4) = -2 + 2\alpha' p_4^2 = 0$) but there is a tachyon for the open string. Actually this can be avoided here while maintaining real Mobius invariance, $x \to (a x + b)/(c x + d)$ or SL(2,R).)

Old Covariant Quantization (OCQ)



We need to factorized the expression on intermediate poles.The best way is to introduce operators acting on the single "tachyon" plane wave state: $|0,p>$

$$[\hat{q}^\mu, \hat{p}^\nu] = i\eta^{\mu\nu} \qquad \hat{p}^\mu|0,p> = p^\mu|0,p>$$

$$[a_n^\mu, a_m^{\nu\dagger}] = \eta^{\mu\nu}\delta_{n,m} \qquad a_n^\mu|0,p> = 0$$

and a "field":

$$X^\mu(\tau) = \hat{q}^\mu + i\hat{p}^\mu\tau + \sum_n \frac{1}{\sqrt{n}}(a_n^{\mu\dagger}\exp[-\tau] + a_n^\mu\exp[\tau])$$

with $x \equiv \exp[-\tau]$

Finding the Spectrum

A short algebraic exercise* for the student will show that the integrand of the N point function takes the form (setting α' = ½)

$$< 0, p_1|V(x_2, p_2)V(x_3, p_3) \cdots V(x_{N-1}, p_{N-1})|0, p_N >= \prod_{1 \leq i < j \leq N} (x_j - x_i)^{p_j p_i}$$

where†

$$V(x,p) =: \exp[ipX] := \exp[ip\hat{q}] \exp[p\hat{p} \log(x)] \exp[ip \sum_n \frac{a^\dagger x^n}{\sqrt{n}}] \exp[ip \sum_n \frac{a_n x^{-n}}{\sqrt{n}}]$$

(*Recall if [A,B] is a c number: $\exp[A] \exp[B] = \exp[B] \exp[A] \exp[[A,B]]$)

- normal ordering gives terms like: $\exp[-\sum_n \frac{p_i p_j}{n} (\frac{x_i}{x_j})^n] = (1 - \frac{x_i}{x_j})^{p_i p_j}$

(† In more stringy language, $X^\mu(\sigma,\tau)$ is the world sheet co-ordinate evaluated at the end σ = 0 or π!)

OCQ result: Negative norm states decouple for d <= 26 and α_0 <= 1 Brower 1970

String Interpretation

The field of the form

$$X^\mu(z) = \hat{q}^\mu + \hat{p}^\mu log(z) + \sum_n \frac{1}{\sqrt{n}}(a_n^\mu z^n + b_n^\mu z^{-n})$$

are general (holomorphic/right moving) solutions to (Euclidean) wave eqn

$$\partial_t^2 X^\mu + \partial_\sigma^2 X^\mu = 0$$

with z = exp[τ + i sigma]:

- Solutions to open use holomorphic and anti-holomorphic solutions to satisfy B.C.
- The close string amplitude integrates over the entire complex plane z
- Constraints to set the world sheet energy momentum tensor are assumed and implemented in a host of cleaver ways to eliminate negative norm solutions.

$$\partial_\sigma X^\mu \partial_\tau X_\mu = \partial_\tau X^\mu \partial_\tau X_\mu - \partial_\sigma X^\mu \partial_\sigma X_\mu = 0$$

String Action

Nambu-Gotto action: Area of world sheet

$$S = -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-det(h)}$$

$$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

or

$$S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{(\partial_\sigma X^\mu\partial_\sigma X_\mu)(\partial_\tau X^\nu\partial_\tau X_\nu) - (\partial_\sigma X^\mu\partial_\tau X_\mu)^2}$$

Polyakov Action: lagrange multiplier metric $\gamma^{\alpha\beta}$

$$S = -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-det(\gamma)}[\gamma^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu]$$

Three methods to handle the deffeomorphism constraints:

- OCQ ignore constraints impose the weakly on the Hilbert space.
- Explicitly solve the constraints (in lightcone gauge).
- Introduce “Fadeev-Popov” ghosts/ BRST to corrector measure.

After QCD *Large N*

- QCD has no coupling constant!
  - Possible small parameters:

$$1/N,\quad n_f/N,\quad m_q/\Lambda_{qcd},\quad \Lambda/M_Q,\ p/\Lambda_{qcd},\quad \Lambda_{qcd}/p,\quad \theta,\ \cdots$$

- 1/N is the most democratic one (flavor and momentum independent).
  - Indeed for mesons (and glueballs) this is precisely the narrow width approximation: $\Gamma_\rho/m_\rho \simeq 0$
  - Baryons/instantons/etc are non-perturbative effects in the 1/N expansion.

- The $N \to \infty$ "defines" the QCD string.

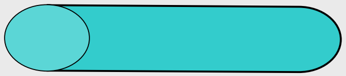
Traditional 1/N Topics

- Topology: ‘tHooft expansion (at fixed $\lambda = g^2 N$)
  - Weak coupling $\lambda \rightarrow 0$
  - Strong Coupling $\beta = 1/\lambda \rightarrow \infty$
- Glueballs and Diffraction in $O(1/N^2)$
- Mesons, Regge and OZI
  - Coupling Constants: g_s, F_π, ...
- Chiral Lagrangian: The η and θ et al.
- Finite T Transition:
  - $\Delta H=1$, $O(1) \rightarrow O(N^2)$
- Non perturbative Effects:
  - Baryons (e.g. Skyrmions), Instantons, Domain Walls ....

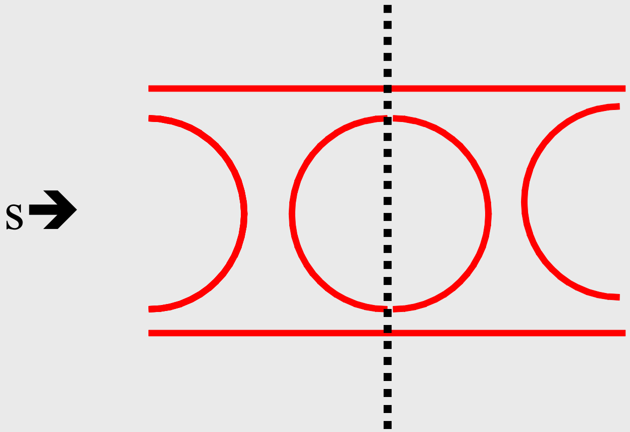
λ =1/N Expansion defines the QCD string

$$A(M\text{-}mesons) = \Sigma_{H,B} \quad (1/N)^{2H-2+B} \ (1/N)^{M/2} \ F_N(\Lambda_{qcd}, M, ..)$$

•Weak and Strong expansion(in g^2N) → *topology* of string perturbation series:

- Expectations:
  - Leading diagrams (cylinder) → stable glueballs 
  - One boundary (open string) → stable mesons of arbitrary spin 
  - Open (closed) string coupling is $g_s \sim 1/N$ ($g_o \sim 1/N^{1/2}$)
  - UV (short distances) exhibits asymptotic freedom
  - IR the chiral Lagrangian at small quark mass and confinement.
  - Non-perturbative effects $O(\lambda^{-1})$
    → Baryons (Mass $\simeq N \Lambda_{qcd} \sim 1/\lambda$)
    → Instantons (Action = $8\pi^2/(g^2N) \sim 1/\lambda$?)

First Unitary Corrections: Pomeron



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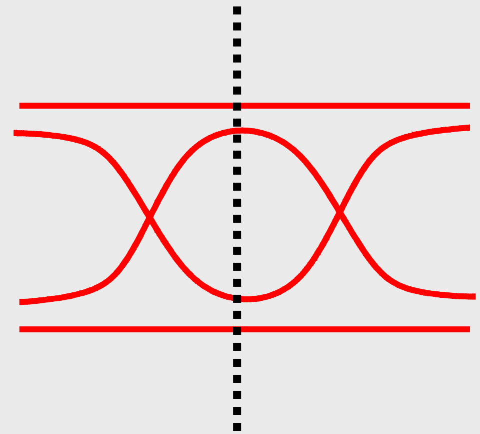
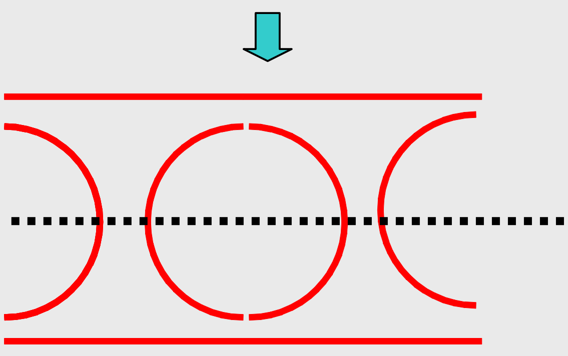


B = 2, M = 4 ➔ $O(1/N^2)$

t channel: Two meson exchange

B=2,M=4 ➔ $O(1/N^2)$

t channel: Single glueball exchange (aka "Pomeron")



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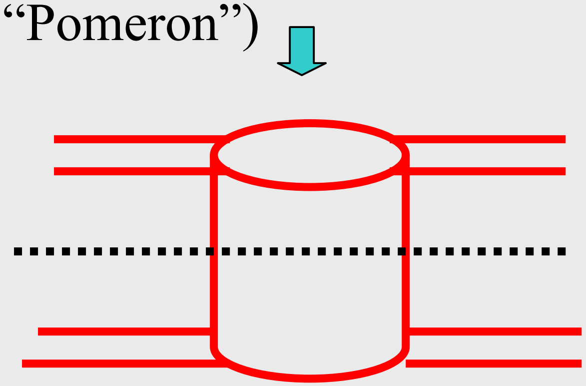


t

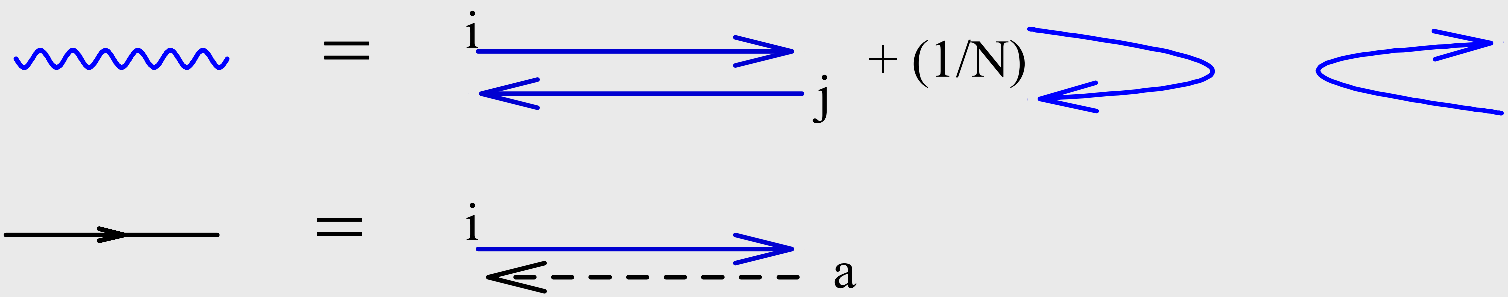
‘tHooft Double Line Expansion

- Two approaches. Weak and Strong coupling

$$S = Tr[(\partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu])^2] + \bar{\Psi}(\gamma_\mu \partial_\mu - igA_\mu)\Psi$$

$A^i_{j,\mu}$

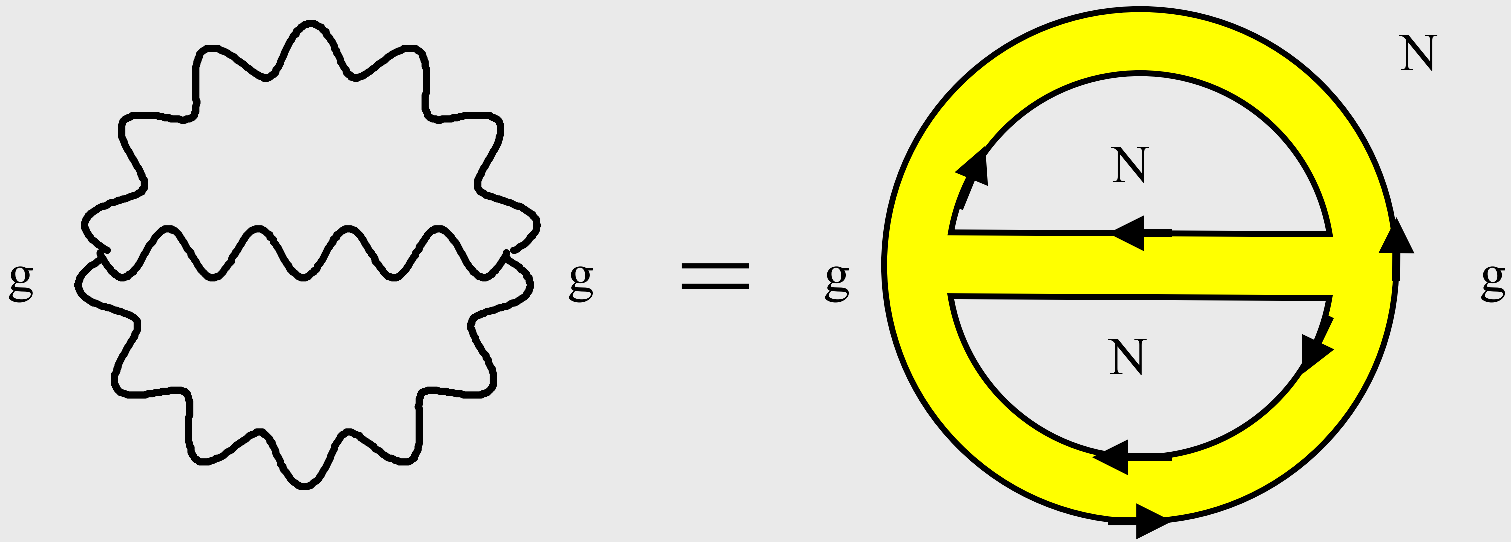
Ψ^i_a



$$Z = \int dA d\bar{\Psi} d\Psi exp[-S]$$

Expand log(Z) in a double series in $\lambda = 1/N$ and $g^2 N$

Weak Coupling Graph



$g^2 N^3 = (g^2 N)\ N^2 = O(N^2)$ for a "sphere"

Thus a 3 vertex gives a factor of $g \sim 1/N^{1/2}$, a face Tr() $\sim$ N.

Comment: All gluons are distinguishable: No n! explosion of graphs; converges in a finite sized box.

Chiral Lagrangian

- Infinite N → $U(n_f)$ Chirality
- Witten-Veneziano: $m^2_\eta = O(1/N)$ and $F^2_\pi = O(N)$
- $m^2_\eta F^2_\pi = 2 n_f \; d^2e_{vac}(\theta)/d\theta^2_{quenched}$ with $e_{vac}(\theta) = N^2 f(\theta/N)$
- $<FF^*> = m_q <\Psi \Psi> \sin(\theta/n_f)$ full QCD
- $<FF^*> = \Lambda^4_{qcd} \sin(\theta/N)$ quenched QCD

Topology of 1/N in *Weak Coupling*

$$\begin{aligned} \text{Edges/Propagators} &= O(1) \\ \text{Vertices} &= O(N^{-V_4-\frac{1}{2}V_3}) \\ \text{Color Faces} &= O(N^F) \\ \text{Quark Boundaries} &= O(n_f^B) \end{aligned}$$

•But $-E + V = -(4/2)V_4 - (3/2)V_3 + (V_4 + V_3) = -V_4 - V_3/2$
•Therefore so the weight ($N^{F-V4-V3/2} = N^{F-E+V} = N^{\chi}$) of the graph is determined by Euler theorem $N^{\chi} = N^{2-2H-B}$

Sphere = $O(N^2)$, Disk = $O(N)$, Torus = $O(1)$, Pretzel = $O(1/N)$ ....

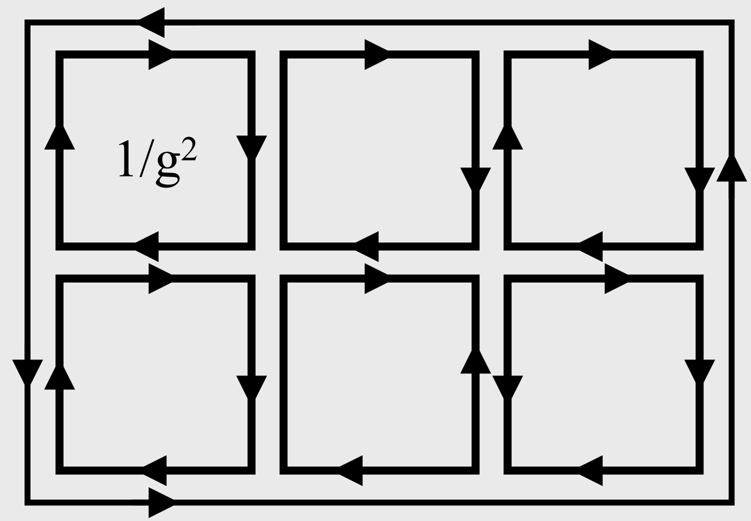
(Rescaling $A \to A/g$ makes it easier --- like strong coupling)

Strong Coupling Expansion

$$S = \frac{1}{g^2} \sum_P Tr[2 - U_P - U_P^\dagger] \qquad U_\mu = \exp[iaA_\mu]$$

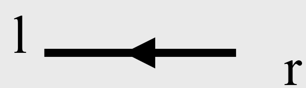
Faces → $U_P \simeq \exp[ia^2 g F_{\mu\nu}]$

Wilson loop



Graphical Notation:

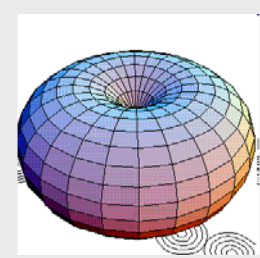
$$U_{\mu\,r}^{\;l} =$$



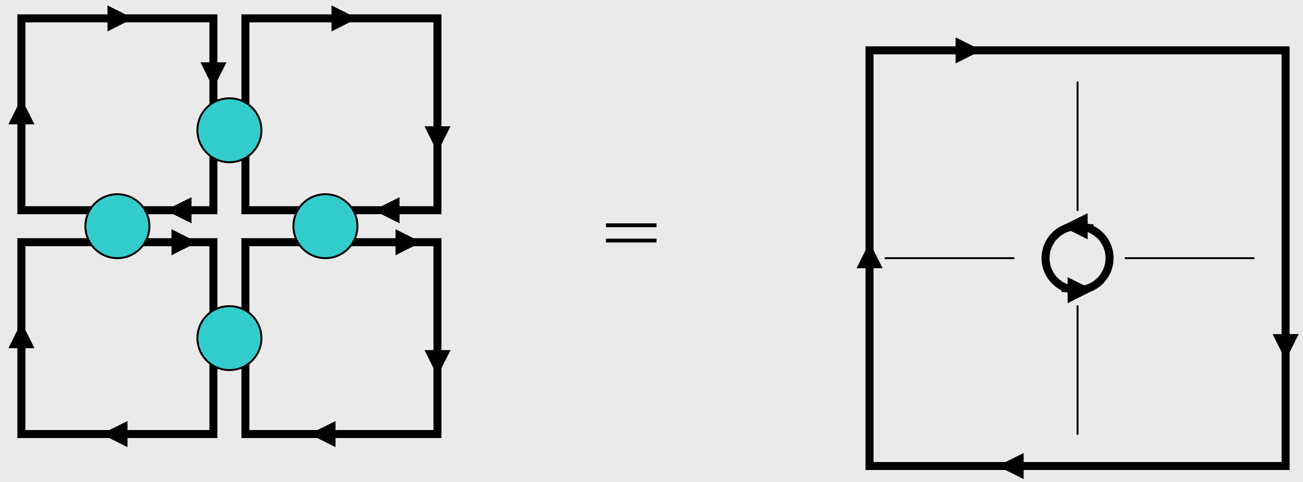
- U corresponds to a SINGLE line.
- Integrate out the links to get

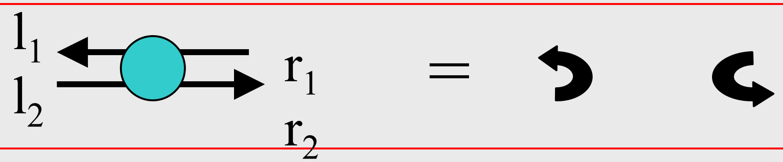
Topology in the Confining Phase
(at strong coupling on the Lattice)

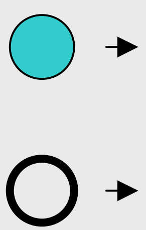
Vacuum graph:



Topology of 1/N in *Strong Coupling*







$$\text{Edges} = \int dU\; U^{l_1}_{r_1} U^{\dagger r_2}_{l_2} = \frac{1}{N}\delta^{l_1}_{l_2}\delta^{r_2}_{r_1} \text{ or } O(N^{-E})$$

$$\text{Vertices} = \delta^r_r = N \text{ or } O(N^V)$$

$1/g^2 \rightarrow$

$$\text{Faces} = O(N^F)$$

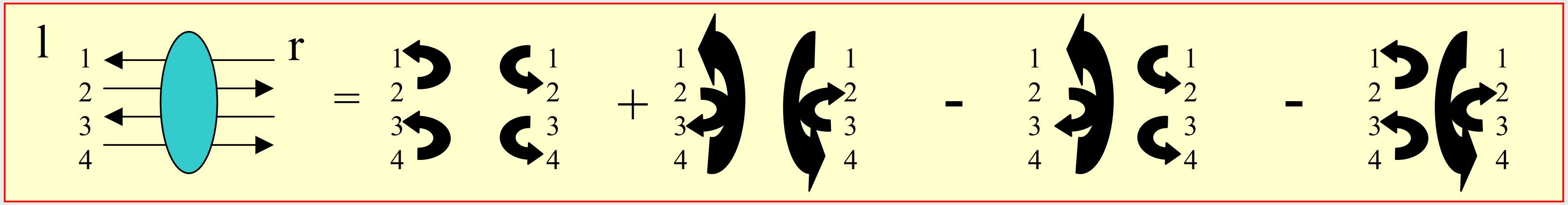
so $\chi = F - E + V$ and therefore as before: $N^\chi = N^{2-2H-B}$

Self-Intersections of Surfaces

- Intersections integral by recursion (Gauge Inv rules like MM Loop equ)
  - (Left/Right Haar invariance ←→ Gauge Invariance) :

$$\int dU\; U^{l_1}_{r_1} U^{\dagger r_2}_{l_2} U^{l_3}_{r_3} U^{\dagger r_4}_{l_4} = \frac{1}{N(N+1)}[\delta^{l_1}_{l_2}\delta^{r_2}_{r_1}\delta^{l_3}_{l_4}\delta^{r_4}_{r_3}$$

$$+(l\&r, 1 \leftrightarrow 3) - \frac{1}{N}[(l, 1 \leftrightarrow 3) + (r, 1 \leftrightarrow 3)]]$$

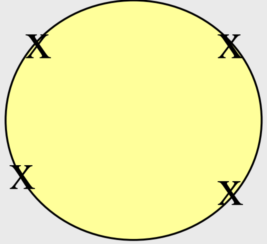


Can this non-local effect be eliminate on a new lattice or in 5-d?

•Baryon Vertex (Non-perturbative string effect):

$$\int dU\; U^{l_1}_{r_1} U^{l_2}_{r_2} \cdots U^{l_N}_{r_N} = \frac{1}{N!}\epsilon^{l_1 l_2 \cdots l_N}\epsilon_{r_1 r_2 \cdots r_N}$$

Coupling Constants et al



B=1, M = 4

- Ext Mesons: $M(x) = N^{-1/2}\ \bar{\Psi}^i\, \Psi_i$ — *Sewing:*

  ($\Delta M = -2$, $\Delta B = 1$)
- Meson 3 point vertex: $(g_s)^{1/2} = O(1/N^{1/2})$ — *(B=1,M=3)*
  - aka open string coupling
- Glueball 3 point vertex: $g_s = O(1/N)$ — (B=3)
  - aka closed string coupling
- $F_\pi = \langle\, \bar{\Psi}\, \gamma_5 \Psi\, |\pi\rangle = O(N^{1/2})$, — (B=1,M=1)
- $\langle \Psi\ \Psi \rangle = O(N)$ — (B=1)
- Eta mass shift : $\Delta\, m^2_\eta = O(1/N)$ — Insertion: (Δ B=1)

  (or H=0, B=2, M = 2)

Lecture II – *String/Gauge Duality*

(preamble: 2 string sol'n and failures in flat space)

- Maldacena's AdS/CFT conjecture
  - D branes and the strong coupling (gravity) limit
  - Breaking Conformal and SUSY → Confinement
  - AdS/CFT Dictionary: Finite T/Instantons/Quarks/Baryons

- Hadronic Physics in the 5th dimension
  - Stringy Deconfinement --- Fat vs Thin Strings
  - IR Physics --- Glueballs as an AdS Graviton (Lecture III)
  - UV Physics --- Parton counting rules at Wide Angles
  - Regge Limit --- Soft vs Hard (BFKL) Pomeron
  - Wilson Loop --- Stretched String beyond the Luscher the term (Lecture III)
  - Hagedorn Transition (deconfinement?) in pp-wave limit. (Skip)
  - Challenges: Friossart Bound, DIS, Microstructure, etc. (future?)

Dual Pion Amplitude (aka NS string)

Veneziano's original guess is actually a Super String 4 point function consistent with Chiral lagragian at low energies if we take the adjust the rho intercept !
$\alpha_\rho(0) = 1/2$:

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = \frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[1-\alpha_\rho(s)-\alpha_\rho(t)]}$$

$$= (1-\alpha_\rho(s)-\alpha_\rho(t))\frac{\Gamma[1-\alpha_\rho(t)]\Gamma[1-\alpha_\rho(s)]}{\Gamma[2-\alpha_\rho(s)-\alpha_\rho(t)]} \sim \alpha'(s+t)$$

In the soft pion limit we see Adler zeros: $p_1 \to 0$, $s \to m^2_\pi$, $t \to m^2_\pi$

(† Neveu-Schwarz "Quark model of dual pions", 1971)

Nambu-Gotto Action ($T_0 = 1/(2\pi\alpha')$, $c = 1$):

$$T_0\, Area = T_0 \int d\tau d\sigma \sqrt{(\partial_\sigma X^\mu \partial_\sigma X_\mu)(\partial_\tau X^\nu \partial_\tau X_\nu) - (\partial_\sigma X^\mu \partial_\tau X_\mu)^2}$$

Pick $\tau = X^0$ = "time" t and \sigma so EOM

$$\partial_t^2 X^\mu - \partial_\sigma^2 X^\mu = 0$$

with constraints

$$\partial_\sigma X^\mu \partial_t X_\mu = 0 \qquad \partial_t X^k \partial_t X_k + \partial_\sigma X^k \partial_\sigma X_k = 1$$

and boundary conditions

Dirichlet B. C. $\partial_t X^k(t)|_{\sigma = 0,\sigma 0} = 0$ (stretched to fixed quark/ D branes)

Free: Neumann B.C. $\partial_\sigma X^k(t)|_{\sigma,\sigma 0} = 0$ (free ends move at speed of light).

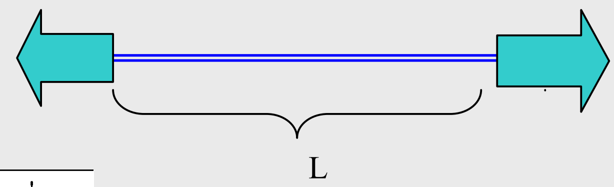
Can scale σ so that energy density $dE = T_0\, d\sigma$!

Two solutions: Stretched & Rotating String

Classical Stretched string:

$X^3 = \sigma$ for $\sigma \in [0,L]$ and $E = T_0\, L$

$$dE = T_0 d\sigma = T_0 ds$$

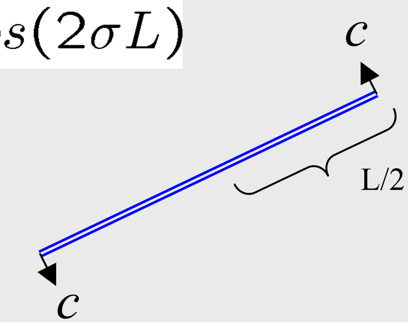


Exact Quantum sol'n: $E = T_0 L \sqrt{1 - \frac{\pi(D-2)}{12 T_0 L^2} + \frac{2\pi a_n^\dagger a_n}{T_0 L^2}}$

Rotating string: $\sigma \in [0,\pi\, L/2]$

$X^1 + i\, X^2 = (L/2)\ \cos(\, 2\sigma/L)\ \exp[i2t/L]$

$$dE = T_0 d\sigma = T_0 ds/\sqrt{1 - v_\perp^2} \quad \text{with} \quad v_\perp(\sigma) = cos(2\sigma L)$$



$J = \alpha'\ E^2$ with $E = \pi\ L\ T_0/2$
BUT angular velocity $\omega = 2/L$ so ends go at speed of light!

Exact Quantum state: $(a_1^{1\dagger} + i a_1^{2\dagger})^J |0, p\rangle$

Failures of the Super String for QCD

Careful quantization of the Super String in flat space leads to

(i) ZERO MASS STATE (gauge/graviton)

(ii) SUPER SYMMETRY

(iii) EXTRA DIMENSION 4+6 = 10

(iv) NO HARD PROCESSES! (totally wrong dynamics)

Stringy Rutherford Experiment

At WIDE ANGLE: s,-t,-u >> 1/α'

$$A_{closed}(s,t) \rightarrow \exp[-\frac{1}{2}\alpha'(s\ln s + t\ln t + u\ln u)]$$

$$A_{closed}(s,t) \sim sin[\pi(,\frac{1}{4}\alpha' t + \alpha_0)] \times A_{open}(s,t,\frac{1}{4}\alpha')A_{open}(t,u,\frac{1}{4}\alpha')$$

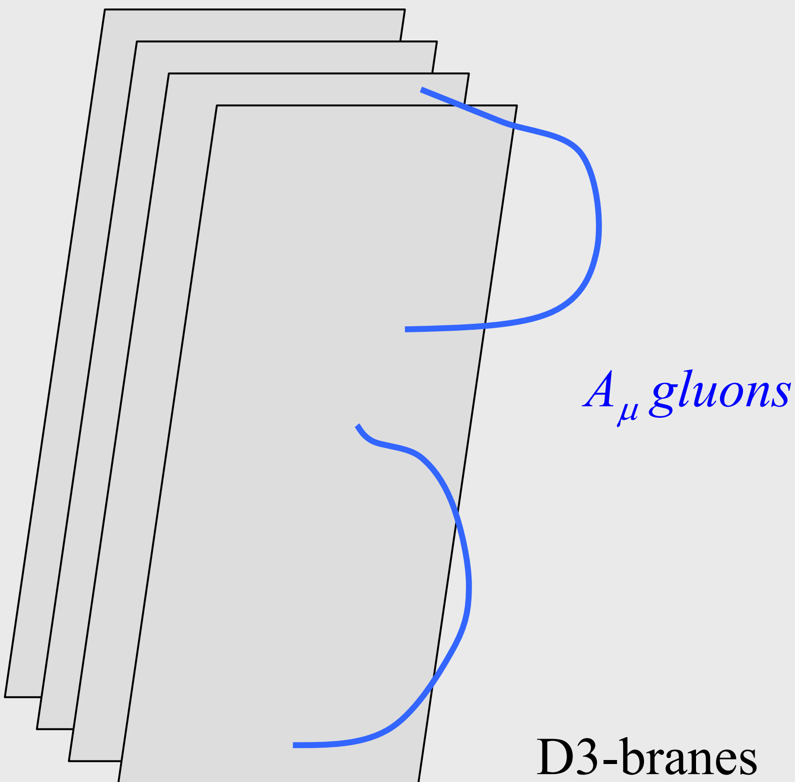
where $A_{open}(s,t,\alpha')) \simeq \dfrac{\Gamma(-\alpha' s - \alpha_0)\Gamma(-\alpha' t - \alpha_0)}{\Gamma(-\alpha' s - \alpha_0 - \alpha' t - \alpha_0)}$

D brane Picture: Two Descriptions

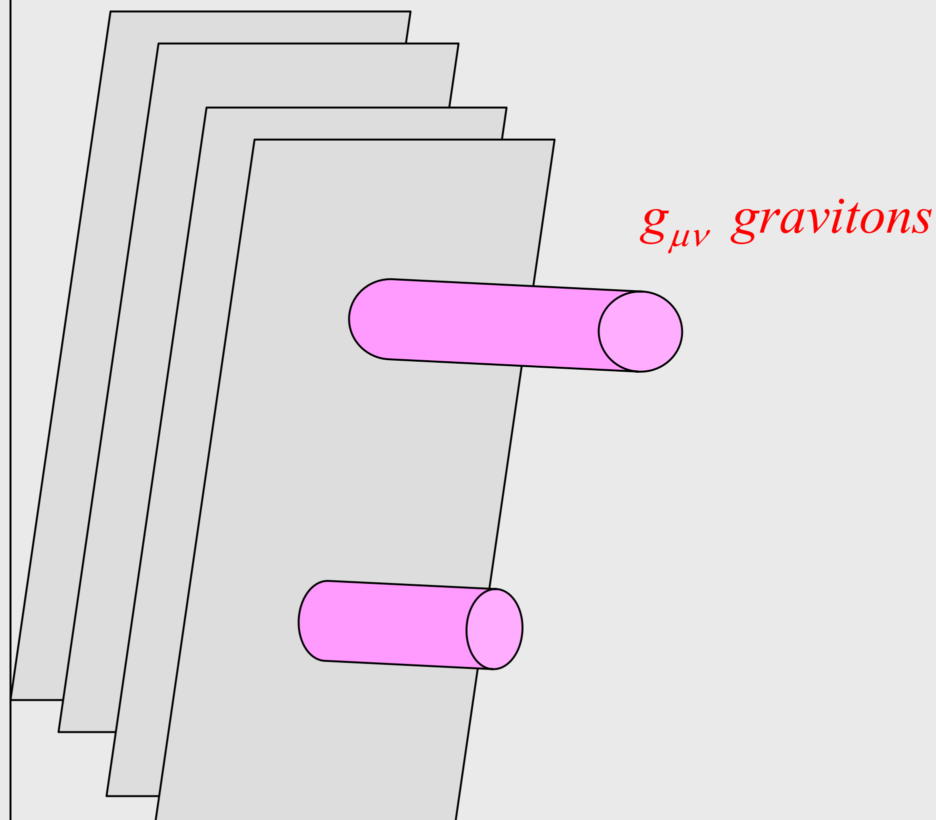
Open stings are Gluons <u>dual</u> to closed string Gravity.

- 3-branes (1+3 world volume) -- Source for open strings and closed strings:

Dynamics of N D3 branes at low energies is (Super) SU(N) YM.



Their mass curves the space (near horizon) into AdS^5 and emits closed string (graviton)



Maldacena's String Counter Revolution

Exact string/gauge dualities are at last known

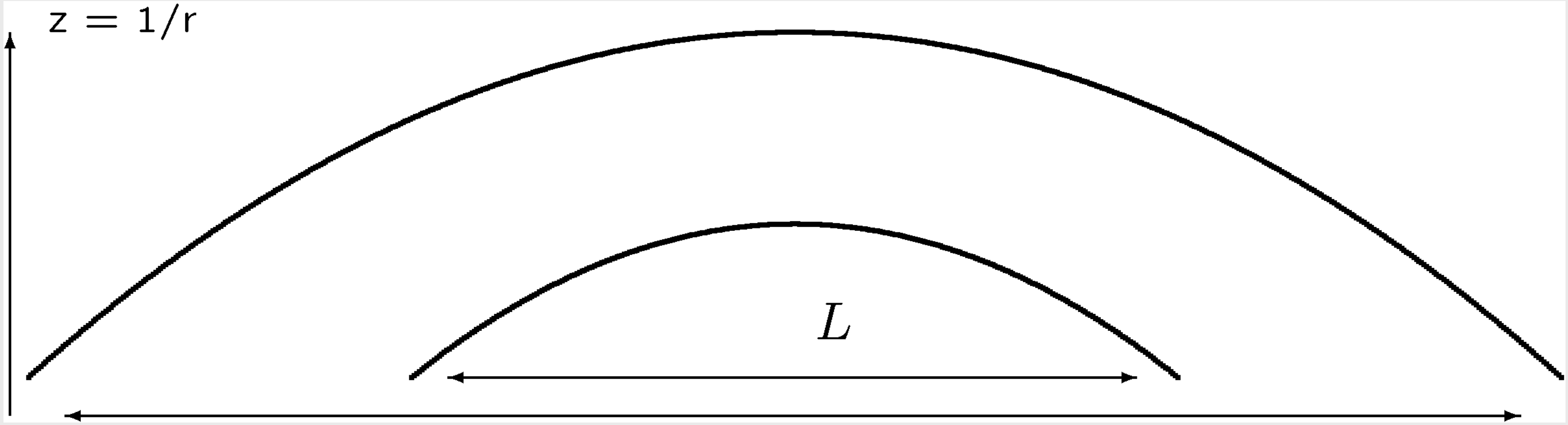
Weak/Strong String(gravity) d+1 ←→ Strong/Weak Yang Mills in d

Simplest example;

IIB strings (gravity) on $AdS^5 x S^5$ ←→ $\mathcal{N} = 4$ SU(N) SYM in d=4

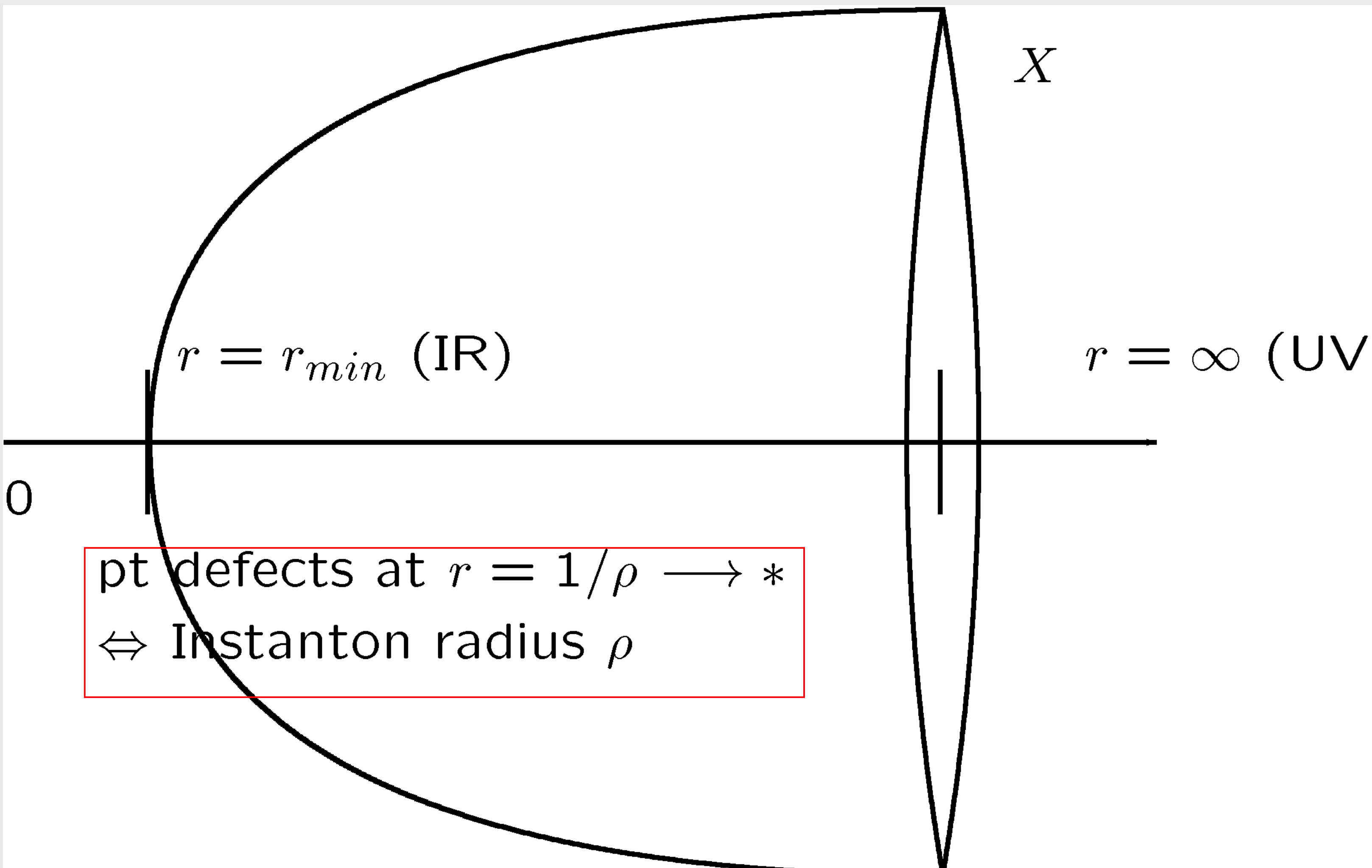
$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}\left(dr^2 + r^2 d^2\Omega_5\right)$$

Wilson loop is found by minimizing the surface area. Get $Q\bar{Q}$ potential V ~ 1/L

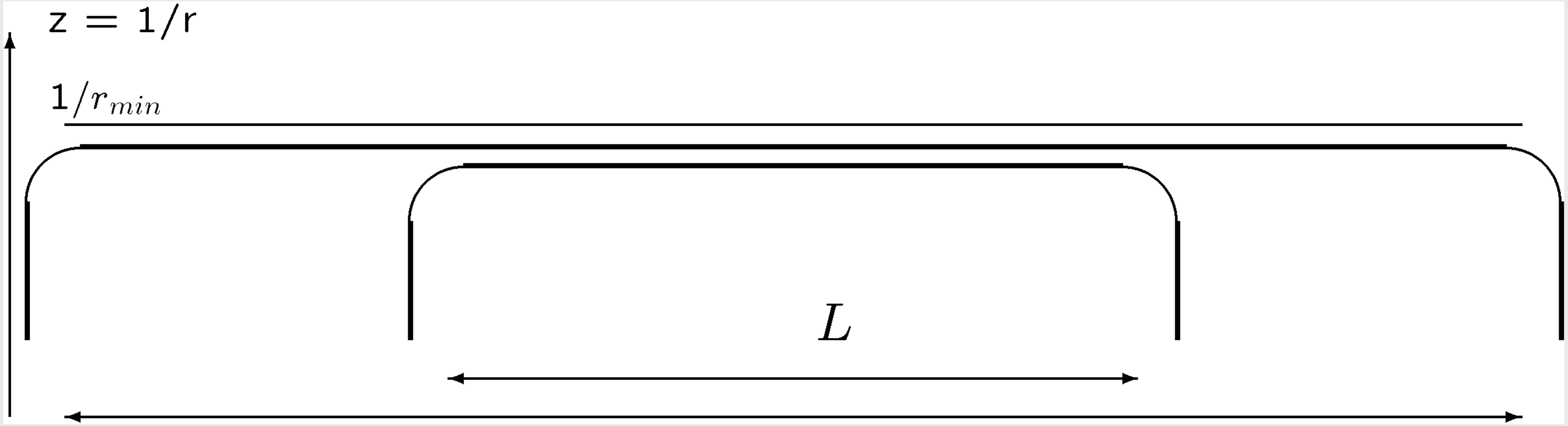


Confinement in the AdS String

IR ``cut-off'' at r > r_{min} in the AdS to break conformal & SUSY.

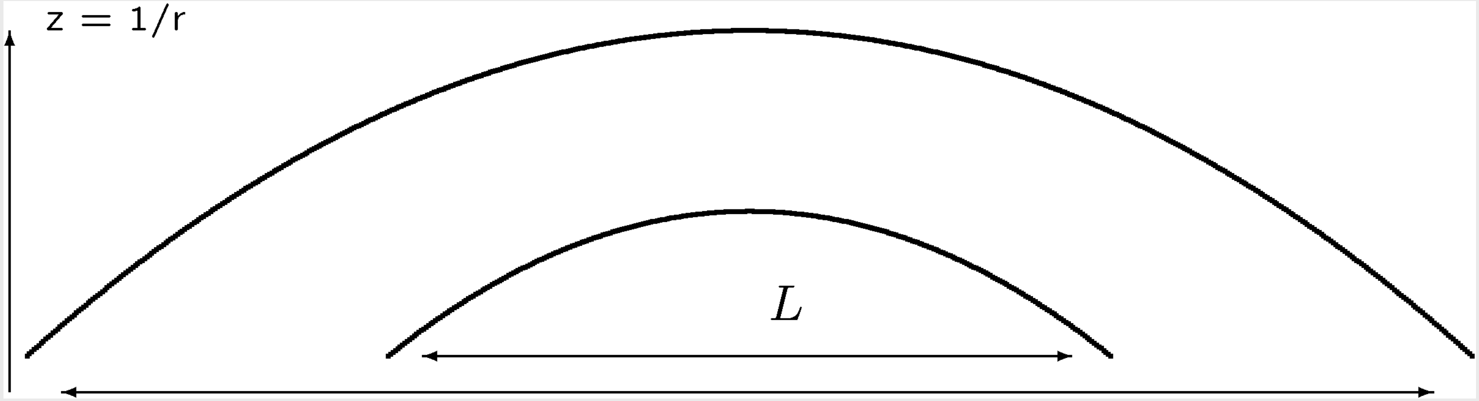


Non-zero QCD string tension and Mass Gap



with $Q\bar{Q}$ potential: $V(L) \sim \alpha'_{qcd} L + c/L + \cdots$,

REPLACES



with $Q\bar{Q}$ potential: $V(L) \sim \text{const}/L$

AdS^{d+2} Black Hole Metric

Two Simple Examples have been studied in detail:

- $AdS^5 \times S^5$ Black Hole background for 10-d IIB string theory
- $AdS^7 \times S^4$ Black Hole background for 11-d M-theory.

AdS Black Hole metrics,

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2[1-(r_{min}/r)^d]}\, dr^2 + \frac{r^2}{R^2}[1-(r_{min}/r)^d]\, d\tau^2 + ds_X^2$$

with antiperiodic Fermions on a compact circle in τ and ds^2_X represents the additional 5 compact dimensions.

$\mu = 1,2,3$ ($\mu = 1,2,3,4$) for 10-d (11-d) string(M-theory).

Note: The AdS^5 radius: $R^4 \simeq g^2 N \alpha'^2$ so strong 'tHooft is weak gravity

QCD Rutherford Experiment

At WIDE ANGLES QCD exhibits power law behavior:

$$A_{qcd}(s,t) \sim \left(\frac{1}{\sqrt{\alpha'_{qcd} s}}\right)^{n-4}$$

where $n = \sum_i n_i$ is the number of ``partons" in external lines.

The OPE gives

$$n = \sum_i \tau_i = \sum_i (d_i - s_i)$$

in terms of the lowest twist τ_i.

Actually QCD is only conformal up to small asymptotic freedom logs.

Wide Angle Scattering

The 2-to-m glueball scattering amplitude $T(p_1,p_2,\cdots, p_{m+2})$ for plane wave glueball:

$$\phi_j(r, X)\,\exp[i x_j^\mu p_j^\mu]$$

scatter via the string(M-theory) amplitude: $A(p_i, r_i, X_i)$ in the 10-d (or 11-d) bulk space (x,r,Y):

$$T(p_i) = \prod_j \int dr_j dY_j \sqrt{-g_i}\;\; \phi_j(r_j, Y_j) \times A(p_1, r_1, Y_1, p_2, r_2, Y_2 \cdots)$$

We now discuss two different approaches to the QCD string that both give the correct parton scaling formula.

- $AdS^5 \times X$ with IR cut-off on $r > r_{min}$ or 10-d IIB string theory
- $AdS^7 \times S^4$\$ Black Hole with horizon $r = r_{min}$ or 11-d M-theory.

This is a check on the underlining universality of Maldacena's duality conjecture.

10-d String theory Approach

Due to the Red Shift in the Warped Co-ordinate , $\Delta s = (R/r)\,\Delta x$,a plane wave glueball, exp[i x p], scatters with a local proper momentum,

$$\widehat{p}_s(r) = \frac{R}{r} p \,,$$

String is UV shifted in the YM's IR. (This is the so called UV/IR connection.)
THUS wide angle scattering IS exponentially suppressed in the region $r \in [r_{min}, r_{scatt}]$

$$\sqrt{\alpha'_s}\, p_s = l_s R\, p/r_{scatt} > 1 \,.$$

HOWEVER there is a small remaining amplitude at large r that that gives the correct conformal scaling of the naive parton model!

$$\phi_i(r) \sim (r_{scatt}/r_{min})^{-\Delta_4} \simeq \Big(\frac{\sqrt{\alpha'_s}p}{\sqrt{r^2_{min}/R^2}}\Big)^{-\Delta_4} \sim (\sqrt{\alpha'_{qcd}}\, p)^{-\Delta_4}$$

E.g for a scalar glueball $\phi \sim r^{-4}$ corresponding to $n_i = 4$ for the YM operator, $Tr[F^2]$, in exact agreement with the parton result.

11-d M theory Approach

Here conformal scaling give $\phi_i(r) \sim (r/r_{min})^{-\Delta_6}$

(e.g now $\Delta_6 = 6$ instead of $\Delta_4 = 4$.)

How can this also agree with the Parton results?

Ans: This is a theory of Membranes in 11-d. When they wrap the 11th coordinate the result is 10-d (IIA) string theory. The local radius of the 11th dimension, $\hat{R}_{11}(r) = rR_{11}/R$,determines local (warped) string parameters

$$\hat{l}_s^2(r) = l_p^3/\hat{R}_{11}(r) \quad \text{and} \quad \hat{g}_s^2(r) = \hat{R}_{11}^3(r)/l_p^3$$

These additional warping factors precisely reproduce the parton results!

$$\left(\frac{r_{scatt}}{r_{min}}\right)^{-\Delta_6^{(i)}} \sim \left(\frac{\sqrt{\alpha'_s}\, p}{\sqrt{r_{min}^3/R^3}}\right)^{-\frac{2}{3}\Delta_6^{(i)}} \sim \left(\sqrt{\alpha'_{qcd}}\, p\right)^{-\frac{2}{3}\Delta_6^{(i)}}$$

Comments

(1) The last expression for M-theory requires the scaling relation:

$$\alpha'_{qcd} \sim \alpha'_s \frac{R^3}{r^3_{min}}$$

The 3rd power is a consequence of minimal (3-d) membrane world volumes in 11-d versus a 2nd power for minimal (2-d) surface in strings in 10-d.

(2) With the appropriate form of the QCD string tension at strong coupling,

$$1/\alpha'_{qcd} \sim (g^2_{YM} N)^{1/2} \Lambda^2_{qcd} \quad \text{for } AdS^5 \text{ IIB strings}$$

$$1/\alpha'_{qcd} \sim g^2_{YM}\, N \Lambda^2_{qcd} \quad \text{for } AdS^7 \text{ M-theory}$$

we get the general results.

Summary on Hard Scattering

(1) AdS^5 Hard Scattering (Polchinski-Strassler):

$$\Delta\sigma_{2\to m} \simeq \frac{1}{s}\, f\left(\frac{p_i \cdot p_j}{s}\right)\frac{(\sqrt{g^2 N})^m}{N^{2m}} \prod_i \left(\frac{\sqrt{g^2 N}\Lambda_{qcd}^2}{s}\right)^{n_i - 1}$$

WHY is it same QCD perturbative result with $g^2N \to (g^2N)$?

---

(2) AdS^7 Hard Scattering (Brower-Tan):

$$\Delta\sigma_{2\to m} \simeq \frac{1}{s}\, f\left(\frac{p_i \cdot p_j}{s}\right)\frac{1}{N^{2m}} \prod_i \left(\frac{1}{\alpha'_{qcd} s}\right)^{n_i - 1}$$

WHY does this only depend on the string tension?

---

(3) Compared with lowest order perturbative results:

$$\Delta\sigma_{2\to m} \simeq \frac{1}{s}\, f\left(\frac{p_i \cdot p_j}{s}\right)\frac{(g^2 N)^m}{N^{2m}} \prod_i \left(\frac{g^2 N\Lambda_{qcd}^2}{s}\right)^{n_i - 1}$$

Soft vs Hard Regge Scattering

Similar arguments can be applied to the Regge limit: s >> -t

$$T(s,t) = \int_{r_{min}}^{\infty} dr\; \beta(tR^3/r^3)(\alpha' s)^{\alpha_s(0)+\alpha'_s tR^3/r^3}.$$

Dominant scattering at large r, gives a BFKL-like Pomeron with almost flat ``trajectory'' (actually a cut in the j-plane)

$$T(s,t) \sim (\alpha' s)^{\alpha_s(0)}/(\log s)^{\gamma+1}$$

The IR region, $r \simeq r_{min,}$ gives soft Regge pole with slope $\alpha'_{qcd} \sim \alpha' R^3/r^3_{min}$

$$T(s,t) \sim \exp[+\alpha' t \log(s)](\alpha'_{qcd} s)^{\alpha_s(0)}$$

The ``shrinkage'' of the Regge peak is caused the soft stringy ``form factor'' in impact parameter:

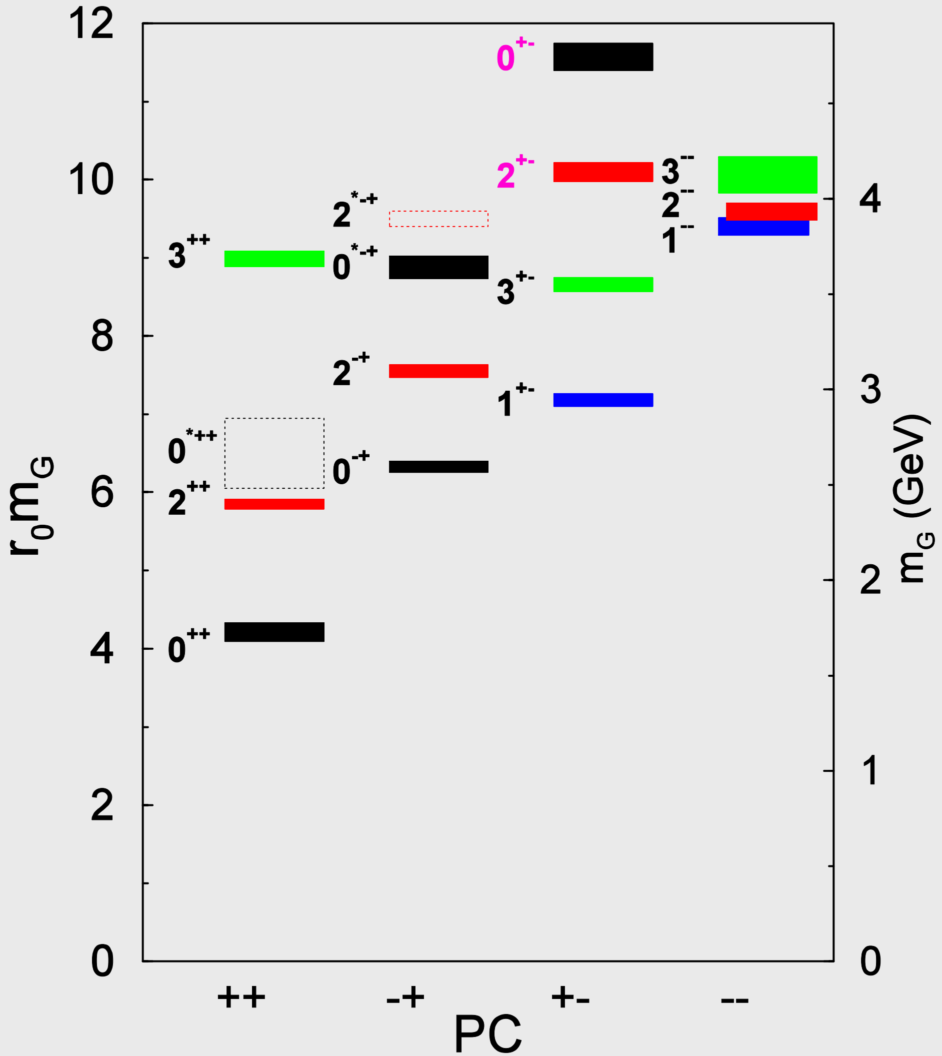
$$< X_\perp^2 > \simeq \alpha'_{qcd} \log(s) \sim \alpha'_s \log(\text{No. of d.o.f})$$

Lecture III – *String vs Lattice Spectra*

- SGRA calculation of Glueball Spectrum
  - d=4 QCD M theory (type IIA) spectrum at strong coupling
  - Comparison with Lattice data and Bag Model

- Nambu action calculation of Stretched String
  - Static quark potential and Luscher term
  - Transverse and "longitudinal" modes
  - Comparison with lattice data

Lattice QCD_4 Glueball Spectrum



Moringstar and Peardon

Gravity vs Y.M on the Brane

11-d Super Gravity:

$$S = -\frac{1}{2\kappa_{11}} \int d^{11}x \sqrt{-g_{11}} \, (R_{11} - |F_4|^2) + \frac{1}{12\kappa_{11}} \int A_3 \wedge F_4 \wedge F_4 + \text{fermions}$$

Born-Infeld dynamics on 3 brane and 4 brane

$$S = \int d^4x \, \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_0 F \wedge F + C_2 \wedge F + C_4) \, ,$$

$$S = \int d^5x \, \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_1 F \wedge F + C_3 \wedge F + C_5)$$

Viewing the gravity fields as coupling constants to the gauge fields we can identify quantum number for them.

IIA Classification of QCD_4

States from 11-d G_{MN}				States from 11-d A_{MNL}		
$G_{\mu\nu}$	$G_{\mu,11}$	$G_{11,11}$	m_0 (Eq.)	$A_{\mu\nu,11}$	$A_{\mu\nu\rho}$	m_0(Eq.)
G_{ij} 2^{++}	C_i $1^{++}{}_{(-)}$	ϕ 0^{++}	4.7007 (T_4)	B_{ij} 1^{+-}	C_{123} $0^{+-}{}_{(-)}$	7.3059(N_4)
$G_{i\tau}$ $1^{-+}{}_{(-)}$	C_τ 0^{-+}		5.6555 (V_4)	$B_{i\tau}$ $1^{--}{}_{(-)}$	$C_{ij\tau}$ 1^{--}	9.1129(M_4)
$G_{\tau\tau}$ 0^{++}			2.7034(S_4)		$G^{\alpha}{}_{\alpha}$ 0^{++}	10.7239(L_4)

Subscripts to J^{PC} refer to $P_\tau = -1$ states

IIA Glueball Wave Equations

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}T_4(r) - (m^2r^3)T_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}V_4(r) - (m^2r^3 - \frac{9}{r(r^6 - 1)})V_4(r) = 0$$

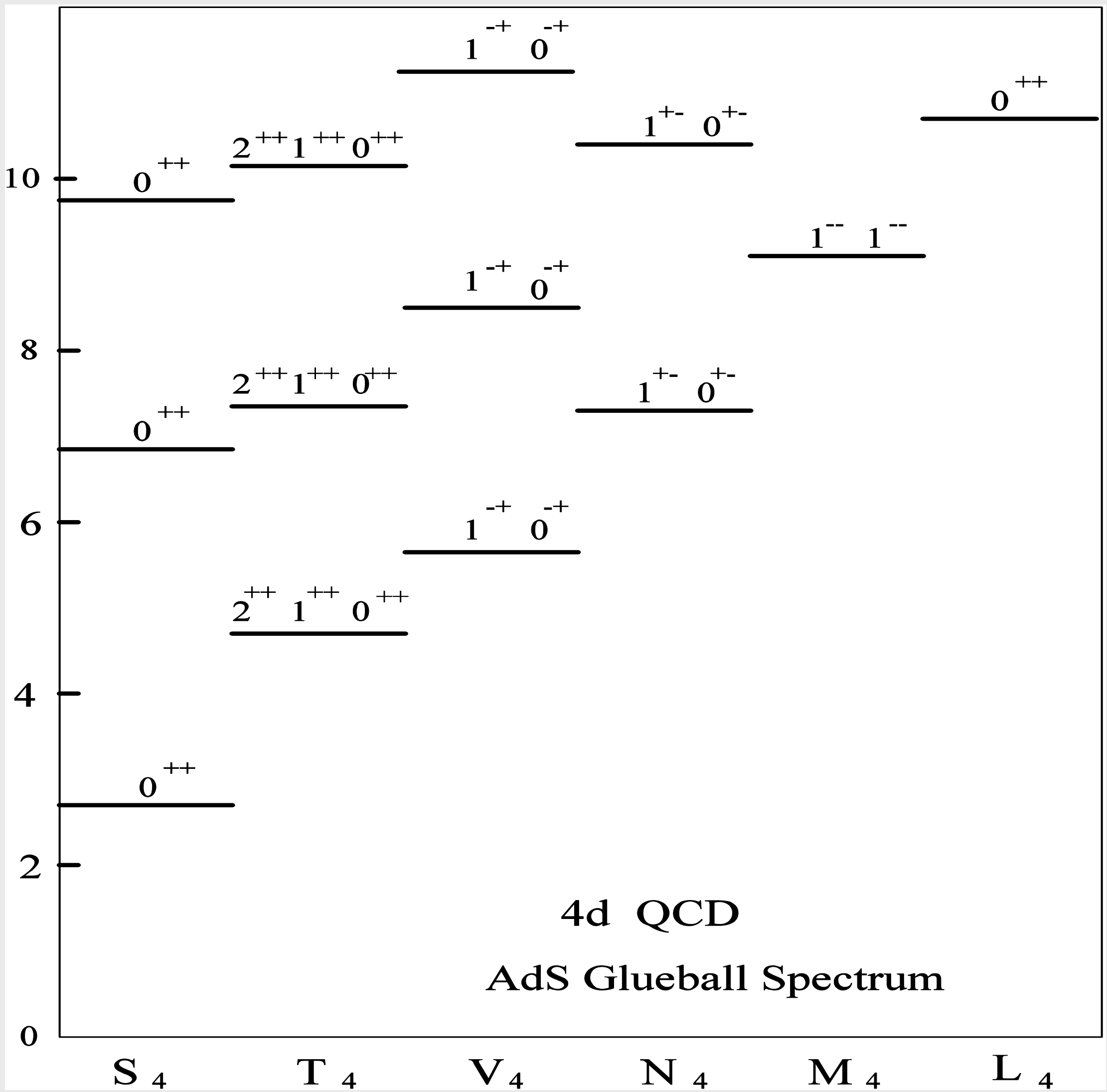
$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}S_4(r) - (m^2r^3 + \frac{432r^5}{(5r^6 - 2)^2})S_4(r) = 0$$

$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}N_4(r) - (m^2r^3 - 27r^5 + \frac{9}{r})N_4(r) = 0$$

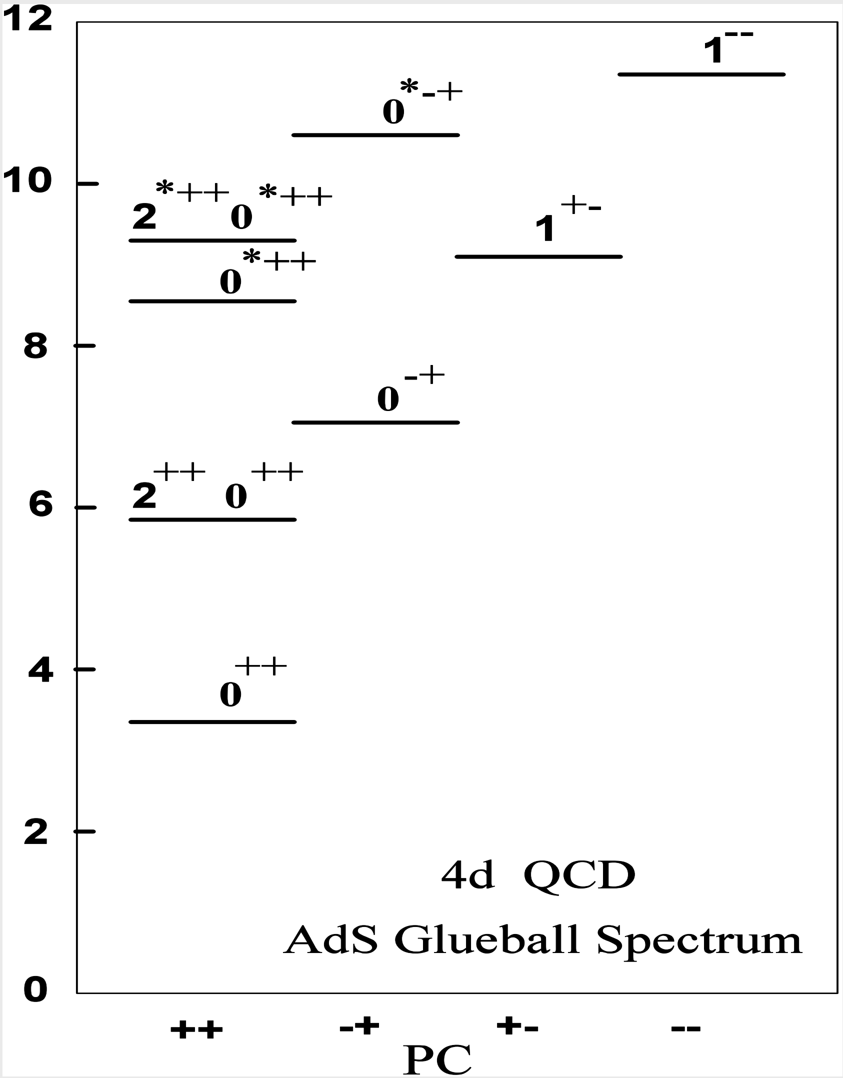
$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}M_4(r) - (m^2r^3 - 27r^5 - \frac{9r^5}{r^6 - 1})M_4(r) = 0$$

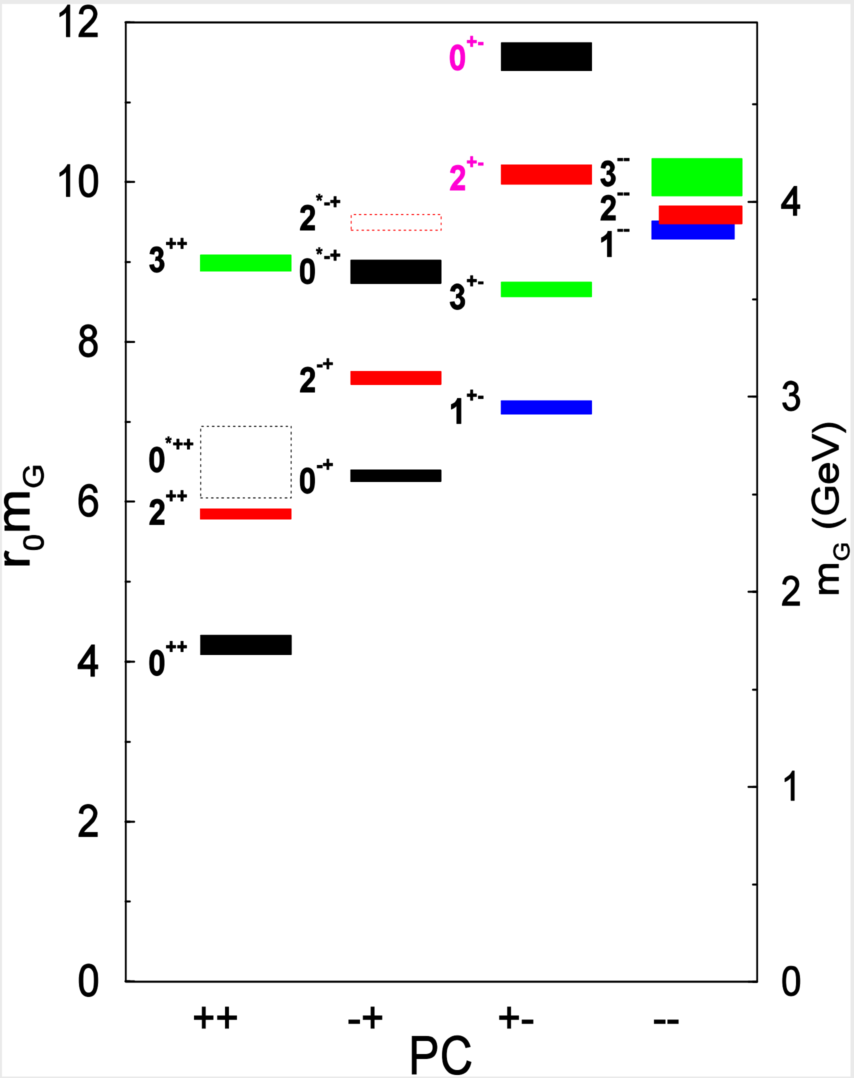
$$-\frac{d}{dr}(r^7 - r)\frac{d}{dr}L_4(r) - (m^2r^3 - 72r^5)L_4(r) = 0$$

AdS Glueball Spectra

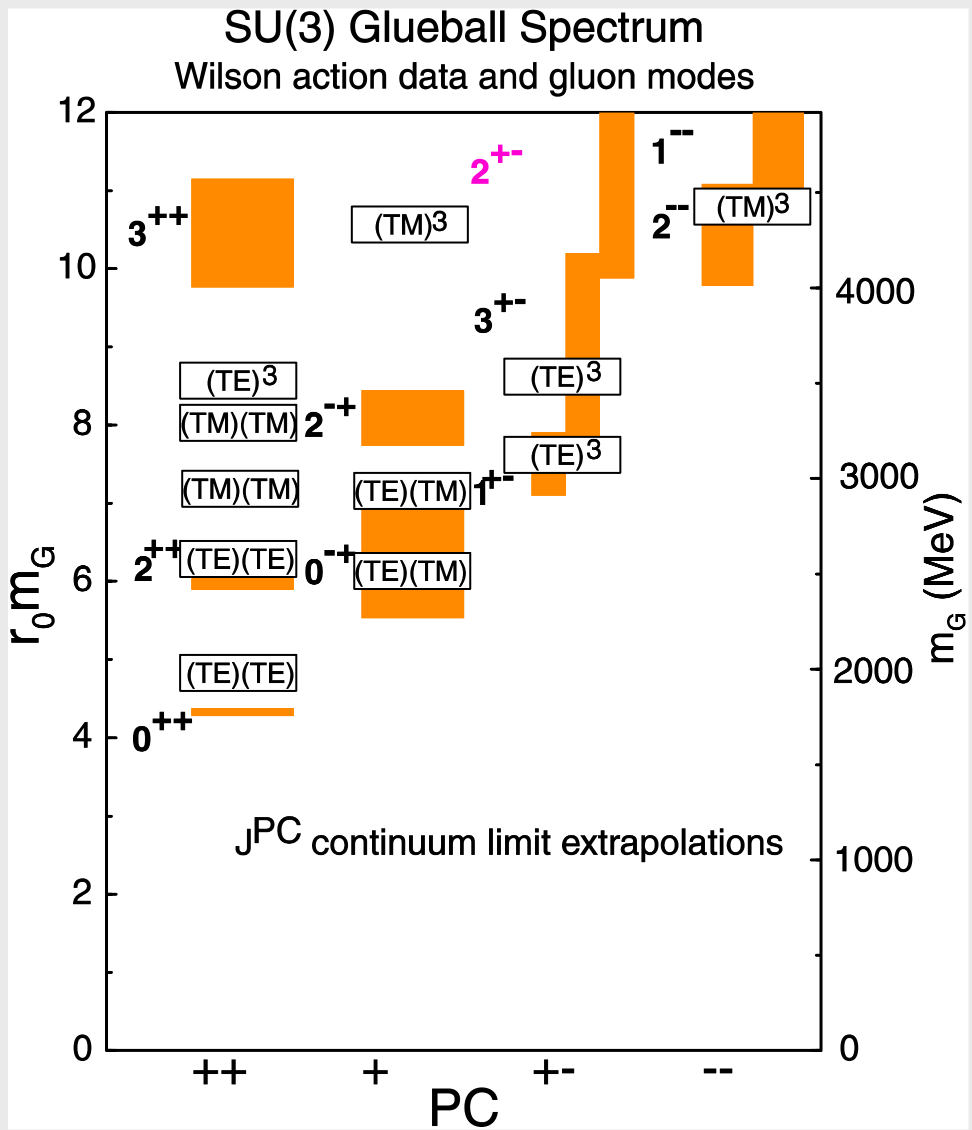


AdS Glueball Spectra vs Lattice Data





Comparison with MIT Bag Calculation



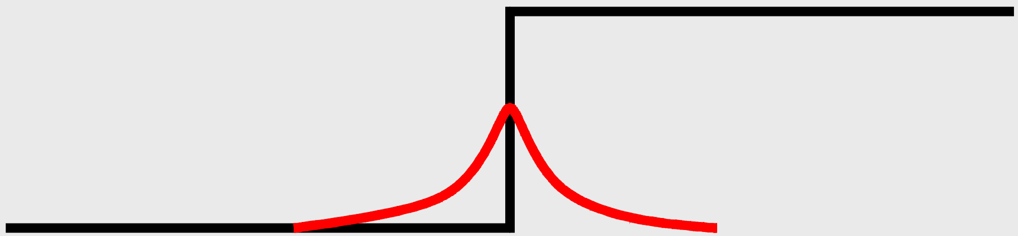
Bag Classification of States

Dimension	State	Operator	Supergravity
d=4	0^{++}	$Tr(FF) = E^a \cdot E^a - B^a \cdot B^a$	ϕ
d=4	2^{++}	$T_{ij} = E_i^a \cdot E_j^a + B_i^a \cdot B_j^a$ – trace	G_{ij}
d=4	0^{-+}	$Tr(F^*F) = E^a \cdot B^a$	C_τ
d=4	0^{++}	$2T_{00} = E^a \cdot E^a + B^a \cdot B^a$	$G_{\tau\tau}$
d=4	2^{-+}	$E_i^a \cdot B_j^a + B_i^a \cdot E_j^a$ – trace	absent
d=4	2^{++}	$E_i^a \cdot E_j^a - B_i^a \cdot B_j^a$ – trace	absent
d=6	$(1,2,3)^{\pm+}$	$Tr(F_{\mu\nu}\{F_{\rho\sigma},F_{\lambda\eta}\}] \sim d^{abc}F^a F^b F^c$	B_{ij} C_{ij}
d=6	$(1,2,3)^{\pm+}$	$Tr(F_{\mu\nu}[F_{\rho\sigma},F_{\lambda\eta}]] \sim f^{abc}F^a F^b F^c$	absent

These are all the local d=4 and d=6 operators:See Jaffe, Johnson, Ryzak (JJR), "Qualitative Features of the Glueball Spectrum", Ann. Phys. 168 334 (1986)

Glueball/Graviton Mixing

M(y) or V(y)



Just like domain wall fermions! $[\gamma_5 \partial_y - M(y)]\Psi(y) = ip_\mu \gamma_\mu \Psi(y)$

Randall-Sundram suggested a kink (aka Planck brane) in the Tensor potential

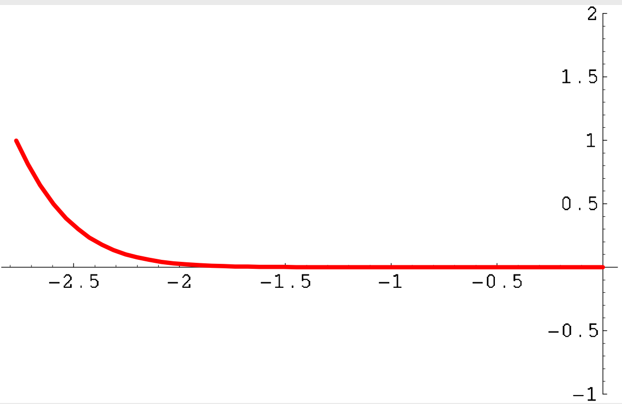
$$[\partial_y - V'(y)][\partial_y + V'(y)]h^{\perp}_{\mu\nu}(y) = p^2 h^{\perp}_{\mu\nu}(y)$$

Again there is a new normalizable zero mode: $h^{\perp}_{\mu\nu} \simeq \exp[-V'(y)]$

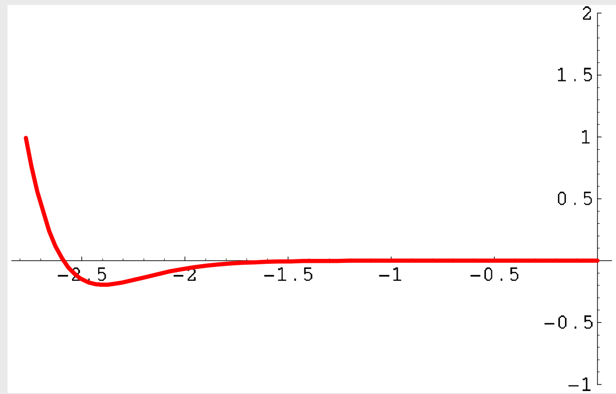
In the AdS black hole the potential is $2\,V(y) = (d+1)|y| + \log(1-1/r^{d+1})$ with the effective potential $W(y) = (V')^2 - V''$ having an attractive delta function deep in the UV.

Glueballs and Graviton are modes of a single closed string.

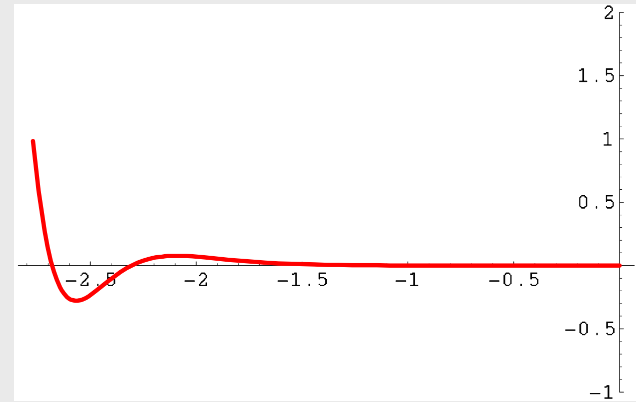
Tensor Glueball/Graviton Wave functions



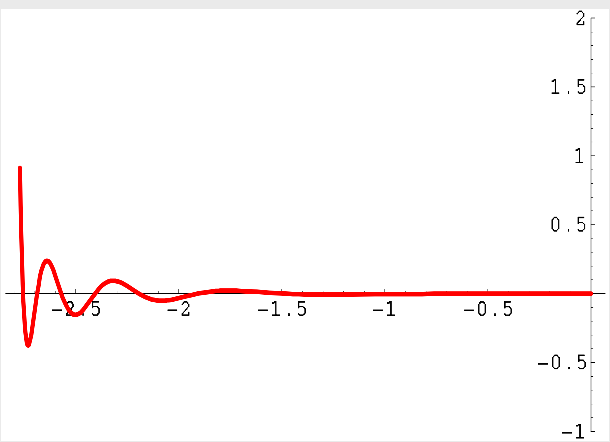
n=0



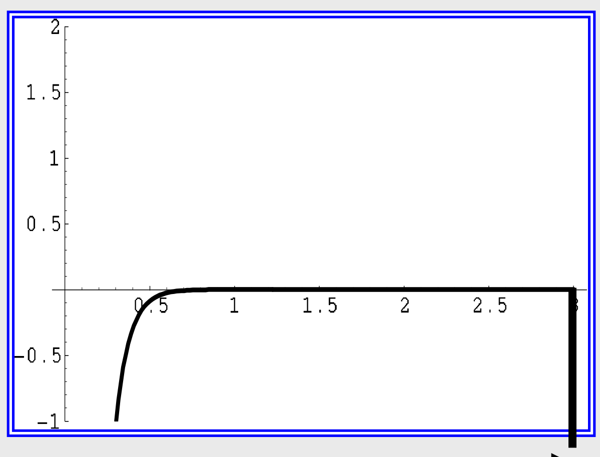
n=1



n=3

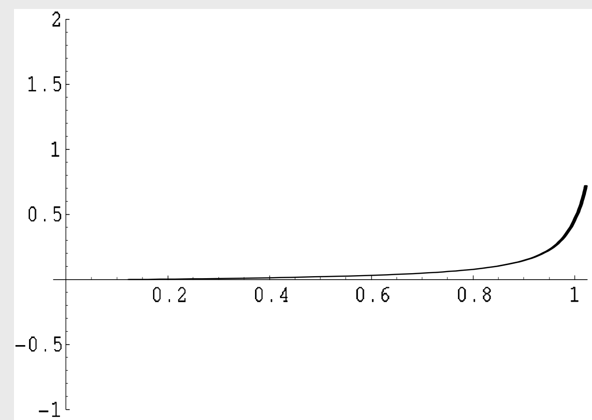


n=8



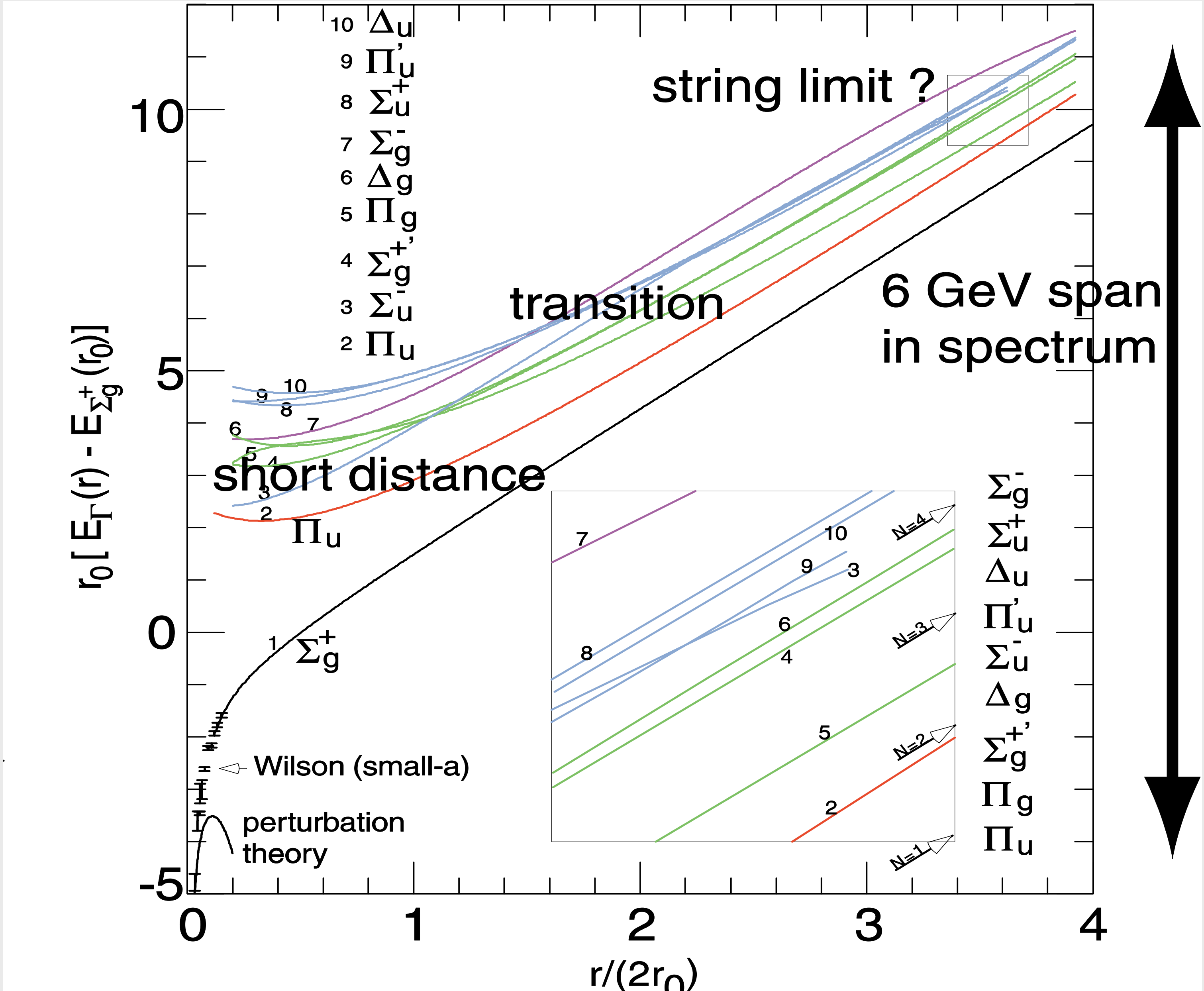
r

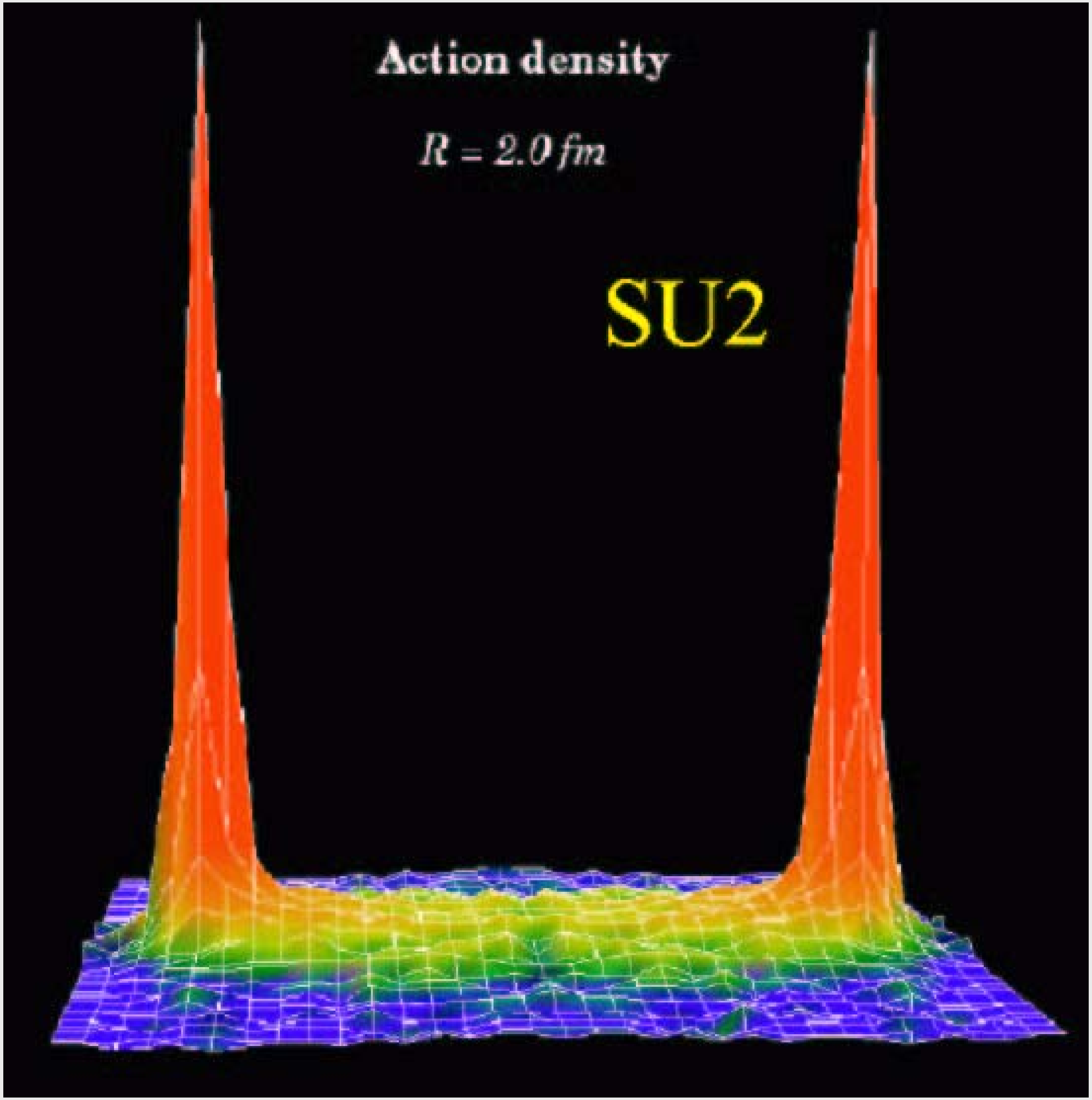
r_{min} Potential



Randall-Sundram graviton

QCD_4 Stretched String Spectrum




Action density
R = 2.0 fm
SU2

The Stretched String: Classification of States

- Nambu Gotto Action (flat space):

$$-T_0 \int d\tau d\sigma \sqrt{(\partial_\sigma X^\mu \partial_\sigma X^\mu)(\partial_\tau X^\nu \partial_\tau X^\nu) - (\partial_\sigma X^\mu \partial_\tau X^\nu)^2}$$

Exact Quantum sol'n:

$$E = T_0 L \sqrt{1 - \frac{\pi(D-2)}{12 T_0 L^2} + \frac{2\pi a_n^\dagger a_n}{T_0 L^2}} = T_0 L - \frac{\pi(D-2)}{12L} + \frac{2\pi a_n^\dagger a_n}{L} + \cdots$$

- Quantum numbers:
  - helicity = Λ
  - Charge conj.: Even/Odd = u/g
  - Transverse Parity = ±

Transverse String excitations

N	m	$\vert n_{m+}, n_{m-}\rangle$	Λ	States
1	1	$\vert 1_{1+}\rangle$, $\vert 1_{1-}\rangle$	1	Π_u
2	2	$\vert 1_{2+}\rangle$, $\vert 1_{2-}\rangle$	1	Π_g
	1	$\vert 2_{1+}\rangle$, $\vert 2_{1-}\rangle$	2	Δ_g
	1	$\vert 1_{1+}, 1_{1-}\rangle$	0	$\Sigma^{+'}_g$
3	1,2	$\vert 1_{1+}, 1_{2+}\rangle$, $\vert 1_{1-}, 1_{2-}\rangle$	2	Δ_u
	1,2		0	Σ^+_u
	1,2	$\vert 1_{1+}, 1_{2-}\rangle + \vert 1_{1-}, 1_{2+}\rangle$	0	Σ^-_u
	3	$\vert 1_{1+}, 1_{2-}\rangle - \vert 1_{1-}, 1_{2+}\rangle$	1	Π'_u
	1	$\vert 1_{3+}\rangle$, $\vert 1_{3-}\rangle$	1	Π'_u
	1	$\vert 1_{1+}, 2_{1-}\rangle$, $\vert 2_{1+}, 1_{1-}\rangle$	3	Φ_u
		$\vert 3_{1+}\rangle$, $\vert 3_{1-}\rangle$		
4	1,3	$\vert 1_{1+}, 1_{3-}\rangle - \vert 1_{1-}, 1_{3+}\rangle$	0	Σ^-_g

The Stretched String in AdS^{d+2} space

- Metric: $ds^2 = V(y)\, dx^\mu\, dx_\mu + dy^2 \quad + W(y)\, d\tau^2 \quad + \ldots$

$$V(y) = r^2/r^2_{min} = [cosh(\frac{d+1}{2}ky)]^{4/(d+1)}$$

- AdS Radius: $R_{ads} = 1/k$

Black Hole Temp ~ $k\, r_{min}$

$$\mathcal{L} = \sqrt{det[G_{\mu\nu}(X) X^\mu_{,i} X^\nu_{,j}]} =$$

$$\sqrt{(V X^\mu_{,\sigma} X^\nu_{,\tau} + y' y^\cdot)^2 - (V X^\mu_{,\sigma} X^\mu_{,\sigma} + y'^2)(V X^\nu_{,\tau} X^\nu_{,\tau} + (y^\cdot)^2)}$$

Classical solution: find $y_{classsical}(z)$ with

$X_1 = X_2 = 0, \quad X^3 = k\,\sigma = z, \quad X^4 = k\,\tau = t,$

$$z = \frac{r_c^2}{k}\int_{r_c}^{r}\frac{dr}{r^2}\frac{W(r/r_{min})}{\sqrt{r^4 - r_c^2}}$$

$$E_0 = \frac{2}{k}\int_{r_c}^{1/\epsilon}\frac{dr\; r^2 W(r/r_{min})}{\sqrt{r^4 - r_c^2}}$$

where $r^2 \sim V(y)$ and $W(r/r_{min}) = rkdy/dr = 1/\sqrt{1-(r_{min}/r)^{d+1}}$

Minimizing in Warped Space

Metric: $ds^2 = r^2\, dx^\mu\, dx_\mu + W(r)\, dr^2/r^2 = V(y)\, dx^\mu\, dx_\mu + dy^2$

Static solution in z-y plane: Action = T × E

$$\text{Energy} = \frac{1}{2\pi\alpha'}\int dz\mathcal{L}_{eff} = \frac{1}{2\pi\alpha'}\int dz\sqrt{V(y)\,\left(V(y)\acute{z}^2 + \acute{y}^2\right)}$$

Euler Lagrange Equations:

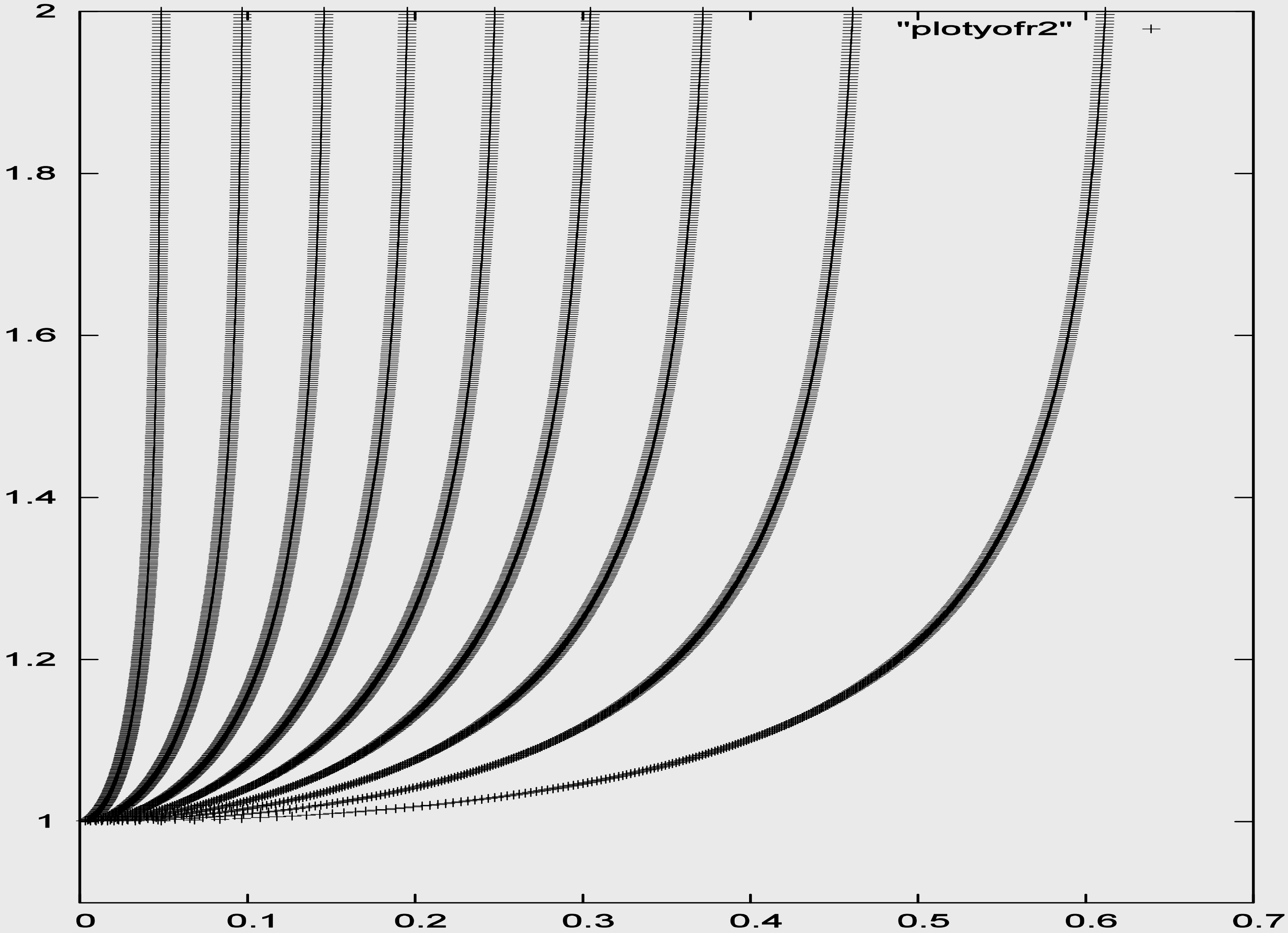
$$\partial_\sigma p_y = \frac{\partial\mathcal{L}}{\partial y} \qquad \partial_\sigma p_z = 0$$

$$p_y \equiv \frac{\partial\mathcal{L}}{\partial\acute{y}} = \frac{\acute{y}\sqrt{V}}{\sqrt{V\acute{z}^2 + \acute{y}^2}} \qquad p_z \equiv \frac{\partial\mathcal{L}}{\partial\acute{z}} = \frac{\acute{z}V^{3/2}}{\sqrt{V\acute{z}^2 + \acute{y}^2}}$$

Due to translation invariance in z

$$\frac{\acute{z}V^{3/2}}{\sqrt{V\acute{z}^2 + \acute{y}^2}} = V(y_c) = V_c\,,$$

Plot of Classical solution $r(z)/r_{min}$ for (r_c- r{min})/$r_c = 0.01^j$ with j = 1,..,9



Ground State Potential Energy

- Renormalize: *subtraction* *$E_0(L) - E_0(L=const)$*

$$E_0 = \frac{2}{2\pi\alpha' k}\left[\int_{r_c}^{\infty} dr\left[\frac{r^2 W(r/r_{min})}{\sqrt{r^4 - r_c^2}} - 1\right] - 1\right]$$

$$\Rightarrow \qquad E_0 = \frac{r_{min}}{2\pi\alpha' k}\, F(k r_{min} L)$$

Large L (confinement)

$$E_0 \to \frac{r_{min}^2}{2\pi\alpha'}\, L + O(L e^{-cL})$$

Small L (coulomb)

$$E_0 \to -\frac{8\pi^2\sqrt{2}}{2\pi\alpha'\Gamma(1/4)^4}\,\frac{1}{L}$$

Curve Fitting to Lattice data

- Fit is essential perfect

$$V(r) - V(r_0) = T_0 r - \frac{g_{eff}^2}{4\pi}\frac{1}{r}$$

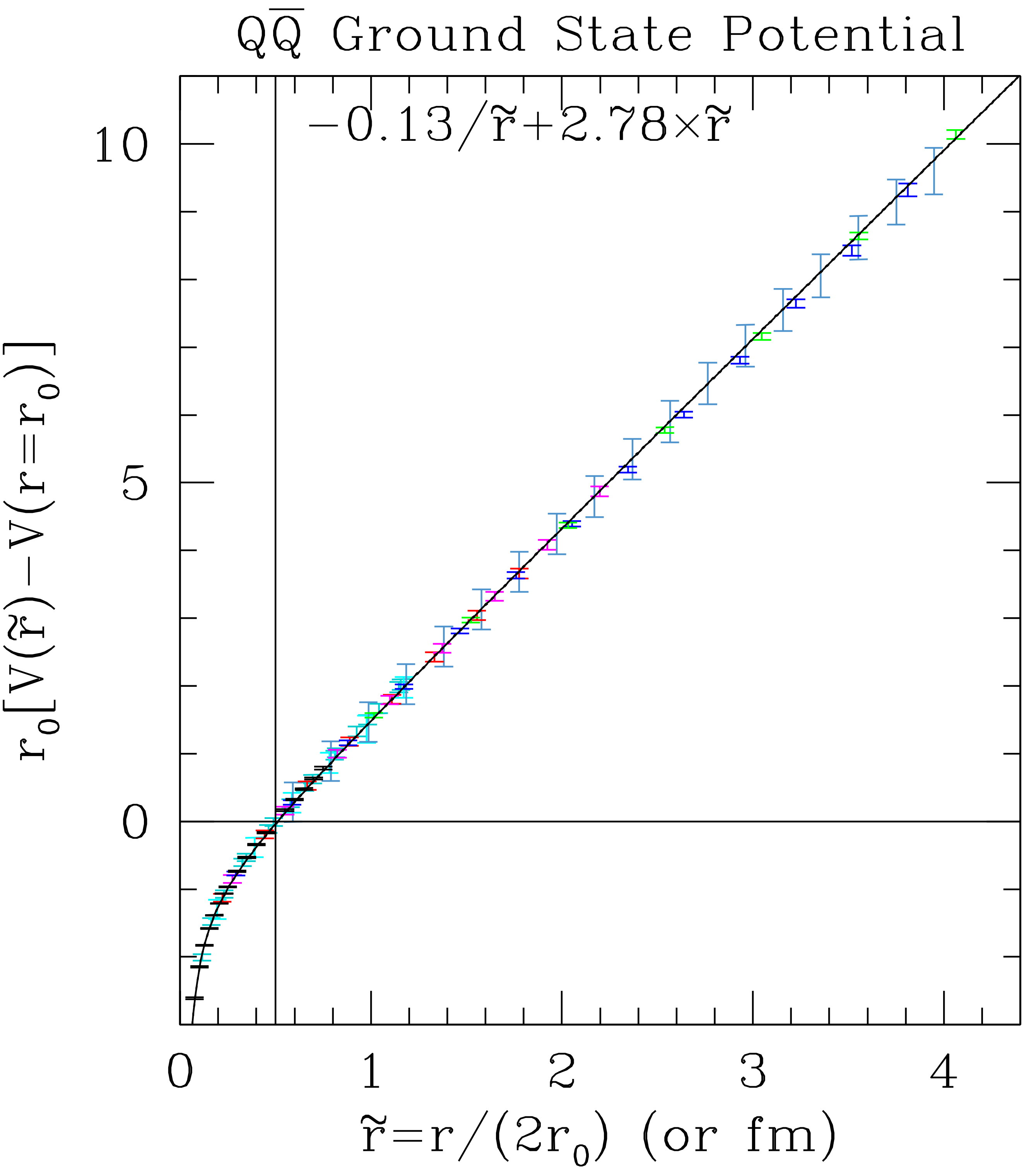
where $T_0 = 5.04$ /fermi^2 and $g^2_{eff}/4\pi = .26^\dagger$

Lattice Summer scale: $r_0 \simeq 0.5$ fermi.

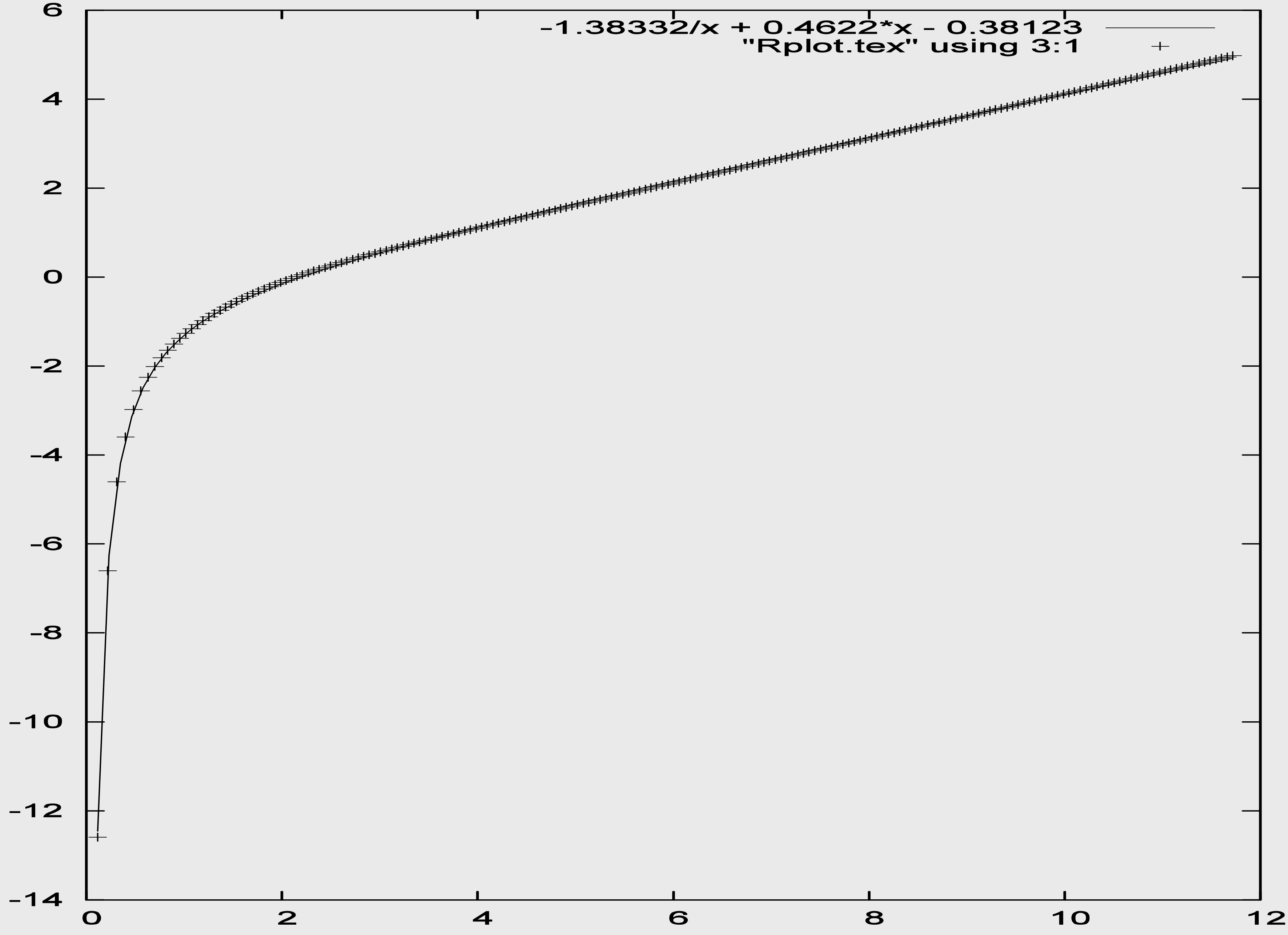
$r^2_0 \; dV(r_0)/dr = 1.65$

Adobe Acrobat Document

† Comment: In strong coupling AdS^5 both term are actually $\sim (g^2_{YM}N)^{1/2}$



QQ̄ Ground State Potential
−0.13/r̃+2.78×r̃
10
5
0
0
1
2
3
4
r̃=r/(2r₀) (or fm)
r₀[V(r̃)−V(r=r₀)]


-1.38332/x + 0.4622*x - 0.38123
"Rplot.tex" using 3:1
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
0
2
4
6
8
10
12

Excited states (Semi-classical limit)

$$E(L) \;=\; \int_{-L/2}^{L/2} dz\,\rho(z) + \frac{1}{2}\int_{L/2}^{L/2} dz[\rho_0(z)(\partial_t X_\perp)^2 + (X'_\perp)^2] + \cdots$$

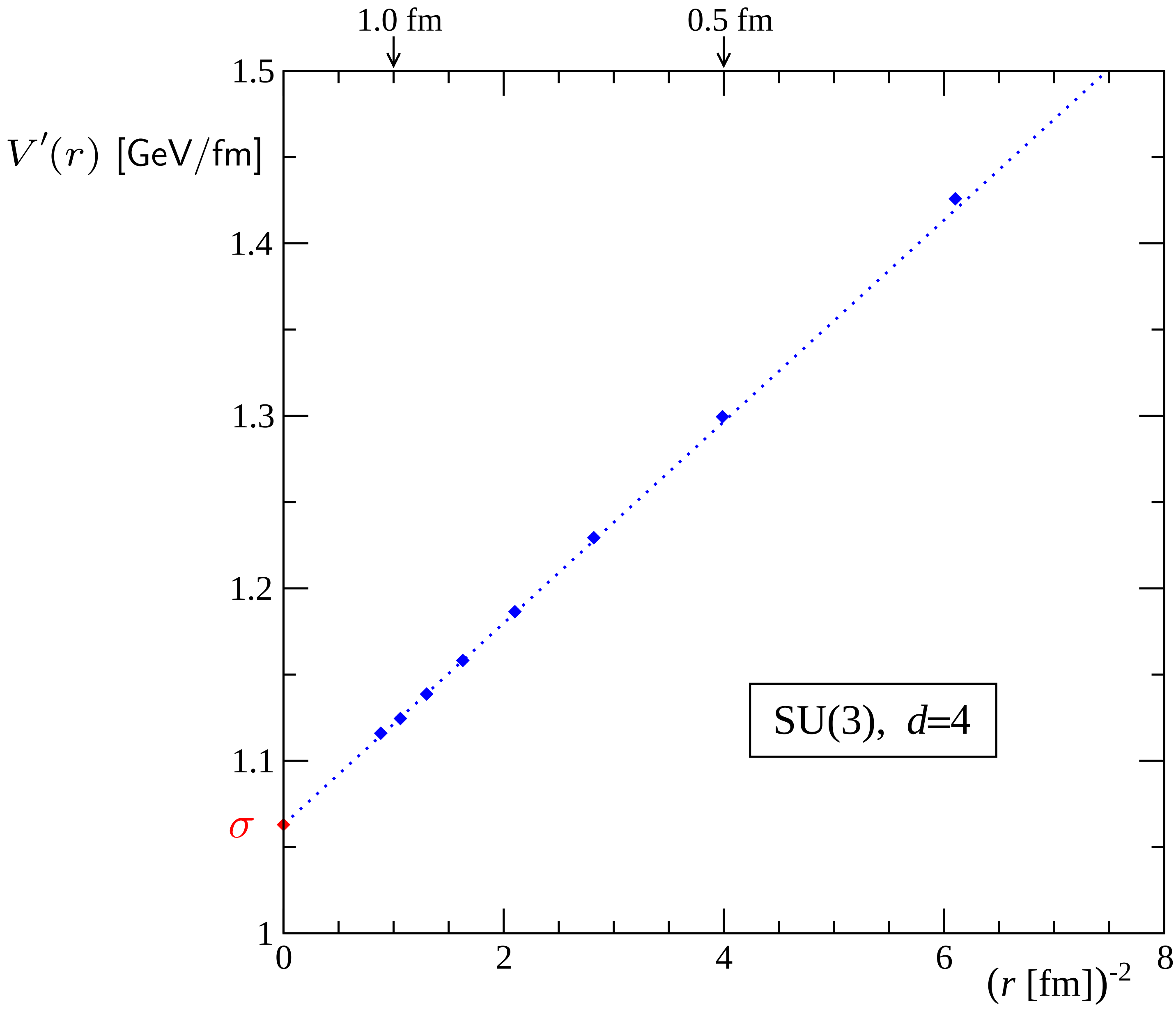
$$\rho_0(z)\partial_t^2 X_\perp - X''_\perp = 0$$

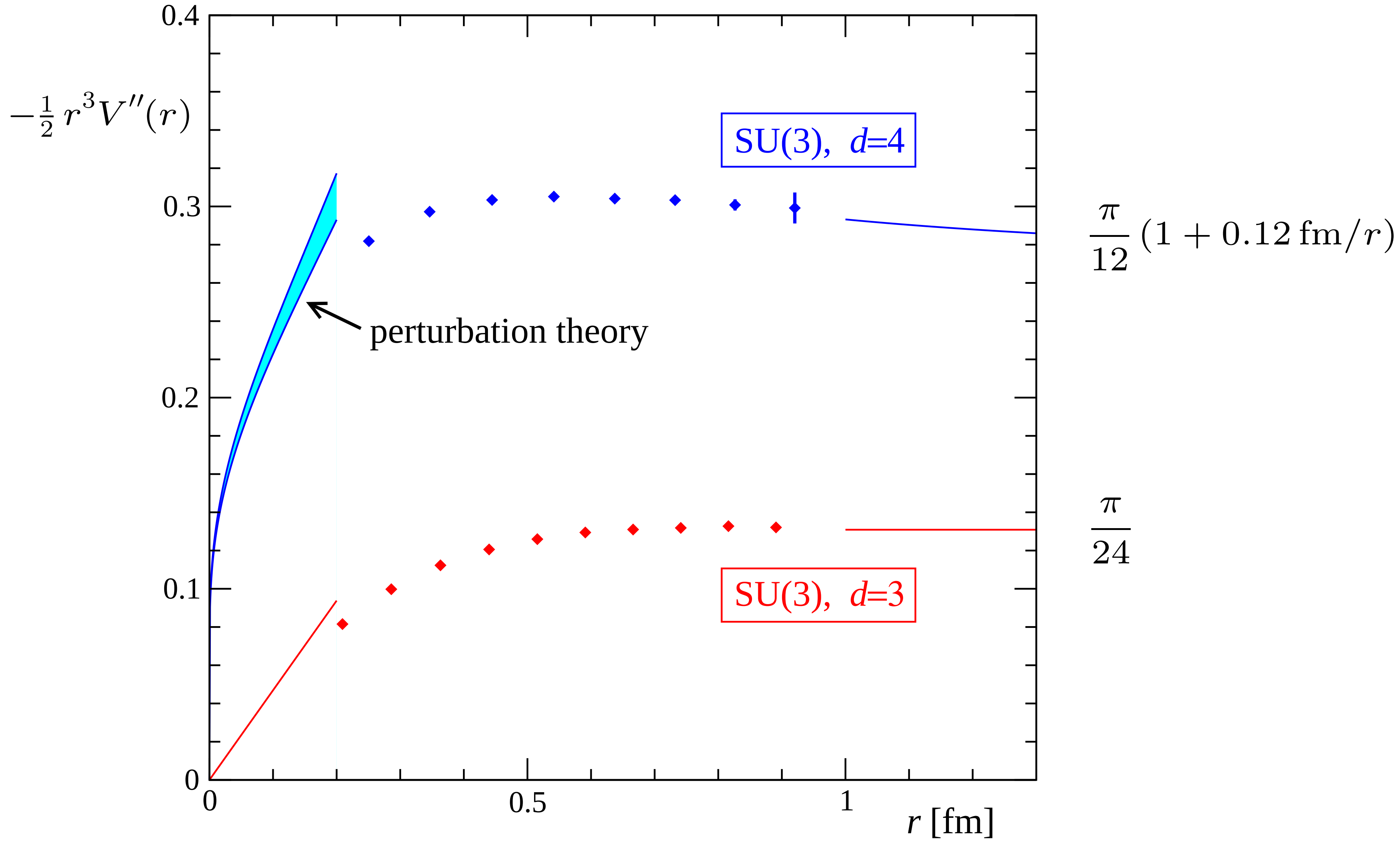
At large L:
$\rho_0(z) \to 1$, $\rho(z) \to 1$
except near end points:

$$\Delta E_n = \frac{n\pi}{L}(1 + \frac{b}{L}) + \cdots$$

b has wrong sign ? Quarks must have size!

$\rho_0(z) = V^2(z)/V^2(0)$


1.0 fm
0.5 fm
1.5
$V'(r)$ [GeV/fm]
1.4
1.3
1.2
1.1
σ
1
0
2
4
6
8
(r [fm])^-2
SU(3), d=4



0.4
0.3
0.2
0.1
0
0
0.5
1
$-\frac{1}{2}\, r^3 V''(r)$
SU(3), d=4
SU(3), d=3
perturbation theory
$\frac{\pi}{12}\,(1 + 0.12\,\mathrm{fm}/r)$
$\frac{\pi}{24}$
r [fm]

Radial (longitudinal) Mode

- Near $r = r_{min}$ (or $y = 0$), $V(y) \simeq r^2_{min} + \text{const}\, y^2$

THE SPECTRUM IS GAUGE INVARIANT

•Choose a gauge with fluctuations only in longitudinal (X^3) or radial (Y) or normal to classical surface, etc

$X^3 = z + \xi$ or $Y = y_{cl}(z) + \xi$, etc

$$-\rho_0(z)\partial_t^2\xi + \xi'' = M^2(z)\xi$$

where $M^2(z) = V''(z) - \frac{3}{2}\frac{V'(z)}{V(z)} \simeq const M^2_{BG}$ except at end points.

$$\Delta E = (d+1) M_{GB} + O(1/L)$$

The Future?

•Major progress to have non-zero set of exact string/gauge dual theories.

•AdS like spaces do have gives some of the qualitative features that elude the Old QCD String

•But the d.o.f. in the UV are far from clear

•Lattice spectral data can definitely help!

•Need to understand better the inverse problem.
(Given a YM theory what space do the string live in.)

•I believe the "string bit" approach working from the perturbative end is a promising approach.